5G Non-Orthogonal Multiple Access (NOMA) Systems

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Abstract

Non-Orthogonal Multiple Access (NOMA) has been recognized as a promising candidate for the fifth generation (5G) of communication systems. Different from the conventional OMA schemes, NOMA multiplexes multiple users on each subcarrier which achieves better performances on power efficiency and user fairness than conventional OMA. Based on the NOMA concepts, we propose 3 schemes of user pairing depending on different criteria. In this report, the performances of the OMA system and all the 3 proposed schemes are analysed by both simulation and analytical approaches. Particularly, the results of different pairing criteria, different pairing orders, and different power allocations are simulated to optimize the performance of NOMA. To verify the optimized performance of the new scheme, some analytical results are included as well.

*Index Terms – NOMA, OMA, user pairing, social privileges (weights).*
I. Introduction

A. Background

With the rapid development of Internet, a growing number of mobile users emerges in recent years. According to the statistic research [1], the number of mobile phone users has reached 4.61 billion in 2016, and it will exceed 5 billion in the next three years. The research graph is shown in Figure 1.

Additionally, as the rise of the Internet of Things (IoT), the Internet will not only be used on computers or phones, but also for other purposes such as self-driving cars, smart city sensor systems, savvy home appliances, or robots [2]. Therefore, there will be an amount of about 1000-fold data traffic increased by 2020 [3]. Furthermore, with the increasing amount internet traffic, power and resource saving will be capturing more and more attentions over the next few years. Hence, meeting the huge demand of data traffic and solving the power efficiency problem of current 4G systems will become key challenges for the evolution of cellular mobile communication. This report will be focusing on the fifth generation (5G) systems which is expected to be launched in 2020 with a requirement of less than 2ms latency [4] [5].
B. Literature and Motivation

Towards the generation history of the cellular mobile communication technologies, different wireless communication schemes have been deployed from the first generation (1G) systems to the fourth generation (4G) systems. According to [3] and [6], these schemes are displayed in Table 1:

<table>
<thead>
<tr>
<th>Generation</th>
<th>Access Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1G</td>
<td>Frequency-Division Multiple Access (FDMA)</td>
</tr>
<tr>
<td>2G</td>
<td>Time-Division Multiple Access (TDMA) &amp; Code-Division Multiple Access (CDMA) [7]</td>
</tr>
<tr>
<td>3G</td>
<td>Code-Division Multiple Access (CDMA)</td>
</tr>
<tr>
<td>4G</td>
<td>Orthogonal Frequency-Division Multiple Access (OFDMA) [8]</td>
</tr>
</tbody>
</table>

Table 1. Generations of cellular mobile communication technologies.

As shown in Table 1, the currently adopted multiple access scheme for 4G communication systems is the orthogonal frequency-division multiple access (OFDMA). According to [8] - [17], in OFDMA systems, a wideband signal has been divided into many narrowband subcarriers, and each user is assigned with an orthogonal subcarrier separated in frequency, and a base station can communicate with each user on the subcarriers associated to the users.

Although 4G systems provide a higher achievable data rate than the previous generations, the high latency sometimes cannot satisfy the requirement of IoT. As the

Figure 2. The latencies of 4G LTE systems
statistical researches from “FierceWireless” website show that the 4G LTE systems have the latencies about 75 milliseconds in 2015 [18]. As [19] mentioned, the IoT devices will not only be remotely controlled and managed by people, but can also communicate with each other. Therefore, some multimedia applications of IoT requires lower transmission latencies that at least less than the human visual delay constraint 10 milliseconds. 5G will provide a latency of 2 milliseconds, which can meet this requirement. As [20] - [22] mentioned that 5G will play an important role in growing the Internet of things.

Apart from the problem on latencies, OFDMA system also has problems on the user fairness. The conventional power allocation strategies such as water filling strategy cause distinct data rates for the users depending on their different channel conditions [23], [24]. As mentioned in [10] and [12], users with the best channel conditions are always selected to maximize the system throughput, and which makes the channel quality of users with poor channel conditions critical. Moreover, with the increasing amount of Internet traffic, 4G systems cannot meet the requirement of power efficiency, it costs too much power and wastes many resources as the orthogonal multiple access requires the individual user to use a single subcarrier. Consider these problems of 4G systems, it will be challenging to improve the cellular mobile communication on the upcoming 5G systems [25], [26]. The targeted requirements for the new generation are to develop a new wireless communication system to accommodate a large amount of Internet traffic and to achieve power efficiency and user fairness. A new appropriate scheme will be proposed to satisfy these requirements.
In this report, non-orthogonal multiple access (NOMA) will be presented as a promising candidate that might be used in the future 5G systems. There are some other potential candidates that have been proposed [3], such as Energy harvesting [27] - [30], massive MIMO [31] - [34], millimeter wave communications [35] - [37] and ultra-dense network [38], [39].

The NOMA scheme is different from the conventional orthogonal multiple access (OMA). The basic concepts and the applications of NOMA are studied in [40] - [48]. According to [23], [24], [49] and [50], NOMA users are divided into some groups, each group is assigned with an orthogonal subcarrier. The users in each group are allocated with different power depending on their channel conditions in order to reach the perfect balance between system throughput and user fairness. Multiple users in one subcarrier are served by one base station in the same time slot, at the same frequency and code channel. The users with different channel conditions have different ways of decoding their messages. For the users with better channel conditions, they will decode the poorer channel users’ messages first, and then employ the successive interference cancellation (SIC) to get their own messages by removing others’ information. On the other hand, users with poorer channel condition are unable to employ SIC, and they can treat other users’ messages as noise to get their own.

