Power-Efficient SWIPT in Distributed Antenna Systems

By

Jinjin Dai

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Abstract

In 2016, the world population grows to 7.3 billion and the worldwide netizens keep growing to 47% of the population. Nearly half of the people around the world are using internet. The internet network is now getting bigger and wider with rapidly growing netizens. To support a high speed and low latency network, communication technology plays a role as the bridge to connect invisible information with each user. The 3rd generation of network was invented to provide high data rate to support things like multi-media and video conferencing and different type of data. Then the 4th generation of network increased the spectrum efficiency to provide much higher data rate. However, the cost of 4G technology is much higher energy consumption. The 5th generation of network emerges and is defined as high speed, low latency, increasing reliable connection and meanwhile more sustainable and greener.

Currently, 4G network has problem with energy or power saving because the 4G tower beams information in all direction and waste power on locations that have no accessing request. On the other hand, 4G network has a working spectrum below 6GHz which is much narrower when compare with new designed 5th generation network which will have a working spectrum from 30GHZ to 300GHz. To build a more sustainable and greener communications, the energy harvesting technology has been proposed and has become a promising way to prolong the lifetime of the networks. A particular research interest in this field is simultaneous wireless information and power transfer system (SWIPT).

This project applies SWIPT to a distributed antenna system (DAS). A brief introduction of the 5G networks and green technologies are given first, then follows with the basic knowledge and literature review of the distributed antenna system (DAS) and simultaneous wireless information and power transfer (SWIPT). The system model and formulation of the simultaneous wireless information and power transfer (SWIPT) in distributed antenna system follows after that. The system formulation is analysed by using convex optimisation method and simulated in MATLAB.
1 Introduction

1.1 A brief overview of the sustainable 5th generation of networks

Here is a graph showing the power of the network people is using nowadays below.

![Figure 1.1 What we do over the network today in one minute [1]](image1)

The 4th generation of technologies defined by International Telecommunication Union (ITU) as IMT-Advanced and the 5th generation as IMT-2020 are shown in the figure below.

![Figure 1.2 Comparison between 4G and 5G with respect to the eight key performance indicators [3]](image2)

It can be seen that the 5th generation of networks is defined to provide more connections to more users with the higher data rate, lower latency, higher spectrum efficiency (SE) and higher energy efficiency [4]. As it is stated in [5] and [6], it is anticipated to connect at least 100 billion
devices worldwide with approximately 7.6 billion mobile subscribers due to the tremendous popularity of smartphones, electronic tablets, sensors, etc. and provide an up to 10 Gb/s individual user experience. To be capable to deal with this large and still expanding demand for wireless communications, several rules that provide feasible solutions are [4]:

- Shorten the distance between the transmitter and receiver (Tx-Rx), meanwhile, increasing utilization of the frequency: this method refers to ultra-dense networks (UDNs) and device-to-device (D2D) communications;
- Utilize the spectrum that has not been used or registered: this method refers to millimetre wave (mmWave) communications and Long-term Evolution using unlicensed spectrum (LTE-U);
- Improving spectral efficiency (SE) by adopting an enormous number of antennas: massive multiple-input multiple-output (M-MIMO).

The Table I below shows comparison among different technologies.

<table>
<thead>
<tr>
<th>Technology</th>
<th>High EE at BS and UT</th>
<th>Coverage</th>
<th>Transmit Power</th>
<th>Circuit Power</th>
<th>Signalling Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>mmWave</td>
<td>BS and UT</td>
<td>200 m</td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>LTE-U</td>
<td>BS and UT</td>
<td>500 m</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>UDNs</td>
<td>UT</td>
<td>10-200 m</td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>D2D</td>
<td>BS</td>
<td>2-100 m</td>
<td>Low</td>
<td>Low</td>
<td>Moderate</td>
</tr>
<tr>
<td>M-MIMO</td>
<td>UT</td>
<td>1000 m</td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

