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SWIPT in Distributed Antenna System with Limited Capacity

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Abstract

This thesis studies the resource allocation algorithm design through using renewable energy. All of the base stations (BS) are connected to the central processor and these base stations (BS) are all recharged by green energy by using energy harvesting from the wind and solar. The central processor plays an important role in this problem and it can control the BS which is near or having enough battery to allocate the energy to that low-battery BS. Firstly, there is a background introduction about the SWIPT network. Secondly, the literature review will be introduced. Furthermore, the system model will be introduced which includes the architecture of the system, channel model, channel state information, achievable data rate, total harvested energy. In the next part, the objective is to maximize the harvested energy while providing power allocation between the base station and efficient communication. Then, we use MATLAB to perform simulation to evaluate the performance of the resource allocation algorithm. Finally, there will be a conclusion.
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1. Introduction

The objective of current and future cellular networks is to supply higher data rate for all of the customers. The interference of cellular network is limited. Multiple user detection and transmission always referred to the background of distributed antenna networks. It is verified and appropriate to solve that interference. Furthermore, it provides wonderful link quality and increases cell coverage. Employing large clusters of cooperating cells can achieve optimistic capacity bounds. It can be used in uplink and downlink of the networks.

There is a serious problem for distributed antenna networks which is a large number of information gathered from or transmitted to included receivers or transmitters. Furthermore, it requires a number of backhaul equipment between the cells. Based on the framework, there will be a proposal algorithm which may improve the cellular network performance in a limited backhaul capacity.
It is necessary for current and future wireless communication networks to supply high data rates, high safety and universal communication with ensured quality of service (QoS). As a result, both transmitters and receivers will consume enormous energy. MIMO techniques is one of the solutions for decreasing networks power consumption. Particularly, multiple user MIMO is an efficient way of achieving the performance gain which is supplied by multiple antennas. For multiple user MIMO, the transmitter has multiple antennas and it serves multiple users.

In wireless power transfer, Nikola Tesla put forward a proposal notion in the 1890s that a transmitter may transmit energy to a receiver through the wireless media. By using the wireless power transmit, it may alleviate the expenditure of planning and install cables throughout the regions. However, there will be a challenge for realizing wireless power transfer. It is inefficiency, because of transmission path loss and low efficiency of radio frequency (RF). Furthermore, old electronic equipment is inefficiency and consumes large power. Due to these problems, wireless power transfer was ignored by so many people.

![Communication system model for SWIPT](Source: Google Images. Image by D. W. K. Ng)

Previous theoretical research for SWIPT has studied that energy and information
can be transmitted perfectly by one same signal without losses. This brings a basic trade-off between information and power transmission. However, the energy and information transmitted at the same time cannot be achieved in reality, because the energy harvesting operation carried out in the radio frequency domain which destroyed the information. In order to realize SWIPT, there is a method that the received signal must split to 2 different parts. The first one is energy harvesting part and another one is information decoding part. The three technologies which can realize the signal splitting into distinct domains will be introduced.

(1) Time Switching (TS)

If Time Switching is used, the receiver will switch in different time. For this situation, the signal splits in a time domain. Therefore, the signal arrived at receiver in one timeslot is employed to decode information or transmit power. The Time Switching technology can equip the receiver with a simple hardware.

(2) Power Splitting (PS)

The Power Splitting technology realizes SWIPT through dividing the received signal into 2 distinct power level parts. The first part is transmitted to the rectenna circuit to harvest energy and the second part is exchanged into baseband to decode information. There is a difference between TS and PS that the PS technology brings a higher receiver complexity and needs the Power Splitting factor $\alpha$ to be optimized. However, it realizes instantaneous SWIPT, since the signal received in 1 timeslot applies to decode information and transmit power.

(3) Antenna Switching (AS)

In general, the function of antenna arrays is generating DC power for reliable equipment operation. Based on this way, the Antenna Switching technology switches every antenna component for decoding/rectifying to realize SWIPT in antenna domain. In AS technique, receivers are divided into two parts that one part is applied to decode information and the other part is applied to harvest energy. There is a requirement for AS technology that the solution should be optimized problem for every communication frame. The objective is to judge the optimal allocation of the antenna
elements.