The throughput for NOMA scheme can be considered as the achievable data rate for each user in one subcarrier. The data rates for NOMA users are calculated in different ways. Take the user pairing as an example, for two downlink users in the single subcarrier, depending on the channel conditions, the user M is pairing with user N,
where N has a better channel condition than M. Then M can treat user N’s messages as noise. According to [51], the data rate for user M can be represented as following:

\[ R_m = \log_2(1 + \frac{|h_m|^2 a_m^2}{|h_m|^2 a_n^2 + \rho}) \]  

(1)

In the above equation, \( h_m \) and \( h_n \) are the channel gain for user M and user N, where \( |h_m|^2 \leq |h_n|^2 \). Also \( a_m \) and \( a_n \) are the power allocation coefficients for the two users. Normally, \( a_m \geq a_n \) since the user with poorer channel condition is usually allocated with more power for user fairness. Constant \( \rho \) denotes the transmit signal to noise power ratio (SNR).

The other user N will decode M’s information and remove them by SIC. The data rate is calculated differently:

\[ R_n = \log_2(1 + \rho a_n^2 |h_n|^2) \]  

(2)

The performance of NOMA system can be indicated by the outage probability [52]; the outage probability is defined as the probability that user’s achieved data rate has not reached the targeted data rate required from the users’ Quality of Service (QoS). Statistically, shown as \( P (R_u < R_t) \), which is a probability formula where \( P \) denotes the probability, \( R_u \) is the achievable user data rate, and \( R_t \) is the targeted data rate.

There are some benefits for NOMA system compared with conventional OMA system, they are displayed as following [53]:

- High achievable data rate
- Large capacity
- Energy efficiency
There are some existing designs employing NOMA system. One of the schemes in [51] it stated is the impact of user pairing with fixed power allocation (F-NOMA). In this situation, the total transmit power is a fixed number. The scheme pairs the users with the best channel condition with the users with the worst channel condition. The results from the paper show that this scheme has a higher throughput compared with the conventional OMA. Also, this difference will become larger when the gap between the channel gains of the paired users is increasing.

Another scheme from [54] represents a randomly deployed method for NOMA users with targeted data rate. The simulations and analytical results show that the randomly deployed users can achieve a lower outage probability than the conventional OMA under well-chosen values of targeted data rate and allocated power.

The authors in [55] studied the power-efficient resource allocation for MC-NOMA systems with QoS constraints. A SIC policy taking consideration of QoS requirements is proposed in their report to optimize the performance of MC-NOMA systems. As QoS is usually defined by a targeted rate and a required outage probability, so we will consider the targeted data rate as a criterion to analyse the performances for NOMA systems and OMA systems.
II. Detailed Design

A. Problem Statement

As we known in the NOMA systems, multiple users are assigned into one orthogonal subcarrier. However, which users should be chosen in the subcarrier, and how much power should be allocated to each user need to be designed. A new design is required, and the performance of this design should also be considered. Hence, the problems can be stated as following:

- How to propose an insightful design scheme by employing NOMA?
- How is the performance of the new design?

B. Concept Generation

There are some hypothetical concepts of solutions that shown in Table 2:

<table>
<thead>
<tr>
<th>Problem statement</th>
<th>Concept generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>How to propose insightful design schemes by employing NOMA?</td>
<td>Randomly paired users with targeted data rates.</td>
</tr>
<tr>
<td></td>
<td>User pairing with targeted data rates depending on users’ channel conditions.</td>
</tr>
<tr>
<td></td>
<td>User pairing with targeted data rates depending on users’ channel conditions and privileges (weights).</td>
</tr>
<tr>
<td>How are the performances of the new designs?</td>
<td>Analysing the performance of the design by working out the analytical results and verifying via simulation.</td>
</tr>
</tbody>
</table>

Table 2. Stated problems and generated concepts.
The three concepts for designing the new scheme are all using user pairing. The differences for them are the pairing criteria. For the first one, two random users will be paired regardless of their channel conditions (priorities). The second concept is to pair the users based on their channel gains, and the third concept for the design is based on the users’ channel conditions as well as their social privileges. There are some reasons for choosing user pairing. If there are many users in one subcarrier, the user with best channel condition will be decoding more users’ messages and get its own messages after that. It would have a large delay to get its own messages, and also cause the computational complexity. User pairing can minimize the delay and the complexity.