These three paradigms mentioned above are capable to increase the throughput from different aspects. However, the massive multiple-input and multiple-output technology deploys a massive number of antennas to achieve desired diversity gain and implement multiplexing while following with an accumulative energy consumption. As a matter of fact, the energy consumption has become the major consideration when designing the wireless communication system in the recent years. The energy efficiency has turned into a remarkably new factor of merit. Note that due to the constraint of the Shannon capacity, only employing technologies to accomplish spectrum efficient system cannot boundlessly improve the energy efficiency. Hence, the issue of power consumption is only to be relieved by the increasing the energy efficiency to a limit extent and is inadequate in order to achieve a sustainable 5G communication system. Therefore, technologies of energy harvesting that provide utilization of renewable resources such as wind energy, solar energy or radio frequency signals such as
interferential signals have drawn the attention of scholars in this field. In the wireless communication network, the energy harvesting technologies allow various components in the wireless networks to be powered up to enable solutions of green energy supply. Therefore, for realising a remarkable improvement in the future work, it is necessary to incorporate the technologies of energy harvesting when designing the wireless communication system.

Now having a basic idea of the fundamental trade-off between the energy efficiency and the spectral efficiency with the help of the general expression of energy efficiency (EE) and spectral efficiency (SE).

\[
SE = K \times B \times N \times \log_2(1 + SNIR(d)) \quad [5]
\]

\[
EE = \frac{K \times B \times N \times \log_2(1 + SNIR(d))}{P_t + P_c} \quad [E.q 1-2]
\]

In which, \( K \) stands for the reuse factor of the frequency

\( B \) stands for the signal bandwidth

\( N \) stands for the spatial multiplexing factor

\( d \) stands for the distance of a signal

\( SNIR \) is the signal-to-interference-plus-noise ratio at the receiver that is inversely proportional to distance \( d \)

\( P_t \) is the consumed transmit power and \( P_c \) is the consumed circuit power

According to the Table I, it can be seen that the transmit power is kept at a low level with compensation on high consumption of the circuit power. With giving the fixed value of the reuse factor, bandwidth, multiplexing factor and link distance, the relation between energy efficiency and spectral efficiency is only relevant to \( P_t \) and \( P_c \).
As the blue line illustrated in Figure 1.3, if the circuit power is negligible, the energy efficiency is inversely proportional to spectral efficiency. If the circuit power is not negligible, the energy efficiency increases first and then decrease with the increase of spectral efficiency. These three lines represented three different conditions converge at an approximately same value at last. Although the actual system is much more complex in reality, the trade-off relation can be found depending on specific system configuration.

Besides the green technologies that fall into the three paradigms mentioned above, the energy harvesting technologies which are prominent approaches to realise green communication have two general pathways: natural energy from renewable resources and RF signals. In the research area of RF energy harvesting, there are two pathways known as simultaneous wireless information and power transfer (SWIPT) and wireless powered communication networks (WPCNs) [7] [8] [9] [10] [11]. The details of SWIPT is given in the next section.

## 2 Background knowledge and Literature Review

### 2.1 Wireless Power Transfer Technology (WPT) and Beamforming

Before SWIPT, wireless power transfer technology (WPT) has been used in a large range of areas and is the precedent of SWIPT. WPT is to deliver electrical energy without using a physical link like cable or wire under the circumstances that this kind of conductors is harmful, not convenient or even impossible. WPT technologies include laser, radio waves, microwaves, photoelectric, capacitive coupling and inductive coupling that will be mentioned below [12]. Generally, a WPT model contains a transmitter, which is used to deliver electric power from the power source by conducting an electromagnetic field that varies with time to radiate energy through the space, and a receiver that is used to receive the power and the receiving circuit extracts and converts the power back to electric current to the devices. The basic structure of the WPT system is shown in the figure below.

![Figure 2.1 Structure of Wireless Power Transfer system](image-url)
WPT basically is categorised by radiation capability that non-radiative refers to near-field and radiative refers or far-field transmission. The near-field transmission uses two methods to transfer energy over short distance. The first one adopts inductive coupling between two conductors winding into coils to generate the magnetic field. The second method transfer power by using capacitive coupling to generate electric fields between circuit nodes. The first method has been commonly applied in near-field wireless power transfer such as charging of phones or medical devices that implant into the body, RFID and etc. These two methods have extremely short propagation range since this kind of power decreases with distance exponentially [13]. The radiative transmission transfers power by pointing to the receiver and beaming the electromagnetic waves that can propagate in space to very long distance. This radiative transmission raises the issue of beamforming which is also the consideration when design a SWIPT system and is briefly introduced in the next paragraph. The only thing that causes a bit concern is that excessive electromagnetic waves might cause harm on living things.