2. Literature review

The successful development of wireless communication networks and technologies has triggered an exponential growth in the number of wireless communication devices worldwide. In the near future, devices embedded with multifunctional sensors and communication chip sets will be able to collect and exchange information via the Internet. Specifically, these smart devices will be connected to computationally powerful central computing systems to provide intelligent services for the daily life such as environmental monitoring, e-health, automated control, energy management, logistic, and safety management. This new concept of interconnecting a massive number of communication and sensing devices is known as the Internet of Things (IoT) [1]. In practice, wind, solar, and geothermal are the major renewable energy sources for generating electricity [2, 3, 4], thereby reducing substantially the reliance on the energy supply from the power grid. Yet, these conventional natural energy sources are usually climate and location dependent which restricts the mobility of smart devices. Besides, most of these energy sources are not available in indoor environments. More importantly, the uncontrollable and intermittent nature of these natural energy sources makes their use in IoT communication networks challenging.

Recently, wireless energy transfer (WET) has emerged as one of the technologies driving IoT networks and has attracted much attention from both academia and industry [5]–[27]. The existing WET technologies can be categorized into three classes: inductive coupling, magnetic resonant coupling, and radio frequency (RF)-based WET. The first two technologies rely on near-field electromagnetic (EM) waves. In particular, these two technologies can provide wireless charging over short distances only due to the required alignment of the magnetic field with the EH circuit. Therefore, in general, near-field techniques do not support the mobility of EH devices. In contrast, RF-based WET [5]–[24] exploits the far-field properties of EM waves
facilitating long distance wireless charging. More importantly, EM waves not only serve as a vehicle for carrying energy, but also for carrying information which enables the possibility of simultaneous wireless information and power transfer (SWIPT) and wireless powered communication (WPC). Specifically, in SWIPT networks, a transmitter broadcasts both information and energy signals to provide information and energy delivery service simultaneously. In wireless powered communication networks (WPCNs), wireless communication devices first harvest energy, either from a dedicated power station or from ambient RF signals, and then use the harvested energy to transmit information signals. Compared to conventional EH, RF-based EH technology provides an on-demand energy replenishment which is suitable for smart wireless communication devices having strict quality of service (QoS) and energy requirements. On the other hand, various “last meter” wireless communication systems, such as Wi-Fi and small cell systems, can be potentially exploited for energy replenishment of battery constrained wireless devices. Nowadays, simple EH circuits are able to harvest microwatts to mill watts of power over the range of several meters for a transmit power of 1 Watt and a carrier frequency of less than 1 GHz [28]. Although the development of WET technology is still in its infancy, there are already some preliminary practical applications of WET such as passive radio-frequency identification (RFID) systems. It is expected that the introduction of RF-based EH to smart communication devices will revolutionize the system architecture and resource allocation algorithm design.

Conventional wireless communication systems are required to provide different types of QoS requirements such as throughput, reliability, energy efficiency, fairness, and timeliness [29]–[32]. On top of this, efficient WET is expected to play an important role as an emerging QoS requirement for RF-based wireless EH communication networks. In practice, for a carrier frequency of 915 MHz, the signal attenuation is 50 dB for every 10-meter of free space propagation. Hence, the efficiency of WET will be unsatisfactory for long distance transmission unless advanced resource allocation and antenna technology are combined. As a result,
various resource allocation algorithms exploiting multiple-antenna technology have been proposed [17]– [24]. Specifically, by utilizing the extra degrees of freedom offered by multiple transmit antennas, a narrow signal beam can be created and can be more accurately steered towards the desired receivers to improve the efficiency of WET. In this chapter, we study the resource allocation algorithm design for two specific RF-based multiple antennae EH communication networks.

3. System Model

3.1 Distributed Antenna System with Central Processor

We focus on the green SWIPT with limited capacity through distributed antenna network. It is a distributed antenna system with central processor and we consider it is a multiuser downlink communication system. As it is shown in Fig. 2, the system includes a central processor (CP), L multi-antenna base stations (BS), K single antenna information receivers (IR) and M single antenna energy harvesting receivers (ER). All of the base stations are connected to the central processor and these base stations are all recharged by green energy by using energy harvesting from the wind and solar. In fact, there is a drawback for green energy harvesting that the green energy harvesting is unstable. Therefore, there will be a problem that the BS does not have enough battery to support the users. The central processor plays an important role in this problem and it can control the BS which is near or having enough battery to allocate the energy to that low-battery. Each BS receives the control signals for resource allocation and data of the information receivers from the central processor through a backhaul link. There are many technologies which can implement the backhaul link, such as digital subscriber line (DSL). However, there may be a limitation for backhaul capacity.
3.2 Channel Model