Recall from the literature review, the two existing schemes using NOMA which are user pairing depending on the channel conditions and randomly deployed users are considered as two of the concepts for the solution. Although these existing designs have good performance, and can save more power, they still have some shortages. For randomly paired users in [54], the channel conditions are not considered at the pairing stage, but only for the data rate calculation. If two users with poor channel conditions are randomly paired together, the actual sum rate would be critical which may not meet the required data rate. For user pairing depending on channel conditions, the existing design in [51] is analysing the performance gain between NOMA and conventional OMA, but how is the performance of the NOMA systems for meeting the QoS requirements. Therefore, in our proposed design, a target data rate is considered as a threshold to determine whether the system has met the requirement or not. This is more useful in practice where all users have their requirements on the data rates. Apart from that,
although the scheme of pairing users depending on their channel conditions has a very high sum rate and each user is fairly treated, but it did not consider about the social weights for these users when pairing them together. For example, there are two users with distinct channel conditions, and they are expected to be paired in one subcarrier. If the user with poorer channel condition is a premium user, the other user with better channel condition is a regular user. In this scheme, the regular user’s priority for user pairing would be higher depending on the channel conditions; the regular user will employ SIC and decode the messages for the premium user. However, in the reality, due to some safety reasons or QoS requirements, the premium user possibly requires a high data rate and does not want others to decode their messages. In this case, another variable social weight is concerned as one of the criteria for pairing in this example. If the premium user has lower channel gain, but with social privilege, the pairing priority would be higher than the regular user. The premium user would choose another user who has poorer channel condition than itself.

Considering the shortages of the existing design, a new design of user pairing with targeted data rates depending on the channel conditions and privileges (weights) is proposed. To compare the performances of all the 3 designs, some simulations will be provided latter in the report and the analytical result of the scheme with the best performance will be solved to verify the simulation results.
C. System model

Based on the proposed schemes, a system model is shown in Figure 3. to elaborate how the NOMA users receiving and decoding messages.

Assume that user m and user n are assigned in one subcarrier. When receiving messages at the same time, both of users should receive a combined signal that have both users’ messages inside. User n is closer to the base station, and its channel condition is better than user m, so it employs successive interference cancellation (SIC) to cancel the user m’s signal and decode its own messages. User m is farther from the base station, and it is allocated with more power than user n. With a poor channel condition, user m is unable to employ SIC, so it decodes its own messages by treating the signal of user n as noise.

Figure 3. The system model of user pairing for NOMA system
III. Simulation Approach

A. Simulation Design

As an improved system for 4G, 5G should have a better performance. The simulation results will demonstrate the performance comparison between NOMA and conventional OMA. We choose one of the proposed schemes for both NOMA and OMA. For OMA, we use TDMA scheme that users share the subcarrier, but they can only communicate at different time periods. But for NOMA, users can receive messages at the same time with different decoding methods.

Recall from the proposed schemes:

- Random user pairing with targeted data rates.
- User pairing with targeted data rates depending on channel conditions.
- User pairing with targeted data rates depending on channel conditions and privileges.

The 3 designed schemes are based on target data rates, so the comparison of their performances can be analysed by keeping target data rate as a constant value. However, the performances with different pairing orders and different allocated power to users are distinct.

For different pairing orders, depending on the pairing criteria, users with high priority are supposed to pair with users with low priority in the case when the pairing criterion is only the channel condition according to the literature review. So some of the methods for pairing orders are analysed for comparison with the traditional way.
(pairing the highest priority user with the lowest priority user). Particularly, the other pairing orders can be stated as following:

- Highest priority user with 2nd highest priority user
- Lowest priority user with 2nd lowest priority user
- 2nd highest priority user with 2nd lowest priority user
- The two users in the middle of priority order

For different power allocation, similar to the pairing orders. According to [23], the user with poorer channel condition is supposed to be allocated with more power. In this design, some more power allocation methods are developed for the performance comparison. These methods are shown as following:

- Equal power allocation for both users.
- Allocating 1/3 power to the user with high priority and 2/3 power to the user with low priority.
- Allocating 1/4 power to the user with high priority and 3/4 power to the user with low priority.
- Allocating 1/5 power to the user with high priority and 4/5 power to the user with low priority.
B. Parameters and Variables Setting

Before simulating the design schemes, some parameters and variables need to be set. Assume there are 10 NOMA users served by one downlink base station with the bandwidth of 200 kHz and carrier frequency of 2 GHz. Set the targeted data rate to be 20 bps per channel use. For simulation, only one subcarrier is considered, 2 of the ten users are paired together into that subcarrier. In simulation, locations of the users are assumed to be uniformly distributed within a disc where the base station is located at the centre of the disc [54]. The range of the distance between the users and base station is from 20 meters to 100 meters. The maximum transmitting power domain is from 10 dBm to 30 dBm. The performances can be analysed by the graphs from the simulations. The x-axis for the graph is the maximum transmitting power, and the y-axis is the outage probability. According to the literature survey, outage probability can be calculated from the probabilities that the simulated user sum rates have not reached the targeted data rate 20 bps/channel. The performance of one scheme is better if the outage probability of that scheme is lower.

There are 4 cases of simulations for this design, to analyse its performance.

Case 1: The simulation for comparison between conventional OMA and the new design.

Case 2: The simulation for user pairing on different pairing criteria (based on the channel gain (H), weights (W), and channel gain over weights (\(H/W\))).

Case 3: The simulation for user pairing on different pairing orders.

Case 4: The simulation for user pairing on different power allocations to the users.
C. Case 1

The performance comparison for different communication systems is shown in Figure 4.