When talking about providing power to a large number of users, antenna array or multiple antennas are used to satisfy the requirement. The simplest explanation of beamforming is how to focus signals towards the receiver on the transmitter side or how to estimate the direction of arrival (DOA) from the source on the receiver side in order to optimise the receiving result rather than affected by too much interference. This technique has had a very long history already and has been studied in many fields, especially for enhancing the desired signal and prevent signal corruption from the noise from the environment. The conventional beamformer works as a spatial filter at the end of the antenna array and the output from each antenna will be weighted depending on specific criterions to form the desired beam pattern [14]. Nowadays, beamforming falls into two categories: fixed beamforming and adaptive beamforming. The fixed beamforming is able to alleviate the interference but not suppress it which is implement by generating beam only on the desired directions and put null in the other directions that may have unwanted noise. Consequently, the spatial filter has fixed coefficients that are independent with the change of the wave through the propagation environment and the incoming data is not used for analysis. However, the fixed beamforming usually need the knowledge of the geometry of the array. The adaptive beamforming is adopted under a more challenging circumstance like multipath that is difficult to mitigate noise when using fixed beamforming. Therefore, the filter has to take the incoming data of the array into account [15].
The figure shown below is the analogue beamforming network at the receiver side. The formula of the array output in this beamforming network is given as:

\[ y(t) = \sum_{k=1}^{N} w_k^*x_k(t) = w^H(t)X(t) \]  

Where \( w = [w_1, \ldots, w_N]^T \in \mathbb{C}^{N \times 1} \) is the coefficients equal to the antenna weights, \( X(t) = [x_1(t), \ldots, x_N(t)]^T \in \mathbb{C}^{N \times 1} \) is the received signal vector.

The knowledge of beamforming is also used when designing a SWIPT system. Some of the previous work on optimising beamforming will be introduced in the literature review later.

2.2 Distributed Antenna System

Distributed antenna system technology has emerged for around more than 30 years and gains its popularity in the next generation communication because it enhances the coverage but having total power reduction and both EE and SE improvement. In the beginning, distributed antenna system was proposed to improve the coverage indoors [17]. As the cellular signal coming from a tower on the outside can get attenuated after passing through the wall of a building. The DAS is applied to clear dead spots inside the building. To construct a DAS, a repeater or some form of a signal source is placed inside the building and connected with a master unit. The master unit then is connected to the remote units with fibre optic cables that carry the RF signals throughout the building to different floors. The remote units on various floors are connected to distributed antennas through additional coaxial cables, splitters and other devices. DAS is not only used indoors but also outdoors to offer wireless coverage in the street or highway. The figure below shows the basic structure of an indoor DAS.
Then it was used in CDMA system [18]. In [17] [18] [19], the studies on DAS has results that it can have user capacity improvement and effective power saving. A DAS combining with MIMO has been proposed in [20], the system model is built by placing each remote antenna in the centre of a hexagon area. It shows that the channel capacity is large at the boundary of two antennas than at the near centre area and the whole system improves the capacity by 3dB when compare to common MIMO system. The downlink gain, other cell interference(OCI) and signal to interference and plus noise ratio(SINR) of DAS in a multicell system have been carried out in [21] since many works of uplink performance have already been researched but not much on the downlink part or in the multicell system. The system model of this paper is built by placing all base stations in the centre and around by six evenly distributed antennas so one cell contains one base station and six antennas. The system model is illustrated in the figure below.

Figure 2.3 Basic structure of indoor DAS

![Figure 2.3 Basic structure of indoor DAS](image)

Figure 2.4 DAS in multicell system structure [21]
By constructing a transmission strategy with two transmission schemes: the blanker transmission that allows signals to transmit through the cell, the single transmission selection scheme that only one antenna or home station in the cell to reduce pathloss, the SINR has been improved and OCI has been reduced. The paper shows that, when compared with a common cellular system, this system has reduced the symbol error rate and improved the capacity. In [22], the figure below is the comparison of the spectrum efficiency of the co-located antenna system and DAS.

![Figure 2.5 Comparison of CAS and DAS [22]](image)

The paper shows that if these two systems all have same antenna density, the spectrum efficiency of DAS has been proved to be better than CAS’s. In [23], it gives the relation between the ergodic capacity and signal to noise ratio and pathloss to analyse how the capacity is affected. This DAS system model is built by connecting distributed antenna units with a processing unit through optical fibres. All unit are separated by large distance so that the channel has both the small-scale and large-scale fading. This work then has been extended to [24] with optimal beamforming scheme and power requirement of each DAU. In [25] [26] [27], the optimisation of energy efficiency of DAS has been researched.

