Asymptotic Tradeoff between Cross-Layer Goodput Gain and Outage Diversity in OFDMA Systems with Slow Fading and Delayed CSIT

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***Abstract*— In this paper, we consider on the asymptotic tradeoff analysis between the *system goodput gain* and the *packet outage diversity gain* in cross-layer OFDMA systems with slow frequency selective fading and delayed CSIT. The OFDMA cross-layer design with delayed CSIT is modeled as an optimization problem where the rate adaptation, power adaptation and subcarrier allocation policies are designed to optimize the system goodput (b/s/Hz successfully received by the mobiles). We derived simple closed-form expressions for the power and rate allocations as well as the asymptotic order of growth in system goodput for general CSIT error σ_e^2. We found that the *system goodput* scales in the order of $\mathcal{O}(\log(\frac{1-\sigma_e^2}{N_d}(\log K + N_d \log\log K)))$ for large K where N_d is the *packet outage diversity* and K is the number of user in the cross-layer OFDMA system. Hence, *double exponentially* larger K is needed to compensate for the penalty in system goodput gain due to CSIT errors σ_e^2 or packet outage diversity N_d for large N_d.**

I. INTRODUCTION

In multiuser OFDMA systems, it is well-known that cross-layer scheduling (by selecting a set of users with the best channel condition for each subcarrier) can substantially increase the system spectral efficiency due to multiuser diversity gain (MuDiv) on system throughput. The *system throughput gain* is the first important aspect in cross-layer designs which is widely known and studied. However, in all these works, the channel state knowledge at the base station (CSIT) is assumed to be perfect. With perfect CSIT, packet errors can be ignored even in slow fading channels by careful rate adaptation as well as applying strong channel coding for the transmitted packets. Hence, system performance is usually evaluated based on *ergodic capacity*. In [1], it is shown that system throughput (ergodic capacity) in cross-layer systems scales with $\mathcal{O}(\log\log K)$ for multi-users systems with perfect knowledge of CSIT at the base station where K is the number of users in the system.

However, in practice, the CSIT can never be perfect due to either the CSIT estimation noise in Time Division Duplex (TDD) systems or the outdate of CSIT due to feedback delay. When the CSIT is imperfect, there will be potential packet transmission error due to channel outage (packet outage). The instantaneous mutual information is not known precisely at the base station due to delayed CSIT, so even applying powerful error correction coding cannot prevent packet error from occurring. Therefore, there is a finite probability that the scheduled data rate exceeds the instantaneous mutual information, causing the transmitted packet to be corrupted. Hence, conventional performance measure by *ergodic capacity* fails to account for the penalty of packet outage. The cross-layer design with delayed CSIT is a relatively new topic. In [2], cross-layer scheduling for OFDMA systems is analyzed using limited feedback in the CSIT. The authors also show that system throughput scales in the order of $\mathcal{O}(\log\log K)$ with one bit feedback. In [3], an opportunistic scheduling approach is proposed with rate feedbacks from the mobiles. However, in all these cases, due to the perfect (but partial) feedback[1] assumption, packet error (packet outage) is not an issue as long as the error correction code is sufficiently strong and hence, these works also considered ergodic capacity as the performance objective.

In reality, with *delayed CSIT* in slow fading channels, packet outage is a key issue and must not be ignored in the cross-layer design or performance analysis. In this case, the cross-layer *packet outage diversity* is important to protect the packet errors due to channel outage and there is a natural tradeoff between the *system goodput gain* and *packet outage diversity* in cross-layer systems. In [4], the authors established a theoretical framework for the fundamental tradeoff between spatial diversity and spatial multiplexing gain in point-to-point MIMO systems. In [5], the authors extended the framework to consider multiuser (uplink) systems. However, in all these works, no knowledge of CSIT is assumed at the base station. Furthermore, flat fading channel is considered and hence, the results cannot be applied in our case with delayed CSIT and frequency selective fading channels.