![Simulation graph for different systems (OMA & NOMA).](image)

In this simulation, the path loss realization and multipath realizations are set to be 100 and 10000 correspondingly. For NOMA, the priority for user pairing is channel gain over weight, and the paired users have the largest priority and lowest priority in equal power allocation. For OMA, only one user is chosen in single subcarrier, and the outage probability calculation is presented in [56], shown as equation (3):

\[ P = \left( \frac{1}{2} \log(1 + SNR) < R_t \right), \]  \hspace{1cm} (3)

where \( R_t \) is the targeted data rate 20 bps/channel, because for OMA, the number of users in one subcarrier is half compared with NOMA. The \( \frac{1}{2} \) pre-log factor in equation (3) is due to orthogonal access in either time or frequency in OMA schemes. Therefore, half of the system resources is utilized for serving the selected users. In Figure 3, the two curves shows the performance for conventional OMA and NOMA. On the same maximum transmit power of 20 dBm, NOMA has a lower outage probability. Compared
with OMA, NOMA system saves about 4 dB power. In another word, OMA used about 2.5 times of power more than NOMA. In addition, OMA system assigns individual user into single subcarrier, but NOMA system assigns two users into one subcarrier, which means the capacity of NOMA system is twice larger than OMA system with the same number of subcarriers and the same required data rate.

D. Case 2

In the considered scenario, users’ locations are assumed to be randomly distributed [54]. To simulate this scenario, H and W are both set to be random variables. When the pairing priority is weight, this pairing condition can be seen as randomly selecting two users as the weights are random numbers, and irrelevant to the data rate calculation. When the priority is $\frac{H}{W}$, W is considered as the social weights; the user selections for pairing is based on the two random numbers. The simulation shows the comparison for user pairing with different criteria (priorities). Their performance can be displayed

![Figure 5. Simulation graph of user pairing based on different criteria.](image-url)
graphically in Figure 5.

For this case of simulation, the path loss realization and multipath realizations are set to be 100 and 50000 correspondingly. The selected users are two users with the largest priority and lowest priority. Each user is allocated with equal power.

The curves in Figure 2 show that user pairing based on channel conditions has the highest sum rate, and the random pairing has the lowest sum rate among the three curves. It is shown by the outage probability on the same power domain 18 dBm. The lowest outage probability means the best performance.

On the same outage probability level, pairing based on channel condition and weights costs a bit more power than the one based on channel condition only. However, as the design section stated, the weights are suggested to be considered as another pairing criterion in practice even though it may cost more power than the existing one.

E. Case 3

The performances can be different if selecting different users based on the values of their priority. Two schemes on the pairing orders have been proposed in Table 3.

<table>
<thead>
<tr>
<th>Schemes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed scheme 1</td>
<td>Largest priority user pairing with lowest priority user</td>
</tr>
<tr>
<td>Proposed scheme 2</td>
<td>Largest priority user pairing with 2(^{nd}) largest priority user</td>
</tr>
<tr>
<td>Baseline 1</td>
<td>Pairing two users in the middle of priority order</td>
</tr>
<tr>
<td>Baseline 2</td>
<td>Lowest priority user pairing with 2(^{nd}) lowest priority user</td>
</tr>
</tbody>
</table>

Table 3. Different schemes in Figure 4.
According to the “Design” section, 5 types of pairing orders will be included in the simulation. Figure 6 shows the simulation results. The path loss realization and multipath realizations are set to be 100 and 10000 correspondingly, and the pairing priority is channel gain over weight. The performance of proposed scheme 1 for user pairing is the best, and the curve coincides with the proposed scheme 2 for the pairing for largest priority user and 2nd largest priority user. Actually, if the largest priority user is chosen, the outage probability will not change too much for which user is selected to pair with it. In other word, the total sum rate mainly depends on the users with good channel conditions (the higher channel gain can achieve higher sum rate), but changes slightly when the other user is selected differently.

In the simulation, both of these pairing orders can be applied because they have the similar performances, and only two users are considered each time. While in the reality, these two orders are different because any of the other 8 users should not be ignored.

Figure 6. Simulation graph for user pairing with different pairing orders.
Consider the same social weight for each user, if the method of pairing two users with best channel conditions is selected, after pairing the best two users among 10, the system will keep pairing the rest 8 users in the same way. The last two users will be the lowest priority user with the 2nd lowest priority user, and the baseline 2 shows the performance which is the worst one. On the other hand, if using the method of pairing the user with the best channel condition with the user with the worst channel condition, the worst performance would happen on the pairing for the middle two users, whose performance is shown in baseline 1. Obviously, baseline 1 has about $10^4$ times lower outage probability than baseline 2 at a maximum transmit power 30 dBm. Therefore, the performance of the second method is better than the first one above. Additionally, it will be unfair if the sum rates for two subcarriers are very distinct.

As a result, the method of user pairing with highest priority and lowest priority users is the best considering the total sum rate as well as the user fairness. These two methods are also compared in [51], which states that the performance will be better if the channel conditions for user pairing are more distinctive.
F. Case 4

In this case, the simulation for user pairing with different power allocations is shown in Figure 7.