2.3 Simultaneous Wireless Information and Power Transfer

Sensor networks like the internet of things(IoTs) or mobile devices usually have a limitation on the power supply. This kind of energy constrained components is designed with either rechargeable battery or disposable battery which is at high cost and not environmentally friendly. As the knowledge of fact, the electromagnetic wave can carry both information and power. Therefore, SWIPT comprises two basic circuits: Energy harvesting and information
decoding. In order to implement SWIPT there are four architecture schemes: separated, time switching, power splitting and antenna switching receivers.

The figure a) shown in the figure above is separated receiver scheme that uses separated antennas connected to EH and ID circuits so that they both is capable to send feedback. In this way, the transmitter side is able to know the information on the receiver side so that to efficiently distribute power to receivers [29]. The top right one is time switching scheme that during $\alpha_iT$ time slot the total power is fed into the EH circuit and during $(1 - \alpha_i)T$ time slot the ID circuit is selected. The bottom left one is the power splitting scheme which used a power splitter after the antennas to divide the power into two streams that $\beta_i$ of the total power is fed in EH circuit and $1 - \beta_i$ of the total power is fed to ID circuit. The bottom right one is antenna switching scheme that antennas are divided into two subsets that one for EH circuit and the other set connected to ID circuit.

As a matter of fact, the energy and information can be carried by signal at the same time, transporting the information and power simultaneously emerged from last decade and gains increasing attention in the wireless communication field. Unlike the harvesting technology in near field using electromagnetic induction introduced in [30], the RF signal based technologies can harvest energy far from the transmitter [31]. The noticeable character of this method is that the energy can always be harvested from any kind of signals broadcasted towards a receiver, even though the signal is an interference signal and does no good to the information encoding [31].
There are many previous studies on SWIPT work on the limit of its performance. The first idea of simultaneous information and power transfer was proposed in [32] in 2008 which establish a single-input and single-output (SISO) noisy channel and gives the function to represent the trade-offs between the rate of the energy harvesting and rate of the information decoding on this line. Later in 2010, with adding the coupled circuit, the single-antenna channel has been extended to frequency selective channels in [33]. By that time, the information decoding and energy harvesting were assumed to be done simultaneously in [32] [33]. With the publication of [34] in 2013, it shows that information decoding is not able to be done independently while energy is extracting at the same time due to the circuit limitation. In [35], the author introduced SWIPT using multiple antennas with two different of types of scheme called time switching and power splitting. The time switching scheme refers to that two user terminals working collocated by switching between information decoding and energy harvesting. In other words, one user terminal performs information decoding in the current time slot and the other one harvest energy from the same signal broadcasted from the transmitter while in the next time slot, they exchange the work they did in the previous time slot. Because of the different power sensitivities of energy harvesting and information decoding, the second scheme based on dynamic power splitting strategy suggests reasonable power stream split from the received signal so that only one user terminal is working.

In [31], the paper uses a multi-user multiple input single output (MISO) system model but with zero-forcing beamforming to optimise the harvested energy. It shows that the EH performance can be improved by having the reduction in signal to interference and noise ratio. In [36], a SWIPT multicasting system also adopts MISO channel but using the power splitting scheme that is mentioned above. The figure below shows the system model.

![Figure 2.7 A MISO power splitting SWIPT system](image-url)
In [31] [36] [37], they all have very similar formulas for the MISO system as the output at the transmitter side is given as:

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{w}_k x_k$$  \hspace{1cm} E.q 2-2

With the knowledge of equation E.q 2-1, it is not difficult to understand that the received signal at $k^{th}$ receiver is given as:

$$y_k = \mathbf{h}_k^H \mathbf{w}_k s_k + n_{A,k} = \mathbf{h}_k^H \mathbf{w}_k s + n_{A,k}, \text{ for } k = 1,2,\cdots,K$$  \hspace{1cm} E.q 2-3