In general, it is not clear how the asymptotic system goodput gain in cross-layer OFDMA system and the packet outage diversity tradeoff with each other. It is also not clear about how would the system goodput be affected by CSIT errors. In this paper, we shall focus on the asymptotic tradeoff analysis between the *system goodput gain* and the *packet outage diver-*

[1]Partial feedback here refers to the limited feedback. Perfect feedback here refers to the assumption that there is no feedback errors or feedback delay in the limited feedback.

sity gain in cross-layer OFDMA systems with slow frequency selective fading and delayed CSIT. The OFDMA cross-layer design with delayed CSIT is modeled as an optimization problem where the rate adaptation, power adaptation and subcarrier allocation policies are designed to optimize the system goodput (b/s/Hz successfully received by the mobiles). Simple closed-form expressions for the power and rate allocations as well as the asymptotic order of growth in system goodput for general CSIT error σ_e^2 and packet outage diversity order N_d are derived. It is found that *double exponentially* larger K is needed to compensate for the penalty in system goodput gain due to CSIT errors σ_e^2 or packet outage diversity N_d for large N_d.

The rest of the paper is organized as follows. Section II outlines the frequency selective fading model, the CSIT error model as well as the packet outage and system goodput. Section III presents the formulation of the cross-layer design as an optimization problem, and the derivation of closed-form solution for rate and power adaptation as well as a low-complexity subcarrier assignment policy. Section IV provides the derivation of the asymptotic tradeoff between system goodput and packet outage diversity for large number of users. Section V concludes with a summary of results.

II. SYSTEM MODEL AND CAPACITY

A. Notation

In this paper, the following conventions are adpoted. $\mathbf{X}$ denotes a matrix and $\mathbf{x}$ denotes a vector. $\mathbf{X}^\dagger$ denotes matrix transpose and $\mathbf{X}^H$ denotes matrix hermitian. " $\doteq$ " denotes *exponential equality*. Specifically, $f(x) \doteq g(x)$ with respect to the limit $x \to a$ if $\lim_{x\to a} \frac{\log f(x)}{\log g(x)} = 1$. " $\dot{\geq}$ " and " $\dot{\leq}$ " are defined in similar manner. Finally, $\Theta(.)$ denotes the *asymptotic tight bound* and $\mathcal{O}(.)$ denotes the *asymptotic upper bound*.

B. Frequency Selective Fading Model

A downlink transmission in OFDMA system is considered. The channel is assumed to be time-invariant, frequency selective channel model. The numbers of resolvable paths are approximately $L = \left\lfloor \frac{W}{\Delta f_c} \right\rfloor$, where W is the signal bandwidth and Δf_c is the coherence bandwidth. For simplicity, we assume uniform power-delay profile so that each path has normalized power given by $1/L$. Consider a time-invariant L-tap delay line channel model, the channel impulse response between the base station and the k-th user is given by:

$$h(\tau; k) = \sum_{n=0}^{L-1} h_{k,n} \delta(\tau - \frac{n}{W}) \tag{1}$$

where $\{h_{k,n}\}$ are modeled as independent identically distributed (i.i.d.) complex Gaussian circularly symmetric random variables with zero mean and variance $\frac{1}{L}$. Therefore, the received signal of the k-th user can be represented as the follow:

$$y_k(t) = \sum_{n=0}^{L-1} h_{k,n} x(t - \frac{n}{W}) + z(t) \tag{2}$$

where $x(t)$ is the transmitted signal from the base station and $z(t)$ is complex white Gaussian noise with density N_0.

Using n_F-point IFFT and FFT in the OFDMA system, the equivalent discrete channel model in the frequency domain (after removing the cyclic prefix with length L) is:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{z}_k \tag{3}$$

where $\mathbf{x}$ and $\mathbf{y}_k$ are $n_F \times 1$ transmit and receive vectors and $\mathbf{z}_k$ is the $n_F \times 1$ i.i.d. complex Gaussian channel noise vector with zero mean and normalized covariance $E[\mathbf{z}_k \mathbf{z}_k^H] = 1/n_F$ (so that the total noise power across the n_F subcarriers is unity). $\mathbf{H}_k$ is the $n_F \times n_F$ diagonal channel matrix between the base station and the k-th user $\mathbf{H}_k = diag\left[H_{k,0}, ..., H_{k,n_F-1}\right]$, where $H_{k,m} = \sum_{l=0}^{L-1} h_{k,l} e^{\frac{-j2\pi lm}{n_F}}, \forall m \in \{0, ..., n_F - 1\}$ are the FFT of the time-domain channel taps $\{h_{k,0}, ..., h_{k,L-1}\}$. Since $H_{k,m}$ is a linear combination of Gaussian random variables, $\{H_{k,0}, .., H_{k,n_F-1}\}$ are circularly symmetric complex Gaussian random variables with zero mean and the correlation between $H_{k,m}$ and $H_{k,n}$ is

$$E\left[H_{k,m} H_{k,n}^H\right] = \frac{1}{L} \frac{1 - e^{\frac{-2j\pi L(m-n)}{n_F}}}{1 - e^{\frac{-2j\pi(m-n)}{n_F}}} = \eta_{k,m,n}. \tag{4}$$