The path loss realization and multipath realizations are set to be 100 and 10000 correspondingly, and the pairing criterion is channel gain. In Figure 7, the performances for different power allocations are very similar, which means the total sum rates will be the same no matter what power allocation is used in this case. When the total allocated power is a constant value, the total sum rate will not change much if the power allocated to the user with better channel condition is less than or equal to the user with poorer channel condition. However, the users in the same subcarrier will have different data rates if the power allocations are different. Different power allocations are applied depending on the users’ requirements.
IV. Analytical Approach

A. Analytical Design

According to the simulation results, the scheme with the best performance is selected to be used in the analytical approach. As the proposed scheme 1 in table 3 achieves the lowest outage probability, it can be considered as the optimized scheme. The case 4 from the simulation results shows that the power allocation is independent of the sum rate. But to achieve user fairness, we decide to allocate 1/5 of the total transmit power to the user with the best channel condition, and 4/5 of the total transmit power to the user with the worst channel condition. Recall from the literature review, the data rate calculations for each user in user pairing system:

\[ R_m = \log_2(1 + \frac{|h_m|^2a_m^2}{|h_m|^2a_n^2 + \rho}) \]  \hspace{1cm} (4)

\[ R_n = \log_2(1 + p|a_n|^2|h_n|^2) \]  \hspace{1cm} (5)

Applying equations (4) and (5), the initial equation for the outage probability of the 2 users sum rate is show in the following equation:

\[ P(R_m + R_n < R_t) = P(\log_2 \left( \frac{1+\rho|h_m|^2}{p|h_m|^2a_n^2 + \rho} \right) + \log_2(1 + p|a_n|^2|h_n|^2)) < R_t). \]  \hspace{1cm} (6)

In the equation (6), we set \( h_n \) and \( h_m \) as random variables with a mean value of 1 which denoting as the Rayleigh fading channel gains for user \( m \) and user \( n \); \( a_n^2 \) denotes the allocated power to the better channel user, which is set to be 1/5. \( R_t \) denotes the target data rate 4 bps/Hz. So the equation (6) represents the outage probability of user pairing with different transmit signal to noise power ratio (SNR). In the analytical results, all the users are considered to be independent and identically distributed (i.i.d.). Instead of the
uniformly distributed within the disc, their distances to the base station are set to be same. As we set $h_n$ and $h_m$ as the rayleigh distributed random variables, then $h_n^2$ and $h_m^2$ should be the exponential distributed random variables. Assume $x$ and $y$ to be the expression of random variables where $x = \rho h_m^2$ and $y = \rho h_n^2$. And their unordered density functions can be shown as following equations:

CDF: $F(x) = 1 - e^{-\frac{x}{\rho}}$;  
PDF: $f(x) = \frac{1}{\rho} e^{-\frac{x}{\rho}}$.  

Order stastics are applied in the analytical approach, as the $h_m$ and $h_n$ are the smallest and the largest value respectively among 10 random variables. So their ordered density functions should be different from the above equations. According to [57] [58] [59], the ordered density functions are given as following:

Joint PDF of $x$ and $y$: $f_{|h_m|^2,|h_n|^2}(x, y) = \omega_1 f(x) f(y) [F(x)]^{m-1} [1 - F(y)]^{M-n} [F(y) - F(x)]^{n-1-m}$.  
Marginal PDF of $x$: $f_{|h_m|^2}(x) = \omega_2 f(x) [F(x)]^{m-1} [1 - F(x)]^{M-m}$.  
Marginal PDF of $y$: $f_{|h_n|^2}(y) = \omega_3 f(y) [F(y)]^{n-1} [1 - F(y)]^{M-n}$.  

Where $\omega_1 = \frac{M!}{(m-1)! (n-1-m)! (M-n)!}$, $\omega_2 = \frac{M!}{(m-1)! (M-m)!}$ and $\omega_3 = \frac{M!}{(n-1)! (M-n)!}$, $f(x)$, $F(x)$, $f(y)$ and $F(y)$ are the density functions from equation (7) and (8).
**Theorem 1:** Suppose the users with the best and the worst channel gain are paired (proposed scheme 1). The probability that it achieves a lower sum rate than the targeted data rate is given by:

\[
P(R_m + R_n < R_t) =
\]

\[
1 - \frac{\omega_3}{\rho} \sum_{i=0}^{n-1} \binom{n-1}{i} (-1)^i \frac{\rho}{M-n+i+1} e^{-\frac{(M-n+i+1)\omega_5}{\rho}} - \omega_1 \left(\sum_{j=0}^{n-1-m} \binom{n-1-m}{j} (-1)^j \frac{\omega_5}{m+j} \omega_6\right),
\]

where \(f(y) = \frac{1}{\rho} e^{-\frac{y}{\rho}}, \ F(y) = 1 - e^{-\frac{y}{\rho}}, \ \omega_1 = \frac{M!}{(m-1)!(n-1-m)!(M-n)!},\)

\[
\omega_3 = \frac{M!}{(n-1)!(M-n)!}, \ \omega_4 = \frac{1}{\alpha n^2} - 2R_t, \ \omega_5 = \frac{2R_{t-1}}{\alpha n^2}, \ \omega_6 = \sqrt{\omega_5 + \frac{(\omega_4+1)^2}{4} - \frac{\omega_4+1}{2}},
\]

\[
\omega = \int_{\omega_6}^{\omega_5} f(y) F(y)^{n-1-m-j} [1 - F(y)]^{M-n} [F(y)^{m+j} - F\left(\frac{\omega_5 - y}{\omega_4 + y}\right)^{m+j}] \ dy.
\]

**B. Analytical Results**

At high SNR, the approximation of the analytical result is shown as following:

\[
P(R_m + R_n < R_t) \approx \frac{\omega_3 \omega_5 n^\alpha}{\rho n^\alpha} - \frac{\omega_1 \omega}{\rho n^\alpha},
\]

where \(\omega = \sum_{i=0}^{n-1-m} \binom{n-1-m}{i} (-1)^i \int_{\omega_6}^{\omega_5} y^{n-1-m-i} \left[ y^{m+i} - \left(\frac{\omega_5 - y}{1+y}\right)^{m+i}\right] \ dy,\)

and \(\omega\) is considered as a constant [51].