We consider it is a frequency flat fading channel and a time division duplexing (TDD) system. In this system, the multi-antenna base stations serve both information receivers and energy harvesting receivers simultaneously in the same frequency band. The wireless information and power transfer from base stations to information receivers and energy harvesting receivers is divided into many time slots. In one time slots, the received signals at information receivers $k \in \{1, \ldots, K\}$ and energy harvesting receivers $m \in \{1, \ldots, M\}$ are given by

$$y_{k}^{IR} = h_{k}^{H} x + n_{k}^{IR},$$

$$y_{m}^{ER} = g_{m}^{H} x + n_{m}^{ER},$$

respectively, where $x \in \mathbb{C}^{N_{L} \times 1}$ represents the transmit vector of the $L$ base stations to the $K$ information receivers and $M$ energy harvesting receivers. $h_{k} \in \mathbb{C}^{N_{L} \times 1}$ is the channel for $L$ base stations and the $K$ information receivers’ information transmission. Furthermore, $g_{m} \in \mathbb{C}^{N_{L} \times 1}$ is the channel for $L$ base stations and $M$ energy harvesting receivers.

Figure 2. Distributed antenna system equipped with a central processor (CP) (Source: Google Images. Image by D. W. K. Ng)
harvesting receivers information transmission. \( n_{m}^{IR} \sim (0, \sigma_{IR}^2) \) and \( n_{m}^{ER} \sim (0, \sigma_{ER_m}^2) \) are the additive white Gaussian noises (AWGN) at the information receivers and the energy harvesting receivers, respectively.

### 3.3 Channel State Information

In this system, we assume that there is a perfect channel state information (CSI) which is available at the transmitter for resource allocation. That means \( h_k, \forall k \in \{1, \ldots, K\} \) and \( g_m, \forall m \in \{1, \ldots, M\} \) are known at the beginning of each time slots through exchanging signals between transmitters and receivers.

### 3.4 Achievable System Data Rate

The energy signal \( w \) is a Gaussian pseudo-random sequence which is known to all the transceivers. Given perfect CSI at the receiver for coherent information decoding, the achievable data rate (bit/s/Hz) between the transmitter and the information receiver is given by

\[
C_{IR_k} = \log_2 \left( 1 + \frac{|h_k^H w|^2}{\sum_{j \neq k} |h_j^H w_j|^2 + \text{Tr}(V_h h_k^H h_k^H) + \sigma_{IR_k}^2} \right),
\]

where \( \frac{|h_k^H w|^2}{\sum_{j \neq k} |h_j^H w_j|^2 + \text{Tr}(V_h h_k^H h_k^H) + \sigma_{IR_k}^2} \) is the receive signal-to-interference-plus-noise ratio (SINR) at information receiver \( k \).

For energy harvesting receiver, the achievable data rate (bit/s/Hz) between the transmitter and the energy receiver is given by

\[
C_{ER_m} = \log_2 \left( 1 + \frac{|g_m^H w|^2}{\sum_{j \neq k} |g_m^H w_j|^2 + \text{Tr}(V_g g_m^H g_m^H) + \sigma_{ER_m}^2} \right),
\]

(3)
where \( \sum_{j=k}^{K} |g_m^H w_j|^2 + Tr(Vg_m g_m^H) + \sigma_{ER_k}^2 \) is the received SINR at energy harvesting receiver \( m \).

3.5 **Total Harvested Energy**

This system harvested energy through using energy harvesting receiver. The information signal \( w \) serves both information and energy. Moreover, the artificial noise signal plays an important role in this system. It can be considered as a kind of energy source to the energy harvesting receiver. The total harvested energy is given by

\[
E_m^{ER} = \mu [Tr(Vg_m g_m^H) + \sum_{k=1}^{K} |g_m^H w_k|^2],
\]

(4)

where \( 0 < \mu \leq 1 \) represents the efficiency of converting the harvested energy to electrical energy.