Observe that $\eta_{k,m,n} = 0$ when $(m-n)L$ is integer multiple of n_F. Hence, we can divide $\{H_{k,0}, .., H_{k,n_F-1}\}$ into $L_s = n_F/L$ groups, where each group has L i.i.d. elements, as follows:

$$\underbrace{\begin{bmatrix} H_{k,0} \\ H_{k,L_s} \\ \vdots \\ H_{k,(L-1)L_s} \end{bmatrix}}_{\mathbf{H}_{k,0}} \underbrace{\begin{bmatrix} H_{k,1} \\ H_{k,L_s+1} \\ \vdots \\ H_{k,(L-1)L_s+1} \end{bmatrix}}_{\mathbf{H}_{k,1}} \cdots \underbrace{\begin{bmatrix} H_{k,L_s-1} \\ H_{k,2L_s-1} \\ \vdots \\ H_{k,LL_s-1} \end{bmatrix}}_{\mathbf{H}_{k,L_s-1}}.$$

In other words, there are L independent subbands (labelled as $j = 0, 1, 2, ..., L-1$) in the n_F-subcarriers with L_s correlated subcarriers in each subband.

C. CSIT error model

For simplicity, we consider TDD systems (with channel reciprocity) and assume the CSIR is perfect but the CSIT is outdated. The estimated CSIT (time domain) at the base station for the k-th user is given by:

$$\hat{h}_{k,l} = h_{k,l} + \Delta h_{k,l} \quad \Delta h_{k,l} \sim CN(0, \sigma_e^2) \;\; l \in \{0, 1, .., L-1\}.$$

Hence, the estimated CSIT in frequency domain (m-th subcarrier) $\hat{H}_{k,m}$ after n_F-point FFT of $\{\hat{h}_{k,0}, ..., \hat{h}_{k,L-1}\}$ is given by:

$$\hat{H}_{k,m} = H_{k,m} + \Delta H_{k,m} \quad \Delta \mathbf{H}_{k,m} \sim CN(0, \sigma_e^2) \tag{5}$$

where $H_{k,m}$ is the actual CSIT of the m-th subcarrier for the k-th user, $\Delta H_{k,m}$ represents the CSIT error which is circular symmetric complex Gaussian (CSCG) random variable with zero mean and variance σ_e^2. The correlation of the CSIT error between the m-th and n-th subcarriers of user k is given by:

$$E\left[\Delta H_{k,m}\Delta H_{k,n}^H\right] = \frac{\sigma_e^2}{L}\frac{1-e^{\frac{-2j\pi L(m-n)}{n_F}}}{1-e^{\frac{-2j\pi(m-n)}{n_F}}}. \tag{6}$$

Finally, the CSI between the K users are i.i.d..

D. Instantaneous Mutual Information and System Goodput

Let B_k denotes the set of subband indices $m = \{0, 1, ..., L-1\}$ assigned to the k-th user. The instantaneous mutual information between the base station and the k-th mobile (given the CSIR $\mathbf{H}_k$) is given by:

$$C_k(\mathbf{H}_k) = \sum_{n=0}^{L_s-1}\sum_{m\in B_k}\log_2\left(1+\frac{n_F p_k\left|H_{k,mL_s+n}\right|^2}{L_s N_d}\right) \tag{7}$$

where L_s is the number of correlated subcarriers in one subband, N_d is the number of independent subbands allocated to the $k-$th user (for diversity protection for packet outage) and p_k is the transmit power allocated to the k-th user. In generally, packet error is contributed by two factors, namely channel noise and the channel outage. In the former case, as long as we can provide sufficient strong channel coding (e.g. LDPC) with sufficiently long block length (e.g. 10Kbytes) to protect the information, it can be shown [6] that Shannon's capacity can be approached to within 0.04 dB for a target FER of 10^{-6}. Hence, packet errors due to the first factor is practically negligible. On the other hand, the channel outage effect is systematic and cannot be eliminated by simply using strong channel coding. This is because the instantaneous mutual information[2] $C_k(\mathbf{H}_k)$ between the base station and k-th user is a function of actual CSI $\mathbf{H}_k$, which is unknown to the base station. Hence, the packet will be corrupted whenever the scheduled data rate r_k exceeds the instantaneous mutual information C_k. In order to account for potential packet errors, we shall consider the *system goodput* (b/s/Hz successfully delivered to the mobile station) as our performance measure. We assume the packet errors are due to channel outage (scheduled data rate r_k exceeds the instantaneous mutual information). The *average total goodput* is given by:

$$U_{goodput}(\mathcal{A},\mathcal{B},\mathcal{P},\mathcal{R}) = \frac{1}{n_F}E_{\hat{\mathbf{H}}}\left\{\sum_{k=1}^{K} r_k \Pr[r_k \le C_k|\hat{\mathbf{H}}]\right\} \tag{8}$$

where $\mathcal{R} = \{r_1, ..., r_K\}$ is the rate allocation policy, $\mathcal{P} = \{p_1, ..., p_K : \sum_k p_k \le P_0\}$ is the power allocation policy , $\{\mathcal{A}\}$ is the user selection policy with respect to the outdated CSIT $\hat{\mathbf{H}}$, $\{\mathcal{B}\}$ is the subband allocation policy with respect to N_d independent subbands.

III. Cross-Layer Design for OFDMA Systems

The optimal power allocation policy $\mathcal{P}^*$, rate allocation policy $\mathcal{R}^*$, user selection policy $\mathcal{A}^*$ and subband allocation policy $\mathcal{B}^*$ are given by:

$$(\mathcal{P}^*,\mathcal{R}^*,\mathcal{A}^*,\mathcal{B}^*) = \arg\max_{\mathcal{P},\mathcal{R},\mathcal{A},\mathcal{B}} U_{goodput}(\mathcal{A},\mathcal{B},\mathcal{P},\mathcal{R})$$

such that

$$\begin{aligned} &P_{out}(k,\hat{\mathbf{H}}) \\ &= \Pr(r_k > \sum_{n=0}^{L_s-1}\sum_{m\in B_k}\log_2(1+\frac{n_F p_k}{L_s N_d}|H_{k,mL_s+n}|^2)|\hat{\mathbf{H}}) \\ &= \epsilon \end{aligned} \tag{9}$$

where L_s is the number of correlated subcarriers in one subband. The key to solve the above optimization problem is on the modeling of the conditional packet outage probability $P_{out}(k,\hat{\mathbf{H}})$. Using similar approach as in [7], we have $P_{out}(k,\hat{\mathbf{H}}) = \Theta\left(F_{\chi_k^2;s^2(B_k);\sigma_e^2/N_d}\left(\frac{(2^{\frac{r_k}{L_s N_d}}-1)L_s N_d}{n_F p_k}\right)\right)$ for high SNR where $F_{\chi_k^2;s^2(B_k);\sigma_e^2/N_d}(x)$ is the cdf of non-central chi-square random variable χ_k^2 with $2N_d$ degrees of freedom, non-centrality parameter $s^2(B_k) = \frac{1}{N_d}\sum_{m\in B_k}|\hat{H}_{k,mL_s+n}|^2$ and variance $\frac{\sigma_e^2}{N_d}$.

A. Rate and Power Allocation Solution

The target packet outage constraint in (9) for high SNR is equivalent to the following:

$$r_k = L_s N_d \log_2\left(1+\frac{n_F p_k}{N_d L_s}F^{-1}_{\chi_k^2;s^2(B_k);\sigma_e^2/N_d}(\epsilon)\right). \tag{10}$$

Using (10) and taking into consideration of the total transmit power constraint P_0, the Lagrangian function of the optimization problem in (9) is given by:

$$\begin{aligned} &L(\{p_k\},\lambda) \\ &= \frac{(1-\epsilon)L_s N_d}{n_F}\sum_{k\in A}\log_2\left(1+\frac{n_F p_k}{N_d L_s}F^{-1}_{\chi_k^2;s^2(B_k);\sigma_e^2/N_d}(\epsilon)\right) \\ &-\lambda(\sum\nolimits_{k\in A}p_k - P_0) \end{aligned} \tag{11}$$

where $\lambda > 0$ is the Lagrange multiplier with respect to the total transmit power constraint. Using standard optimization techniques, the optimal power allocation and optimal rate allocation are given by (for all $k \in A(\hat{\mathbf{H}})$):

$$p_k^* = \frac{L_s N_d}{n_F}\left(\frac{1-\epsilon}{\lambda}-\frac{1}{F^{-1}_{\chi_k^2;s^2(B_k);\sigma_e^2/Nd}(\epsilon)}\right)^+$$

$$r_k^* = \left[L_s N_d\log_2\left(\frac{(1-\epsilon)F^{-1}_{\chi_k^2;s^2(B_k);\sigma_e^2/Nd}(\epsilon)}{\lambda}\right)\right]^+.$$