**Proof:** See the appendix.
C. Verification

The verification for the simulation and analytical results is shown in Figure 8. It shows that the exact expression of analytical result of Theorem1 matches with the simulation result. Note that this simulation is only used for verification, the users have been set to be independent and identically distributed. In another word, it is a simplified simulation to keep all assumptions the same as we made in the analytical approach. The end of the simulation curve is a little bit mismatching with the analytical result (dashed line), and the reason is that the outage probability at the point fluctuated because the lack of simulation samples. As also shown in this figure that with the infinite SNR value, the analytical equation can be approximated as a linear line. And at the high SNR, the approximation matches with the exact expression.

Figure 8. Verification for the simulation and analytical results
V. Improvement

Although case 3 of simulation has stated the user fairness can be achieved by pairing the highest priority user with the lowest priority user. It only considers the fairness on subcarriers. However, will the users in one subcarrier achieve user fairness? From the simulation results, it is hard to answer this question. But from the data rate calculating equations, the data rates for the users with poor channel conditions will be critical if the allocated power is less than or equal to the users with good channel conditions. Although the total sum rates are very similar with different power allocations, each user’s data rate in the same subcarrier will be different [60].

Jain’s fairness measure provides an algorithm of measuring the user fairness in one subcarrier. Jain’s fairness Index [61] can be presented as equation (14):

\[ J(x) = \frac{\left( \sum_{i=1}^{M} x_i \right)^2}{M \sum_{i=1}^{M} x_i^2}, \]  

where \( M \) is number of users in one subcarrier, \( x_i \) is the data rate of user \( i \) from a particular power allocation. The fairness index \( J(x) \) is a continuous value with a range of \([1/M, 1]\), where \( 1/M \) and 1 happens on the least fair and the fairest allocations correspondingly.

Jain’s fairness on each user in one subcarrier is analysed by calculating the data rate in Figure 9. The pairing priority is channel gain over weight and user 1 has higher priority than user 2.
From Figure 11 and Table 4, the data rates for users have a great difference in the equal power allocation, as the power allocated to user 2 should not be great than the power
to user 1. Applying Jain’s Index equation (14), when number of user is 2 (M = 2), J(x) equals to 0.541 which is the least fair condition in Jain’s fairness measure.

In contrast, when we allocate a small portion (e.g. 1/7200) of the total power to the higher priority user and the rest of the power to the lower priority user, the users’ data rates are very similar. Applying equation (14), J(x) equals to 1, which is the fairest condition in Jain’s fairness measure.

Normally, users’ data rates in one subcarrier do not have to be the same, they depend on the users’ QoS requirements, which can give the system the required data rate for each user. The system can pair these users based on their channel conditions and social weights, and use different power allocations to meet the targeted data rate for the individual user. Moreover, some Opportunistic scheduling algorithms of resource allocations for user fairness are provided in [62] - [70].

VI. Conclusion

In conclusion, the simulation results show that the user pairing based on channel conditions can achieve a higher sum rate as well as a larger capacity than conventional OMA. In other word, this design scheme can accommodate much more users to meet the huge demand of internet traffic for the following years, and it also saves more power than the conventional OMA when the required data rates are same. 3 proposed NOMA schemes based on user pairing have been studied as well, and their
performances are shown in the simulation. The scheme of pairing the users with the best and the worst channel conditions achieves the best performance among the 3 proposed schemes and its performance has been verification via analytical results. Although the scheme of considering social weights achieves a bit lower sum rate than the method of user pairing based on users’ channel conditions, considering users’ importance in the society is the feature of this design, which makes NOMA system more realistic. Moreover, the performances of user pairings for different pairing orders and different power allocations have been compared by simulations. User pairing for the highest priority user and lowest priority user can achieve the highest sum rate as well as fairness for subcarriers. Also, another simulation result shows that with different power allocation, the total sum rate the keeps constant, but each user’s data rate is different. When a small portion of the power is allocated to the higher priority user and the rest of the power to the lower priority user, fairness for the two users in one subcarrier can be achieved. Different power allocations will be applied if the user fairness among the two users in the same subcarrier is required or each user has own requirement. In the future, the analytical results are suggested to be solved for better verifications.
VII. Appendix

Proof for Theorem 1: According to the equation (6), the outage probability of user pairing for Theorem can be expressed as following [51]:

\[ P(R_n + R_m < R_t) = 1 - P \left( \log_2 \left( \frac{1 + \rho |h_m|^2}{\rho |h_m|^2 a_n^2 + 1} \right) + \log_2 (1 + \rho a_n^2 |h_n|^2) > R_t \right). \]  