The signal at $k^{th}$ antenna is equal to $\mathbf{h}_k^H$, the conjugated channel vector times $s$, the data sent from the transmitter times the beamforming vector $\mathbf{w}$ plus the AWGN $n_{A,k}$. After that, $\rho_k$ portion of the power is provided to ID circuit and $(1 - \rho_k)$ of the total power is provided to EH circuit. Therefore, the signal fed to ID circuit is expressed as below:

$$y_{l,k} = \sqrt{\rho_k} (\mathbf{h}_k^H \mathbf{w}_k s + n_{A,k}) + n_{P,k}, \text{ for } k = 1,2,\cdots,K$$  \hspace{1cm} E.q 2-4

The signal before the $k^{th}$ EH circuit is given as:

$$y_{E,k} = \sqrt{1 - \rho_k} (\mathbf{h}_k^H \mathbf{w}_k s + n_{A,k}), \text{ for } k = 1,2,\cdots,K$$  \hspace{1cm} E.q 2-5

The signal to noise ratio(SNR) of the ID at $k^{th}$ antenna is given as the ratio of the power of signal divided by the power of noise, which is given as below in [36]:

$$SNR_k = \frac{\rho_k |\mathbf{h}_k^H \mathbf{w}_k|^2}{\rho_k \sigma_{A,k}^2 + \sigma_{P,k}^2} = \frac{\rho_k |\mathbf{h}_k^H \mathbf{w}_k|^2}{\sigma_k^2}$$  \hspace{1cm} E.q 2-6

In [31], the SINR ratio is given as:

$$SINR_k = \frac{\rho_k |\mathbf{h}_k^H \mathbf{w}_k|^2}{1 + \sum_{i \neq k} \rho_i |\mathbf{h}_k^H \mathbf{w}_i|^2}$$  \hspace{1cm} E.q 2-7

The energy harvested given in [36] is expressed as:

$$Y_k = \xi_k (1 - \rho_k)(|\mathbf{h}_k^H \mathbf{w}_k|^2 + \sigma_{A,k}^2)$$  \hspace{1cm} E.q 2-8

in which $\xi_k \in (0,1]$ is the energy efficiency.

One of the problem formulation given in [36] is given as below,

$$\min_{b,\{0<p_k<1\}\forall k} |\mathbf{w}_k|^2 \text{ s.t.}$$  \hspace{1cm} E.q 2-9

$$\frac{\rho_k |\mathbf{h}_k^H \mathbf{w}_k|^2}{\sigma_k^2} \geq \Gamma_k, \forall k.$$  \hspace{1cm} E.q 2-10

$$(1 - \rho_k)(|\mathbf{h}_k^H \mathbf{w}_k|^2 + \sigma_{A,k}^2) \geq \eta_k, \forall k.$$  \hspace{1cm} E.q 2-11
to guarantee a reliable information transfer by ensuring the SNR value above a threshold value and enough energy harvested by ensuring a power at each EH is within the constraint. This paper also considers perfect channel state information (CSI) and imperfect CSI.

In paper [38] [39], the SWIPT in MIMO Hybrid Analog-Digital Beamforming system is proposed and it uses an algorithm to solve the hybrid beamforming. In [40], it also studies the energy efficiency of the MIMO SWIPT which is applied to a two-way amplify-and-forward (AF) relay system. This paper also uses an iterative algorithm based on constrained concave convex procedure (CCCP) to obtain the optimal solution. The structure of the system is shown in the figure below.

![Figure 2.8 System Model of the two-way AF relays system [40]](image)

This paper gives the formula of the received signal at the very end as \( y = H_1 W_1 s_1 + H_2 W_2 s_2 + n_a \) and studies the EE optimisation problem.

3 System Model and Problem Formulation

According to the basic knowledge and literature reviews of DAS and SWIPT, DAS has benefits of larger defined coverage, less dead spots in the coverage and lower overall power with same coverage since the link availability is more frequent. A challenge when applying SWIPT is that the energy transmission decreases with the distance. However, DAS can exploit the spatial diversity and shorten the distance between transmitter and receivers. Therefore, this project is planning on studying the power optimisation of a system combining the DAS with SWIPT. The studies of energy efficiency optimisation of SWIPT in IoT distributed antenna system and in DAS solely are proposed in [41] and [42] respectively. Both of these two papers use power splitter before the receiver. They analyse the Karush–Kuhn–Tucker (KKT) conditions first and then give the formulas of the problem. After that, an algorithm to solve the problem in [42] is
carried out as shown in the figure on the left below. The simulation parameter is given in the figure on the right below.