4. **Problem Formulation and Optimization**

4.1 **Energy Harvesting Optimization**

In this SWIPT with distributed antenna system, we aim to maximize the harvested energy while providing power allocation between the base station and efficient communication. The resource allocation algorithm design is formulated as the following optimization problem:

\[
\text{maximize } E_m, \quad E_m = \sum_{m=1}^{M} Tr[(\sum_{k=1}^{K} w_k w_k^H + v)G_m]
\]

s.t. \( C1: \sum_{j \neq k} |h_k^H w_k|^2 + Tr(v H_k) + \sigma_{Br_k}^2 \geq \Gamma_{req_k}, \forall k, \)
C2: $\sum_{k=1}^{K} |w_k|^2 + Tr(v) \leq P_{\text{max}}, \forall k,$

C3: $|R_n w_k|^2 + Tr(R_n v) \leq P_{n}^{\text{max}}, \forall n \in \{1,...,N_t\},$

$\min_{\nu, \sigma \in \Omega} \mu[Tr(\sum_{k=1}^{K} w_k + \nu)G_m)] \geq P_{m}^{\text{min}}, \forall m$

C4: $\min_{m \geq \text{req}} \Gamma^{\text{min}}_{\text{req}}$ in C1 indicates the minimum requirement on the signal-to-noise-ratio(SINR) of the receiver for decoding. In practice, the central processor sets $\Gamma^{\text{min}}_{\text{req}} > 0$. Constraint C2 means the total transmit power must be less than $P_{\text{max}}$ which specifies the maximum transmit power allowance. In C3, the $P_{n}^{\text{max}}$ represents the maximum transmit power per antenna. The $P_{m}^{\text{min}}$ in C4 is the minimum transmit power which is required for energy harvesting receiver. The constraint C5 specifies the transmit power for any base station must be less than the total harvested energy. In C6, $V$ is a positive semidefinite Hermitian matrix.

4.2 Semidefinite Program Relaxation

The problem in (5) is a non-convex optimization problem. In particular, the $E_m$ and constraint C1 are non-convex which cannot do maximization. In order to get the convex formulation, we can rewrite (5) in an equivalent formulation:

$$\max_{W_k, V \in \mathbb{H}^{N_t \times K}} \sum_{m=1}^{M} \sum_{k=1}^{K} Tr(W_k G_m) + \sum_{m=1}^{M} Tr(V G_m)$$
\text{s.t.} \quad C1: \quad \text{Tr}(H_k W_k) \geq \Gamma_{\text{req}}^{\min} \left[ \sum_{j \neq k} \text{Tr}(H_k W_j) + \text{Tr}(V H_k) + \delta_{R_k}^2 \right],

C2: \quad \sum_{k=1}^{K} \text{Tr}(W_k) + \text{Tr}(V) \leq P_{\text{max}},

C3: \quad \text{Tr}(W_k R_n) + \text{Tr}(R_n V) \leq P_n^{\max}, \forall n \in \{1, \ldots, N_i\},

\min_{\lambda_i, \mu_i} \mu [\text{Tr}(\sum_{k=1}^{K} W_k + \nu) G_m] \geq P_m^{\min}, \forall m

C4: \quad H_k = \text{Tr}(\text{H}_k W_k) \equiv \text{Tr}(\text{H}_k W_k),

C5: \quad \text{Tr}(w^H B_i) \leq E m, \forall i \in \{1, \ldots, I\},

C6: \quad V \geq 0,

C7: \quad W_k \geq 0, \forall k,

C8: \quad \text{Rank}(W_k) \leq 1, \quad (6)

where \( W_k \) and \( V \) are matrices. Furthermore, \( W_k = w_k w_k^H \). For the reformulation (6), the only non-convexity is due to constraint C8. Particularly, C8 is a combinatorial constraint which requires a brute force search for finding an optimal maximization. To circumvent the non-convexity, the semidefinite programming (SDP) relaxation is used to (6) by relaxing constraint C8 which is \( \text{Rank}(W_k) \leq 1 \). When constraint C8 is removed from the (6) formulation, we will get:

\[ \begin{align*}
\text{maximize} & \quad \sum_{m=1}^{M} \sum_{k=1}^{K} \text{Tr}(W_k G_m) + \sum_{m=1}^{M} \text{Tr}(V G_m) \\
\text{s.t.} & \quad \text{Tr}(H_k W_k) \geq \Gamma_{\text{req}}^{\min} \left[ \sum_{j \neq k} \text{Tr}(H_k W_j) + \text{Tr}(V H_k) + \delta_{R_k}^2 \right],
\end{align*} \]

C2: \quad \sum_{k=1}^{K} \text{Tr}(W_k) + \text{Tr}(V) \leq P_{\text{max}},

C3: \quad \text{Tr}(W_k R_n) + \text{Tr}(R_n V) \leq P_n^{\max}, \forall n \in \{1, \ldots, N_i\},

\min_{\lambda_i, \mu_i} \mu [\text{Tr}(\sum_{k=1}^{K} W_k + \nu) G_m] \geq P_m^{\min}, \forall m