B. User and Subcarrier Allocation Solution

The combinatorial search for A and $\{B_k\}$ are coupled among the n_F subcarriers due to the constraint that each B_k should contain N_d independent subbands. As a result, we shall propose a low complexity *greedy* combinatorial search algorithm to obtain the admitted user set A^* and the subcarrier allocation sets $\{B_k^*\}$. The proposed algorithm is shown to achieve close-to-optimal performance by numerical simulation in figure 2. The algorithm is summarize as follow:

[2]The instantaneous mutual information represents the maximum achievable data rate for error free transmissions.

Step 1: Initialize $A^* = \emptyset$, $B_k^* = \emptyset$, a user selection list $A_{selection}$ which include all user indices and a subband selection list $B_{selection}$ which include all independent subband indices.

Step 2: Initialize a temporary list T_k for all user in $A_{selection}$ to store subband indices.

$$T_k = \arg \max_{|T_k|=N_d} \left(\sum_{m \in B_{selection}} |\widehat{\mathbf{H}}_{k,mL_s}|^2 \right)$$

Step 3: Select user $k = \arg \max_{k \in A_{selection}} \left(\sum_{m \in T_k} |\widehat{\mathbf{H}}_{k,mL_s}|^2 \right)$.

Step 4: Put the selected users into set A^* and the corresponding allocated subbands into set B_k^*.

Step 5: Remove the selected users and the selected subbands from $A_{selection}$ and $B_{selection}$ and repeated step 2 until all the independent subbands are allocated to users.

IV. Asymptotic Tradeoff Analysis for High SNR

We shall first introduce the following important lemma based on *ordered statistics*.

Lemma 1 (Extreme Value Theorem): Let $\{Q_1, ..., Q_K\}$ be a set of K i.i.d. central chi-square random variables with $2n$ degrees of freedom and variance σ_q^2 and $Q^* = \max_k Q_k$. We have $\Pr\left[Q^* = \Theta\left(\sigma_q^2 \left(\log K + n \log\log K\right)\right)\right] \geq \Theta\left(\frac{1}{\log K}\right)$ for large K.

Proof 1: Please refer to appendix A.

As a result, the average system goodput is given by:

Theorem 1 (Asymptotic System Goodput for High SNR):

$$\begin{aligned} &\mathcal{E}_{\widehat{\mathbf{H}}}[G^{**}_{goodput}(\widehat{\mathbf{H}})] \\ &= \mathcal{O}\left[(1-\epsilon)\log_2\left(F^{-1}_{\chi_k^2;\widetilde{s}^2;\sigma_e^2/N_d}(\epsilon)P_0\right)\right]. \end{aligned} \quad (12)$$

Proof 2: Please refer to appendix B.

A. Frequency Diversity at Small Target Packet Outage Probability ϵ

It can be shown that for a given s^2, the inverse cdf of non-central chi-square X is given by $F_X^{-1}(\epsilon) \doteq \epsilon^{1/N_d}\sigma_X^2(N_d!)^{1/N_d}\exp\left(\frac{s^2}{N_d\sigma_X^2}\right)$ for small ϵ. As a result, using (12), the average packet outage probability $\overline{P_{out}(k)}$ scales with the SNR P_0 (at a given average goodput) in the order of:

$$\overline{P_{out}(k)} = E_{\widehat{\mathbf{H}}}\left[P_{out}(k,\widehat{\mathbf{H}})\right] = \mathcal{O}\left(P_0^{-N_d}\right) \quad (13)$$

for sufficiently small ϵ. Hence, N_d diversity order protection is achieved on packet outage.

