Cross-Layer Optimization for OFDMA System with Imperfect CSIT in Quasi Static Channel

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***Abstract*—Asymptotic performance analysis of orthogonal frequency division multiple access (OFDMA) is conducted in this work. We take a cross-layer approach and analyze how the system performance scale with some important parameters such as number of user, CSIT quality and order of diversity. It is well studied that system goodput scale with number of user K in the order of $\mathcal{O}(\log\log K)$ due to multiuser diversity (*system goodput gain*) under perfect CSIT assumption. However, in quasi static fading channels with imperfect CSIT, channel outage happens even if capacity achieving coding is applied at the base station. In this case, cross-layer scheduler can protect the packet by increasing the order of diversity (*packet outage diversity*) . So, there is a trade-off between system goodput gain and packet outage diversity.**

I. Introduction

The orthogonal frequency division multiple access (OFDMA) is a promising candidate and access scheme of high speed wireless communication system, such as Wi-Max and future Forth Generation (4G) technology [1][2]. OFDMA achieves high spectral efficiency by dividing the total available bandwidth to narrow subbands in an efficient way. This special structure of OFDMA allows users to spread their information across different subbands to avoid frequency selective deep fading. In an OFDMA systems, by selecting a set of users with best channel condition, it can be shown that the system throughput scale with the number of users in the order of $\mathcal{O}(\log\log K)$[3]. In[4], cross-layer scheduling for OFDMA systems is analyzed using limited feedback in the CSIT. The authors also show that system throughput scales in the order of $\mathcal{O}(\log\log K)$ with one bit feedback. Yet, in all these works, ergodic capacity is the key performance measure and perfect CSIT is assumed. With perfect CSIT, mis-scheduling will not happen. By carefully adjust the power and rate, channel outage can be avoided. However, in practice, perfect CSIT is somewhat unrealistic because of feedback delay or estimation error. On the other hands, in quasi static fading channel, ergodic capacity is no longer a meaningful performance measure since there is no significant channel variation across the encoding frame, thus channel outage occurs with finite probability. To include the effect of packet error due to channel outage, system goodput (effective throughput) is a more meaningful performance measure which is defined as average bits/sec/Hz successfully delivered to the user.

In this paper, the OFDMA cross-layer design with imperfect CSIT is modeled as an optimization problem where the rate adaptation, power adaptation and subcarrier allocation policies are designed to optimize the system goodput. Simple closed-form expressions for the power and rate allocations as well as the asymptotic order of growth in system goodput with respect to number of user are given in the paper, which provide some important insights on the tradeoff of cross-layer *system godoput gain* and *diversity protection gain*.

The rest of the paper is organized as follows. In Section II, we outline the OFDMA system model. In Section III, we define *system goodput* and formulate the cross-layer design as an optimization problem. In Section III-B, we shall give closed-form solution for rate and power adaptation and discuss a low-complexity subcarrier assignment policy. In Section IV, we shall analyze the asymptotic tradeoff between system goodput and packet outage diversity for large number of users. In Section V, we conclude with a summary of results.

II. System Model

In this paper, the following conventions are adopted. $\mathbf{X}$ denotes a matrix and $\mathbf{x}$ denotes a vector. $\mathbf{X}^{\dagger}$ denotes matrix transpose and $\mathbf{X}^{H}$ denotes matrix hermitian. " $\doteq$ " denotes *exponential equality*. Specifically, $f(x) \doteq g(x)$ with respect to the limit $x \to a$,$a = \{0, \infty\}$, if $\lim_{x\to a} \frac{\log f(x)}{\log g(x)} = 1$." $\dot{\geq}$ " and " $\dot{\leq}$ " are defined in similar manner. Finally, $\mathcal{O}(.)$ denotes the *asymptotic upper bound*.