**Algorithm 1** Optimal algorithm for problem (P1)

1. Set the channel gain as \( h_1 > h_2 > \cdots > h_N \).
2. Compute \( p_{1}^{*} \) using (17).
3. while \( i \leq N \) do
   4. if \( p_{i-1} = P_{i-1} \) then
      5. Compute \( p_{i}^{*} \) using (15) with the DA ports power obtained in previous iteration.
   6. else
   7. Set \( p_{i}^{*} = 0 \).
   8. end if
9. end while
10. Obtain \( \alpha^{*} \) in (16).

**Figure 3.1 Optimal algorithm**

**Figure 3.2 Simulation Parameters**

With those hints from the completed works introduced above, this project will be implemented in a similar study process by giving the structure of the system model in the figure below first.

**Figure 3.3 Architecture of SWIPT in the multiuser distributed antenna system**

The system considers \( N \) distributed antenna ports with single antenna and \( k \) multi-users. Some of the users requesting for energy harvesting are shown as the headphone user in the figure above and some requesting information transmission are shown as the phone user in the figure above. The distributed antenna ports are wired with a central processor.

Similar to equations of the system introduced above, the received signal at ID receiver is given as:

\[
y_k = h_k^H (\sum_{k=1}^{K} w_k s_k) + n_k, \quad \text{for} \; k = 1, 2, \cdots, K
\]

**Equation 3.1**

in which the \( h_k^H \in \mathbb{C}^{1 \times N} \) is the ID channel vector of the \( k^{th} \) user, \( w_k \in \mathbb{C}^{N \times 1} \) is the beamforming vector for the \( k^{th} \) user, \( s_k \) is the transmitted signal with information symbols and \( n_k \) is the additive Gaussian white noise. The received signal at the EH receiver is expressed as:
\[ y_{EHj} = g_j^H(\sum_{k=1}^{K} w_k s_k) + n_k, \text{ for } k = 1, 2, \ldots, K \]  

in which \( g_j^H \) is the EH channel vector of the \( k^{th} \) user.

The equations of this model are:

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} P_k \\
\text{s.t.} & \quad \text{SINR}_k \geq \Gamma_{\text{required}} \\
& \quad \sum_{k=1}^{K} P_{EHj} \geq P_{\text{min}}
\end{align*}
\]

The transmitted power \( P_k \) is expressed as:

\[ P_k = ||w_k||^2 \]

The SINR is the ratio of signal power divided by interference signal power plus noise power:

\[ \text{SINR}_k = \frac{S_k}{l_j + \Delta_k} \]

in which \( l_k = \sum_{n \neq k} l_{nk} \) is the sum of interference signal power from transmitters other than \( k^{th} \) transmitter. The signal power and interference signal power are given as:

\[ S_k = |h_k^H w_k|^2 \]

\[ l_j = \sum_{j \neq k} |h_k^H w_j|^2 \]

The power harvested by \( k^{th} \) user is given by:

\[ P_{EHj} = |g_j^H w_k|^2 \]

The conditions are updated to:

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} ||w_k||^2 \\
\text{s.t.} & \quad \frac{|h_k^H w_k|^2}{\sum_{j \neq k} |h_k^H w_j|^2 + \sigma^2} \geq \Gamma_{\text{required}} \\
& \quad \sum_{j=1}^{K} |g_j^H w_k|^2 \geq P_{\text{min}}
\end{align*}
\]
4 Optimisation Solution

To solve this problem, the convex optimisation will be introduced in this section. Convex optimisation or convex programming is the subfield of mathematical optimisation. The mathematical optimisation has few well-known types: convex programming, non-linear programming, quadratic programming and etc. Convex optimisation is about to solve minimizing a convex function with a convex set [43]. The linear programming and the least squares are two widely known subtypes of convex programming. Linear programming (LP) is the special case of convex programming that all constraints are linear functions. LP is not able to be solved by easy analytical equations but using some effective algorithms such as Dantzig’s simplex method. The convex programming can be classified as problems without constraints or with constraints of equalities and inequalities. The problem (the equation that stands for minimising power) without constraints can be solved by setting the equation of the problem to zero. Any problem that has constraints of equality can be recast to equivalent non-constrained problem so that it can be solved by setting the problem to zero again. Inequality constrained problem is solved with an interior-point method by applying Newton’s method to convert unequal constraints to constraints of equality [44]. Some problems are called hidden convex problem that the formula of the problem is a non-convex function but can be transformed to a convex programming.