Consider the second part of equation (15) as the addressed probability. Applying some algebraic methods, the addressed probability can be manipulated as following:

\[ P \left( \log_2 \left( \frac{1 + \rho |h_m|^2}{\rho |h_m|^2 a_n^2 + 1} \right) + \log_2 (1 + \rho a_n^2 |h_n|^2) > R_t \right) = P \left( \frac{1}{a_n^2} - 2^{R_t} \rho |h_m|^2 + \rho |h_n|^2 + \rho^2 |h_m|^2 |h_n|^2 > \frac{2^{R_t-1}}{a_n^2} \right). \]  

Equation (16) can be simplified as following:

\[ P(\omega_4 x + xy + y > \omega_5). \]  

As \( y \) is always greater than \( x \), if \( \omega_4 < 0 \), consider \( y \) in 3 cases:

1. \( y > \omega_5 \)
2. \( -\omega_4 < y < \omega_5 \)
3. \( y < -\omega_4 \)

Let the probabilities of equation (17) with the given 3 cases be \( Q_1 \), \( Q_2 \) and \( Q_3 \). And \( P(\omega_4 x + xy + y > \omega_5) = Q_1 + Q_2 + Q_3 \).
**Case 1:**

Proof below shows when $y > \omega_5$, the probability of $\omega_4 x + xy + y > \omega_5$ is always equal to 1.

**Proof of Case 1:**

Assume $y = \omega_5$, then $\omega_4 x + xy + y = (\omega_4 + \omega_5)x + \omega_5$. (18)

As the $x$ and $y$ are always positive numbers, and $\omega_4 + \omega_5 = \frac{1-a_n^2}{a_n^2} \times 2^R_t > 0$.

Therefore, equation (18) is always greater than $\omega_5$.

Given case 1, the probability of equation (17) is shown as following:

\[
Q_1 = P(\omega_4 x + xy + y > \omega_5 | y > \omega_5) \times P(y > \omega_5) = P(y > \omega_5). \tag{19}
\]

According to the ordered density function equation (11), $Q_1$ is given by:

\[
Q_1 = P(y > \omega_5) = \int_{\omega_5}^{\infty} f_{|h_n|^2}(y) \, dy = \int_{\omega_5}^{\infty} \omega_3 f(y)[F(y)]^{n-1}[1 - F(y)]^{M-n} \, dy \\
= \frac{\omega_3}{\rho} \sum_{i=0}^{n-1} \binom{n-1}{i} (-1)^i \frac{\rho}{M-n+i+1} e^{-\frac{(M-n+i+1)\omega_5}{\rho}} \tag{20}
\]

**Case 2:**

The probability of equation (17) with given case 2 is expressed in equation (20):

\[
Q_2 = P(\omega_4 x + xy + y > \omega_5 | -\omega_4 < y < \omega_5) \times P(-\omega_4 < y < \omega_5). \tag{21}
\]

When $\omega_4 < 0$, the $y$ region in this case should be $-\omega_4 < y < \omega_5$. According to the joint PDF of $x$ and $y$ from equation (9), the first part of $Q_2$ is given by:

\[
P(\omega_4 x + xy + y > \omega_5 | -\omega_4 < y < \omega_5) = \int_{\omega_4 x + xy + y > \omega_5} f_{|h_n|^2}(x, y) \, dx \, dy \tag{22}
\]
The region of $x$ in the double integral is \( \frac{\sigma_5 - y}{\sigma_4 + y} < x < y \), \( (23) \)

which implies \( y > \frac{\sigma_5 - y}{\sigma_4 + y} \). \( (24) \)

Solve equation (24), we can get \( y > \sqrt{\sigma_5 + \frac{(\sigma_4 + 1)^2}{4} - \frac{\sigma_4 + 1}{2}} \), Assume that \( \sigma_6 = \sqrt{\sigma_5 + \frac{(\sigma_4 + 1)^2}{4} - \frac{\sigma_4 + 1}{2}} \). So \( y > \sigma_6 \). \( (25) \)

The regions for $x$ and $y$ are given by:

\( \frac{\sigma_5 - y}{\sigma_4 + y} < x < y; \) \( (26) \)

\( \sigma_6 < y < \sigma_5. \) \( (27) \)

The region of $y$ in this case should be changed into \( \sigma_6 < y < \sigma_5, \) as \( \sigma_6 > -\sigma_4 > 0. \)

Therefore, probability is \( Q_2 \) given by:

\[
Q_2 = \int_{\sigma_6}^{\sigma_5} \int_{\frac{\sigma_5 - y}{\sigma_4 + y}}^{y} \frac{\sigma_1 f(x) f(y) [F(x)]^{m-1} [1 - F(y)]^{M-n} [F(y) - F(x)]^{n-1-m}}{x} \, dx \, dy.
\]

Applying binomial expansion, equation (28) can be evaluated in equation (29):

\[
Q_2 = \sigma_1 \left( \sum_{j=0}^{n-1-m} \left( \frac{n-1-m}{m+j} \right) \frac{(-1)^j}{m+j} \sigma \right),
\]

\( (29) \)

where \( \sigma = \int_{\sigma_6}^{\sigma_5} f(y)F(y)^{n-1-m-j}[1 - F(y)]^{M-n} [F(y)^{m+j} - F \left( \frac{\sigma_5 - y}{\sigma_4 + y} \right)^{m+j}] \, dy. \)
Case 3:

When \( y < -\omega_4 \), \( \omega_4 x + xy + y \) is always small than \( \omega_5 \), proof is shown as following:

Proof of case 3:

Assume that \( y = -\omega_4 \), \( \omega_4 x + xy + y = -\omega_4 \).