A. Frequency Selective Fading Channel Model and Imperfect CSIT Model

We consider a downlink transmission in OFDMA system. The channel is assumed to be time-invariant, frequency selective channel model. The number of resolvable paths are approximately $L = \left\lfloor \frac{W}{\Delta f_c} \right\rfloor$, where W is the signal bandwidth and Δf_c is the coherence bandwidth. Consider a time-invariant L-tap delay line channel model, the channel impulse response between the base station and the k-th user is given by:

$$h(\tau;k) = \sum_{n=0}^{L-1} h_n^{(k)} \delta(\tau - \frac{n}{W}) \quad (1)$$

where $\{h_n^{(k)}\}$ are modeled as independent identically distributed (i.i.d.) complex Gaussian circularly symmetric random variables with zero mean and variance $\frac{1}{L}$. Thus, the received signal of the k-th user can be represented as the follow:

$$y_k(t) = \sum_{n=0}^{L-1} h_n^{(k)} x(t - \frac{n}{W}) + n(t) \quad (2)$$

where $x(t)$ is the transmitted signal from the base station and $n(t)$ is complex white Gaussian noise with density N_0.

Using n_F-point IFFT and FFT in the OFDMA system, the equivalent discrete channel model in the frequency domain (after removing the cyclic prefix with length L) is:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k \quad (3)$$

where $\mathbf{x}$ and $\mathbf{y}_k$ are $n_F \times 1$ transmit and receive vectors and $\mathbf{n}_k$ is the $n_F \times 1$ i.i.d. complex Gaussian channel noise vector with zero mean and normalized covariance $E[\mathbf{n}_k \mathbf{n}_k^H] = 1/n_F$ (so that the total noise power across the n_F subcarriers is unity). $\mathbf{H}_k$ is the $n_F \times n_F$ diagonal channel matrix between the base station and the k-th user $\mathbf{H}_k = diag\left[H_0^{(k)}, ..., H_{n_F-1}^{(k)}\right]$, where $H_m^{(k)} = \sum_{l=0}^{L-1} h_l^{(k)} e^{\frac{-j2\pi lm}{n_F}}, \forall m \in \{0, ..., n_F - 1\}$ are the FFT of the time-domain channel taps $\{h_0^{(k)}, ..., h_{L-1}^{(k)}\}$. Since $H_m^{(k)}$ is a linear combination of Gaussian random variables, $\{H_0^{(k)}, .., H_{n_F-1}^{(k)}\}$ are circularly symmetric complex Gaussian random variables with zero mean and the correlation between $H_m^{(k)}$ and $H_n^{(k)}$ is

$$E\left[H_m^{(k)} H_n^{(k)H}\right] = \frac{1}{L} \frac{1 - e^{\frac{-2j\pi L(m-n)}{n_F}}}{1 - e^{\frac{-2j\pi(m-n)}{n_F}}} = \eta_{k,m,n} \quad (4)$$

Observe that $\eta_{k,m,n} = 0$ when $(m-n)L$ is integer multiple of n_F. Hence, we can divide $\{H_0^{(k)}, .., H_{n_F-1}^{(k)}\}$ into $L_s = n_F/L$ groups, where each group has L i.i.d. elements, as follows:

$$\underbrace{\begin{bmatrix} H_0^{(k)} \\ H_{L_s}^{(k)} \\ \vdots \\ H_{(L-1)L_s}^{(k)} \end{bmatrix}}_{\mathbf{H}_0^{(k)}} \underbrace{\begin{bmatrix} H_1^{(k)} \\ H_{L_s+1}^{(k)} \\ \vdots \\ H_{(L-1)L_s+1}^{(k)} \end{bmatrix}}_{\mathbf{H}_1^{(k)}} \cdots \underbrace{\begin{bmatrix} H_{L_s-1}^{(k)} \\ H_{2L_s-1}^{(k)} \\ \vdots \\ H_{LL_s-1}^{(k)} \end{bmatrix}}_{\mathbf{H}_{L_s-1}^{(k)}}$$

In other words, there are L independent subbands (labelled as $m = 0, 1, 2, ..., L-1$) in the n_F-subcarriers with L_s correlated subcarriers in each subband.