The problem which is given in the previous section have inequality constraints. To solve this kind of problem with inequality constraints, a method called convex optimisation will be applied. Here is a standard form of solution. Consider a nonlinear optimisation problem that has the form as below:

Optimise \( f(x) \)

Subject to

\[
g_i(x) \leq 0, \quad i = 1, 2, \ldots, m
\]
\[
h_i(x) = 0, \quad i = 1, 2, \ldots, m
\]

Where \( x \) is the optimisation variables and \( f \) is the objective problem which could be minimisation or maximisation. The KKT multipliers \( \mu_i (i = 1, 2, \ldots, m) \) and \( \lambda_i (i = 1, 2, \ldots, m) \) are introduced for finding \( x \) that satisfy some regular conditions which are shown as below.

For minimisation:

\[
-\nabla f(x^*) = \sum_{i=1}^{m} \mu_i \nabla g_i(x^*) + \sum_{j=1}^{m} \lambda_i \nabla h_j(x^*)
\]

For maximisation:

\[
\nabla f(x^*) = \sum_{i=1}^{m} \mu_i \nabla g_i(x^*) + \sum_{j=1}^{m} \lambda_i \nabla h_j(x^*)
\]

Defining an auxiliary function as:
\( \mathcal{L}(x, \lambda) = f(x) + \sum_{i=1}^{m} \mu_i g_i(x) + \sum_{j=1}^{m} \lambda_j h_j(x) \)

The two terms can be either added or subtracted. This function is called Lagrangian function and transfers the problem with constraints to none-constraint problem. The optimal solution and values \( \mu \) and \( \lambda \) could be found by taking differentiation with respect to \( x \) on both sides. However, this method can only be applied to problem and conditions that are all convex functions. In case of this project, the formulations contain vectors, matrices and even non-convex term. Therefore, a method called semidefinite programming (SDP) relaxation will be applied to the formulations first [45] [46] [47]. The semidefinite programming is a subfield of convex optimisation and is used for problem having context of matrix and inequalities.

Since,

\[ \|w_k\|^2 = w^Hw, \forall k \in [1,K] \]
\[ = \text{Tr}(w^Hw) \]
\[ = \text{Tr}(ww^H) = \text{Tr}(W_k) \]
\[ \|w_k\|^2 = w^Hw, \forall k \in [1,K] \iff \text{Rank}(W_k) = 1 \]

The problem can be converted to a linear form and with two more constraints:

\[
\begin{align*}
\text{minimize} & \sum_{k=1}^{K} \text{Tr}(W_k) \\
\text{s.t.} & \quad \frac{\text{Tr}(H_k W_k)}{\sum_{j \neq k} \text{Tr}(H_k W_k) + \sigma^2} \geq \Gamma_{\text{req}} \\
& \quad \sum_{j=1}^{K} \text{Tr}(G_j W_k) \geq P_{\text{min}}, \forall j \in [1,K] \\
& \quad -W_k \leq 0, \forall k \in [1,K] \\
& \quad \text{Rank}(W_k) \leq 1, \forall k \in [1,K]
\end{align*}
\]

The fourth constraint can be removed for more optimal solutions. Rearrange to standard form, the problem becomes to:

\[
\begin{align*}
\text{minimize} & \sum_{k=1}^{K} \text{Tr}(W_k) \\
\end{align*}
\]
s.t. C1:  
\[ \Gamma_{req} \left[ \sum_{j \neq k} Tr(H_k W_k) + \sigma^2 \right] - Tr(H_k W_k) \leq 0 \]

C2:  
\[ P_{min} - \sum_{j=1}^{K} Tr(G_j W_k) \leq 0, \forall j \in [1, K] \]

C3:  
\[ -W_k \leq 0, \forall k \in [1, K] \]

Now obtain the Lagrangian function as:

\[ L = \sum_{k=1}^{K} Tr(W_k) + \lambda_1 \left[ \Gamma_{req} \left( \sum_{j \neq k} Tr(H_k W_k) + \sigma^2 \right) - Tr(H_k W_k) \right] \]

\[ + \sum_{j=1}^{J} \mu_j \left[ P_{min} - \sum_{j=1}^{K} Tr(G_j W_k) \right] - \sum_{k=1}^{K} Tr(Y_k W_k) \]

Solving the \( \frac{\partial L}{\partial W_k} = 0 \),

\[ \frac{\partial L}{\partial W_k} = I_k - \lambda_1 H_k + \sum_{j \neq k} \lambda_j \Gamma_{req} H_j - \sum_{j=1}^{J} \mu_j G_j - Y_k = 0 \]

where \( Y_k \) is a dual variable that is related to the third constraint.