B. Cross-Layer Goodput Gains at Large K

For a given ϵ, the inverse cdf of X can be expressed as $F_X^{-1}(\epsilon) \doteq \mathcal{O}(s^2)$ asymptotically for large s^2. As $s^2(B_k) = \frac{1}{N_d}\sum_{m\in B_k}|\hat{H}_{k,mL_s+n}|^2$ is central chi square distributed with $2N_d$ degree of freedom. Hence, by using the Lemma 1 and

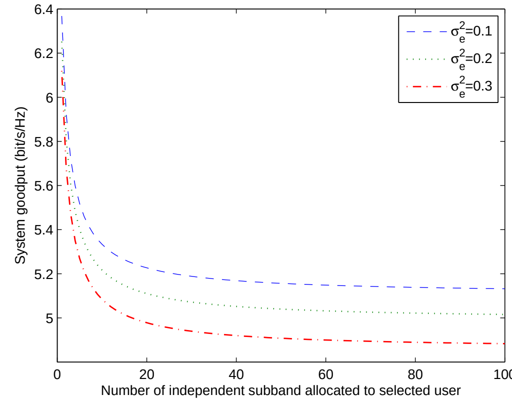

Fig. 1. Asymptotic system goodput versus packet outage diversity order N_d.

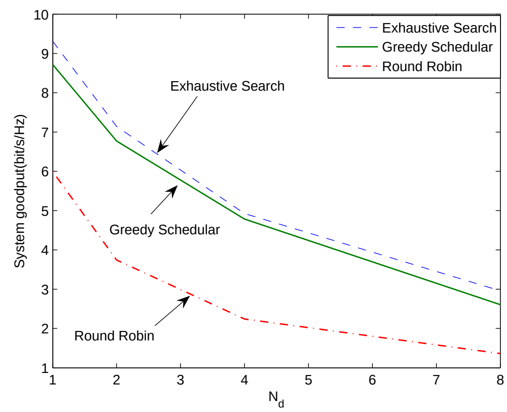

Fig. 2. A comparison of the average system goodput versus number of order diversity with different scheduler at High SNR=20 dB, $\sigma_e^2 = 0.01$, $N_F = 1024$

replacing σ_q^2 by $\frac{1-\sigma_e^2}{N_d}$, the cross-layer goodput gain in (12) reduces to:

$$\begin{aligned} &E[G^{**}_{goodput}(\hat{\mathbf{H}})] \quad (14) \\ &= \mathcal{O}\left[(1-\epsilon)\log\left(P_0\frac{1-\sigma_e^2}{N_d}\left(\log K + N_d\log\log K\right)\right)\right]. \end{aligned}$$

Comparing with the well-known cross-layer throughput gain of $\mathcal{O}(\log\log K)$, we observe that for large N_d, the cross-layer goodput gain (at diversity order of N_d) is reduced to $\mathcal{O}(\log\log\log K)$ because for large N_d, the fluctuation of the aggregate CSI of each user (sum of $2N_d$ i.i.d. random variables) is reduced substantially. Hence, *double-exponentially* larger number of users are needed to compensate for the loss in cross-layer goodput gain due to N_d diversity or CSIT error σ_e^2. Figure 1 shows the asymptotic system goodput versus the number of independent subbands allocated to user with different CSIT errors σ_e^2 at ϵ=0.01 and P_0=20 dB. For large N_d, the cross-layer goodput gain is decreased substantially (scales in the order of $\mathcal{O}(\log\log\log K)$ instead of the conventional $\mathcal{O}(\log\log K)$). On the other hand, the average packet outage probability scales in the order of $\mathcal{O}(P_0^{-N_d})$. From these results, we can deduce that there is a natural tradeoff between packet outage diversity order N_d and the cross-layer goodput gain.

V. Conclusion

In this paper, we explore the asymptotic trade-off between cross-layer goodput gain and packet outage in OFDMA downlink system, with delayed CSIT in slow fading frequency selective channel. We formulate the cross-layer design as a mixed convex and combinational optimization problem. Due to the delayed CSIT, it is critical to account for potential packet errors (due to channel outage) and we consider total system goodput as our optimization objective. By allocating N_d independent subbands to a user, the packet outage probability drops in the order of SNR^{-N_d}. On the other hand, the *system goodput* scales in the order of $\mathcal{O}(\log\left(\frac{1-\sigma_e^2}{N_d}\left(\log K + N_d \log\log K\right)\right)$ for large K in high SNR. Hence, *double exponentially* larger K is needed to compensate for the penalty in system goodput gain due to packet outage diversity N_d or CSIT errors σ_e^2 for large N_d.