C4: \quad H_k = \text{Tr}(\text{H}_k W_k) \equiv \text{Tr}(\text{H}_k W_k),

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The formulation (7) is a convex SDP. From the basic principles of optimization theory, if obtained solution \( W_k \) for the relaxed problem admits a rank-one matrix, then it is the optimal solution of the original problem in (6). Moreover, the optimal \( W_k \) can be obtained by performing an eigenvalue decomposition of \( W_k \). However, the constraint relaxation may not be tight since it is possible that \( \text{Rank}(W_k) > 1 \). In the following, we reveal the tightness of the adopted SDP relaxation in (7) through examining of the dual problem and the Karush-Kuhn-Tucker (KKT) conditions of (7). The Lagrangian function of (7) is given by

\[
\mathcal{L}(W_k, V, \alpha, \beta, \phi, \gamma, \delta, \Theta, Y, \Phi) = f_0(W_k, V) + f_1(W_k, V, h_k, \mu, R_n) + f_2(W_k, B_i, E_m), \quad \text{where} \quad (8)
\]

\[
f_0(W_k, V) = \sum_{m=1}^{M} \sum_{k=1}^{K} \text{Tr}(W_k G_m) + \sum_{m=1}^{M} \text{Tr}(V G_m), \quad (9)
\]

\[
f_1(W_k, V, h_k, \mu, R_n, \Phi) = -\sum_{m=1}^{M} \text{Tr}(Y V G_m)
\]

\[
+ \alpha \{ -\text{Tr}(H_k W_k) \Gamma_{\text{req}} \} + \text{Tr}(V H_k) + \delta_{I_{R_k}}^2 \}
\]

\[
+ \beta \{ -[\sum_{k=1}^{K} \text{Tr}(W_k) + \text{Tr}(V)] + P_{\text{max}} \}
\]

\[
+ \sum_{j=1}^{J} \phi_j \{ -[\text{Tr}(W_k R_n) + \text{Tr}(R_n V)] - P_{n}^{\text{max}} \}
\]

\[
+ \sum_{n=1}^{N} \gamma_n \{ -\mu[\text{Tr}(\sum_{k=1}^{K} W_k + v) G_m] + P_{m}^{\text{min}} \}, \quad \text{and} \quad (10)
\]
\[
f_2(W_k, B_i, E_m, \Phi) = \sum_{k=1}^{K} \sum_{i=1}^{J} \beta_k [\text{Tr}(W_k B_i) - E_m],
\]
(11)

Here, \( \alpha \geq 0 \) is the dual variable for the minimum required SINR of the energy harvesting receiver in C1. Moreover, dual variable \( \beta \geq 0 \) is for the maximum transmit power allowance in C2. \( \phi \) is the vector of the maximum transmit power per antenna in C3 where \( \phi_j \geq 0, \ j \in \{1, \ldots, J\} \). \( \gamma \), with elements \( \gamma_n \geq 0 \), is the dual variable vector for the minimum transmit power constraint in C4. Matrix \( Y \geq 0 \) is the dual variable for the semi definiteness constraint on matrix \( W_k \). Therefore, the dual problem for the SDP relaxed problem is given by

\[
\max_{\Phi \geq 0} \min_{W_k, V \in \mathbb{H}^{n \times n}, h_k, j, \gamma, \delta, \theta, \Phi} f(W_k, V, \alpha, \beta, \phi, \gamma, \delta, \theta, Y, \Phi).
\]
(12)

Next, we reveal the tightness of the SDP relaxation adopted in (7) in the following theorem.

**Theorem 1:** Assuming the channel vectors of the IR, \( h \), and the ER, \( g_j \), \( j \in \{1, \ldots, J\} \), can be modeled as statistically independent random variables, then the solution of (7) is rank-one, i.e., \( \text{Rank}(W_k) = 1 \), with probability one.

**Proof:** Please refer to Appendix.

Thus, the SDP relaxation is tight whenever the channels satisfy the condition stated in Theorem 1. Furthermore, the optimal solution of the problem is obtained in each iteration and the dual variables are also obtained.

### 5. Results

In this section, we use MATLAB to perform simulation to evaluate the performance of the resource allocation algorithm.

In Figure 3, it shows the average harvested power versus the minimum required SINR for energy harvesting receiver. In particular, with the SINR increased, the
average harvested power decreased. The reason is that the beam will shift to information receiver when SINR increases. From the simulation, the average harvested power is obvious increase when the proposed algorithm applied to the system.