Therefore,

\[ Y_k = -\lambda_1 H_k + I_k + \sum_{j \neq k} \lambda_j \Gamma_{req} H_j - \sum_{j=1}^{J} \mu_j G_j \]

As it can be seen that \( Y_k \) can be rearrange to a form as

\[ Y_k = -a H_k + B \quad \text{E.q 4-1} \]

where \( a = \lambda_1 \) and \( B = I_k + \sum_{j \neq k} \lambda_j \Gamma_{req} H_j - \sum_{j=1}^{J} \mu_j G_j \).

To prove that the fourth condition listed above is removable, suppose that \( B \) is a positive semidefinite matrix, then there will be a vector \( V \) that has one eigenvalue. Assume \( V = \nu \nu^H \) and multiply E.q 4-1 by \( U \) on the both sides, the equation becomes to

\[ Tr(Y_k V) \leq -Tr(H_k V) + Tr(BV) \]

Since the statistically independent property of \( H_k \) and \( \sum_{j \neq k} H_j \), here exists that \( Tr(Y_k V) < 0 \) and \( Tr(H_k V) > 0 \). Therefore, the matrix \( B \) must be a positive semidefinite matrix and its rank has a value of \( N_T \).
Since the subadditivity property of rank \( \text{rank}(a + b) \leq \text{rank}(a) + \text{rank}(b) \), replace \( a \) with \( Y_k \) and \( b \) with \( H_k \), it becomes to
\[
\text{rank}(Y_k) \geq \text{rank}(B) + \text{rank}(H_k) = N_T - 1
\]
Hence, there exists \( \text{Rank}(W_k) \leq 1 \) at same time.

### 5 Simulation Results

Before the simulation, there are few assumptions list below:

- The TGn path loss model [48] for indoor communication is adopted with directional transmit and receive antenna gains of 10 dB
- The reference distance of the path loss model is 2 metres and there are \( K \) receivers uniformly distributed between the reference distance and the maximum service distance of 10 metres
- The system bandwidth is \( B=200kHz \)
- A carrier centre frequency of 470MHz

Using the KKT conditions in the box below,

| C1: & C2: & C3: |
|---|---|---|
| \( \text{minimize} \sum_{k=1}^{K} \text{Tr}(W_k) \) & \( \text{C1:} \quad \Gamma_{req} \sum_{j \neq k} \text{Tr}(H_k W_k) + \sigma^2 \) & \( \text{C3:} \quad -W_k \leq 0, \forall k \in [1,K] \) |
| s.t. & \( \text{Tr}(H_k W_k) \leq 0, \forall j \in [1, K] \) & |

Here is plot of average total transmitted power to the minimum required SNIR when the number of antennas is 3, 6 and 9.
As the minimum required SNIR increases, the average the transmitted power provided to the system increases. As the number of antennas increases, the power provided decreases since the beamforming of each antenna becomes narrower and more focusing and the distance between two adjacent lines will become narrower since the channel hardening.
With fixed number of information receiver, when the number of the energy harvesting receivers increases, the transmitted power increases. As the number of antennas increases, the transmitted power also decreases.

6 Conclusion

In this project, the SWIPT is integrated with a distributed antenna and the transmitted power provided to the system is analysed by using convex optimisation to evaluate the system formulations. The main challenge of the optimisation part is to convert the original formulation to a form that is associated with the trace of the matrix. Prove the rank of beamforming matrix $W$ is less and equal to 1 under the situation that $H_K, G_j$ are statistically independent. As a result, the power increases when minimum required SNIR increases and the number of EH receivers increases but decreases when the number of antennas increases. Therefore, the number of antennas is an important factor when design such a system as the more antenna is installed, the more cost is spent.
7 Bibliography