According to equation (18) that \( \omega_4 + \omega_5 > 0 \), \( -\omega_4 < \omega_5 \) which makes the probability of \( \omega_4 x + xy + y \) never be greater than \( \omega_5 \) in the condition that \( y < -\omega_4 \).

Therefore, the probability of \( \omega_4 x + xy + y > \omega_5 \) with given \( y < -\omega_4 \) is 0:

\[
P(\omega_4 x + xy + y > \omega_5 | y < -\omega_4) = 0.
\]  

(31)

The probability of \( Q_3 \) is evaluated as following:

\[
Q_3 = P(\omega_4 x + xy + y > \omega_5 | y < -\omega_4) \times P(y < -\omega_4) = 0.
\]  

(32)

The overall outage probability of Theorem1 can be evaluated as following:

\[
P(R_n + R_m < R_t) = 1 - Q_1 - Q_2 - Q_3
\]  

(33)

\[
= 1 - \frac{\omega_5}{\rho} \sum_{i=0}^{n-1} \binom{n-1}{i} (-1)^i \frac{\rho}{M-n+i+1} e^{\frac{(M-n+i+1)\omega_5}{\rho}} - \omega_1 \left( \sum_{j=0}^{n-1-m} \binom{n-1-m}{j} (-1)^j \frac{\rho}{m+j} \frac{\omega_5}{\omega_4} \right),
\]

where \( \omega = \int_{\omega_6}^{\omega_5} f(y)F(y)^{n-1-m-j}[1-F(y)]^{M-n}[F(y)^{m+j} - F \left( \frac{\omega_5-y}{\omega_4+y} \right)^{m+j}] \, dy.\)
High SNR approximation:

As from equation (20), \( Q_1 = \frac{\sigma_3}{\rho} \sum_{i=0}^{\infty} \left( \begin{array}{c} n-1 \\ i \end{array} \right) (-1)^i \frac{\rho}{M-n+i+1} e^{\frac{(M-n+i+1)\sigma_5}{\rho}}. \) The series expansion of the exponential function is applied in this equation.

\[
Q_1 = \sum_{j=0}^{\infty} \frac{(-1)^j \sigma_3 \sigma_5^j}{j! \sigma_5^j} \sum_{i=0}^{\infty} \left( \begin{array}{c} n-1 \\ i \end{array} \right) (-1)^i (M - n + i + 1)^{j-1}. \tag{34}
\]

\( Q_4 \) is equal to 1 if \( \sigma_5 = 0 \), so in equation (33), \( \sigma_3 \sum_{i=0}^{\infty} \left( \begin{array}{c} n-1 \\ i \end{array} \right) (-1)^i \frac{1}{M-n+i+1} = 1. \) So

\[
Q_1 = 1 + \sum_{j=0}^{\infty} \frac{(-1)^j \sigma_3 \sigma_5^j}{j! \sigma_5^j} \sum_{i=0}^{\infty} \left( \begin{array}{c} n-1 \\ i \end{array} \right) (-1)^i \sum_{l=0}^{j-1} \left( \begin{array}{c} j-1 \\ l \end{array} \right) (M - n + 1)^{j-1-l}. \tag{35}
\]

According to [51], after applying the binomial coefficients, the probability of \( Q_1 \) can be approximated as following:

\[
Q_1 \approx 1 + \frac{(-1)^n \sigma_3 \sigma_5^n}{n! \sigma_5^n} (-1)^{n-1} (n - 1)! = \frac{\sigma_3 \sigma_5^n}{n \sigma_5^n} \tag{36}
\]

According to [51], the approximation of two functions in \( Q_2 \) are shown as following:

\[
f(y) = \frac{1}{\rho} e^{-\frac{y}{\rho}} \approx \frac{1}{\rho}, \text{ and } F(y) = 1 - e^{-\frac{y}{\rho}} \approx \frac{y}{\rho}. \] So

\[
F\left(\frac{\sigma_5-y}{\sigma_4+y}\right) = 1 - e^{-\frac{\sigma_5-y}{\rho(\sigma_4+y)}} \approx \frac{\sigma_5-y}{\rho(\sigma_4+y)}. \]

And \( Q_2 \) can be rewritten as following:

\[
Q_2 \approx \frac{\sigma_3}{\rho^n} \sum_{i=0}^{n-1-m} \left( \begin{array}{c} n-1-m \\ i \end{array} \right) (-1)^i \int_{\sigma_6}^{\sigma_5} y^{n-1-m-i} \left[ y^{m+i} - \left( \frac{\sigma_5-y}{1+y} \right)^{m+i} \right] dy \tag{37}
\]

As from equation (32), \( Q_3 = 0 \), so the approximation of equation (33) can be written as following:

\[
P(R_n + R_m < R_t) = 1 - Q_1 - Q_2
\]

\[
\approx \frac{\sigma_3 \sigma_5^n}{n \rho^n} - \frac{\sigma_3}{\rho^n} \sum_{i=0}^{n-1-m} \left( \begin{array}{c} n-1-m \\ i \end{array} \right) \int_{\sigma_6}^{\sigma_5} y^{n-1-m-i} \left[ y^{m+i} - \left( \frac{\sigma_5-y}{1+y} \right)^{m+i} \right] dy \tag{38}
\]
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