![Figure 3. Average harvested power (dBm) versus the Minimum required SINR (bit/s/Hz) for energy harvesting receiver.](image)

**Figure 3. Average harvested power (dBm) versus the Minimum required SINR (bit/s/Hz) for energy harvesting receiver.**

In Figure 4, it illustrates average achievable data rate versus the minimum required SINR for information receiver. For the baseline 1, it’s the MRT scheme with respect to the channel of information receiver. The average data rate and SINR are positive correlation. Therefore, with the SINR increased, the average data rate increases steadily. However, with the SINR increased, the average data rate increases dramatically when the proposed algorithm is applied to the system.
Figure 4. Average achievable data rate (bit/s/Hz) versus the Minimum required SINR (bit/s/Hz) for information receiver.

6. Conclusion

This paper introduced a network system with distributed antenna for green SWIPT in a limited capacity. The objective is to maximize the harvested energy while providing power allocation between the base station and efficient communication. Moreover, the central processor plays an important role in sharing harvested energy between the base station. The formulation is a non-convex optimization problem. To get the optimal solution, we use the SDP relaxation to change the formulation to convex one. The simulation results illustrate the proposed resource allocation algorithm performs close to the optimal scheme.

Appendix
Proof of Theorem 1

Since the SDP relaxation is used for (6) and the formulation turns out to be convex problem. In the following, we focus on those KKT conditions for Theorem 1 proof:

\[ Y^* \geq 0, \quad \alpha^*, \beta^*, \phi_j^*, \gamma_n^*, \delta^*, \theta^* \geq 0 \]

(13)

\[ Y^*W^* = 0, \]

(14)

\[ Y^* = I_{N_T} + \sum_{n=1}^{N_T} \gamma_n \Psi_n - \sum_{j=1}^{j} \phi_j G_j - (\alpha^* + \beta^*)H \]

(15)

where \( A = I_{N_T} + \sum_{n=1}^{N_T} \gamma_n \Psi_n - \sum_{j=1}^{j} \phi_j G_j \) and \( Y^*, \alpha^*, \beta^*, \phi_j^*, \gamma_n^*, \delta^* \) and \( \theta^* \) are the optimal lagrange multipliers for (12). If \( \text{Rank}(Y^*) = N_T - 1 \), then the optimal \( W^* \neq 0 \) must be a rank-one matrix and it can be got by performing eigenvalue decomposition of \( W^* \). What we want to prove is that \( A \) is a full-rank matrix with rank \( N_T \) whenever the condition stated in Theorem 1 is satisfied. For given set of optimal dual variables, the dual problem in (12) can be expressed as

\[
\min_{W_k, V \in \mathbb{H}^{N_T}} \mathcal{L}(W_k, V, \alpha^*, \beta^*, \phi^*, \gamma^*, \delta^*, \theta^*, Y^*, \Phi^*) \quad .
\]

(16)

Assume \( A^* \) is not positive definite and \( W_k = tww^H \), where \( t > 0 \) is a scaling parameter and \( W \) is the eigenvector with respect to one of the non-positive eigenvalues of \( A^* \). Substituting \( W_k = tww^H \) into (16), we can get

\[
\text{Tr}(tA^*ww^H) - t\text{Tr}(ww^H(Y^* + (\alpha^* + \beta^*)H)) + \Delta
\]

(17)

\( \Delta \) represents a collection of the variables that are independent of \( W \). Moreover, the channel vectors \( h \) and \( g \) are assumed to be statistically independent. We set
\( t \to \infty \), \( t \text{Tr}(ww^H(Y^* + (\alpha^* + \beta^*)H) \to -\infty \) and \( \Gamma_{\text{req}} > 0 \). Therefore, \( A^* \) is a positive definite matrix with probability one, i.e., \( \text{Rank}(A^*) = N_T \).

According to (15), we can get

\[
\text{Rank}(Y^*) + \text{Rank}((\alpha^* + \beta^*)H) \\
\geq \text{Rank}(Y^* + (\alpha^* + \beta^*)H) \\
= \text{Rank}(A) = N_T \Rightarrow \text{Rank}(Y^*) \geq N_T - 1. \tag{18}
\]

Therefore, \( \text{Rank}(Y^*) \) is either \( N_T - 1 \) or \( N_T \). As result, \( \text{Rank}(Y^*) = N_T - 1 \) and \( \text{Rank}(W^*) = 1 \) hold with probability one. The optimal \( w^* \) can be obtained by performing eigenvalue decomposition of \( W^* \).
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