Appendix

Appendix A: Proof of Lemma 1

Consider a sequence of i.i.d. random variable x_k, having central chi-square distribution with degree of freedom $2n$. Each x_k is characterized by the CDF of $F(x) = 1 - e^{-\frac{x}{\sigma^2}} \sum_{m=0}^{n-1} \frac{1}{m!}\left(\frac{x}{\sigma^2}\right)^m$; the PDF of $f(x) = \frac{1}{\sigma^{2n}\Gamma(n)} x^{n-1} e^{-\frac{x}{\sigma^n}}, x \geq 0$, where σ^2 is the variance of the complex Gaussian random variables. Define the growth function $g(x) = \frac{1-F(x)}{f(x)}$ and we have

$$\lim_{x\to\infty} g(x) = 1. \tag{15}$$

From [8] and [9], we have the following expression

$$\begin{aligned} &\log[-\log F^K(b_K + y g(b_K))] \\ &= -y + \frac{y^2}{2!} g'(b_K) + \frac{y^3}{3!}[g(b_K) g^{(2)}(b_K) - 2g'^2(b_K)] \\ &+ ... + \frac{e^{-y} + ...}{2K} + \frac{5e^{-2y+...}}{2K} + ... - \frac{e^{-3y}}{8K^3} + ... + ... \end{aligned} \tag{16}$$

where b_K is given by $F(b_K) = 1 - \frac{1}{K}$, i.e. $e^{-\frac{b_K}{\sigma^2}} \sum_{m=0}^{n-1} \frac{1}{m!}\left(\frac{b_K}{\sigma^2}\right)^m = \frac{1}{K}$.

In the other words, b_K is the solution of $\frac{b_K}{\sigma^2} - \log \sum_{k=0}^{n-1} \frac{1}{m!}\left(\frac{b_K}{\sigma^2}\right)^m = \log K$.

So

$$\begin{aligned} &\frac{b_K}{\sigma^2} - \log\left(\frac{1}{(n-1)!}\left(\frac{b_K}{\sigma^2}\right)^{n-1}\right) \\ &-O\left(\log\left(\frac{1}{(n-2)!}\left(\frac{b_K}{\sigma^2}\right)^{n-2}\right)\right) \\ &\doteq \log K. \end{aligned} \tag{17}$$

Then the equation is equivalent to:

$$\frac{b_K}{\sigma^2} - (n-1)\log\left(\frac{b_K}{\sigma^2}\right) - (n-2)\,O\left(\log\left(\frac{b_K}{\sigma^2}\right)\right) \doteq \log K.$$

Thus, $b_K = \sigma^2\left(\log K + (n-1)\log\log K + O\left(\log\log\log K\right)\right)$ as it satisfy the above equation. Noticing the CDF of $\tilde{x} = \max_{1\leq k\leq K} x_k$ is given by $F^K(\tilde{x})$, then substituting y as $\pm\log\log K$ in equation (16) and from equation (15),

$$\Pr\left\{-\log\log K \leq \max_{1\leq k\leq K} x_k - b_K \leq \log\log K\right\} \geq 1 - O\left(\frac{1}{\log K}\right).$$

Appendix B: Proof of Theorem 1

Given the CSIT $\widehat{\mathbf{H}}$, the conditional average goodput of the k-th user ($k \in A^*(\widehat{\mathbf{H}})$) for high SNR P_0 after cross-layer scheduling is given by:

$$\begin{aligned} &G^{**}_{goodput}(\hat{\mathbf{H}}) \\ &= \frac{(1-\epsilon)L_s N_d}{n_F} \sum_{k\in A^*} \log_2\left(\frac{F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon) P_0 n_F}{N_d L_s |A^*|}\right). \end{aligned} \tag{18}$$

Consider selecting one user with the largest $s^2(\hat{\mathbf{H}}; B_k^*)$ from K users. Using the result in Lemma 1, we have $s^2(\widehat{\mathbf{H}}; B_k^*) = \mathcal{O}\left(\frac{1-\sigma_e^2}{N_d}\left(\log K + N_d \log\log K\right)\right)$ with probability 1 (for sufficiently large K). Assume that $K \gg |A|$ and if we ignore the inter-dependency (or coupling constraint) in the user selection result between different users, we have $s^2(\widehat{\mathbf{H}}; B_k^*) = \mathcal{O}\left(\frac{1-\sigma_e^2}{N_d}\left(\log K + N_d \log\log K\right)\right)$ with probability 1 for all other users $k \in A^*$. Hence, the result follows by direct substitution into (18).

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