For simplicity, we consider TDD systems (with channel reciprocity) and assume the CSIR is perfect but the CSIT is not. The estimated CSIT (time domain) at the base station for the k-th user is given by:

$$\hat{h}_l^{(k)} = h_l^{(k)} + \Delta h_l^{(k)} \quad \Delta h_l^{(k)} \sim CN(0, \sigma_e^2) \; l \in \{0, 1, .., L-1\}$$

Hence, the estimated CSIT in frequency domain (m-th subcarrier) $\hat{H}_m^{(k)}$ after n_F-point FFT of $\{\hat{h}_0^{(k)}, ..., \hat{h}_{L-1}^{(k)}\}$ is given by:

$$\hat{H}_m^{(k)} = H_m^{(k)} + \Delta H_m^{(k)} \quad \Delta \mathbf{H}_m^{(k)} \sim CN(0, \sigma_e^2) \quad (5)$$

where $H_m^{(k)}$ is the actual CSIT of the m-th subcarrier for the k-th user, $\Delta H_m^{(k)}$ represents the CSIT error which is circular symmetric complex Gaussian (CSCG) random variable with zero mean and variance σ_e^2. The correlation of the CSIT error between the m-th and n-th subcarriers of user k is given by:

$$E\left[\Delta H_m^{(k)} \Delta H_n^{(k)H}\right] = \sigma_e^2 \frac{1 - e^{\frac{-2j\pi L(m-n)}{n_F}}}{1 - e^{\frac{-2j\pi(m-n)}{n_F}}} \quad (6)$$

Finally, the CSI between the K users are i.i.d.

B. Instantaneous Mutual Information and System Goodput

Let B_k denotes the set of subband indices $m = \{0, 1, ..., L-1\}$ assigned to the k-th user. Hence, the instantaneous mutual information between the base station and the k-th mobile (given the CSIR $\mathbf{H}_k$) is given by:

$$C_k = \sum_{n=0}^{L_s-1} \sum_{m \in B_k} \log_2(1 + \frac{n_F p_k |H_{mL_s+n}^{(k)}|^2}{L_s N_d}) \quad (7)$$

where L_s is the number of correlated subcarriers in one subband, N_d is the number of independent subbands allocated to the $k-$th user and p_k is the transmit power allocated to the k-th user.

In general, packet error is contributed by channel noise and the channel outage. As long as we can provide sufficient strong channel coding (e.g. LDPC) with sufficiently long block length (e.g. 10Kbytes) to protect the information, it can be shown in [5] that packet errors due to the noise is practically negligible. On the other hand, the channel outage effect is systematic and cannot be eliminated by using strong channel coding. This is because the instantaneous mutual information[1] $C_k(\mathbf{H}_k)$ between the base station and k-th user is a function of actual CSI $\mathbf{H}_k$, which is unknown to the base station. Hence, the packet will be corrupted whenever the scheduled data rate r_k exceeds the instantaneous mutual information C_k. Hence, for simplicity, we shall model the packet error solely by the probability that the scheduled data rate exceeding the instantaneous mutual information (i.e. packet error due to the channel outage only).

In order to account for potential packet errors, we shall consider the *system goodput* (b/s/Hz successfully delivered to the mobile station) as our performance measure. Since packet errors (due to channel outage) is very important to the overall goodput performance, we shall require *diversity* to protect the information from channel outage to enhance the chance of successful packet delivery to the mobile receivers in the presence of outdated CSIT. By assigning N_d independent subbands to a mobile user, we sacrifice the cross-layer goodput gain to trade for N_d order diversity protection on the packet outage probability. We first define the instantaneous goodput of a packet transmission for user k as $\rho = \frac{r_k}{n_F}\mathbf{1}(r_k \leq C_k)$, where $1(.)$ is an indicator function which is 1 when the

[1]The instantaneous mutual information represents the maximum achievable data rate for error free transmissions.

event is true and 0 otherwise. The *average total goodput*[2] is defined as the total average b/s/Hz successfully delivered to the K mobiles (averaged over multiple scheduling slots) and is given by:

$$U_{goodput}(\mathcal{A},\mathcal{B},\mathcal{P},\mathcal{R}) = \frac{1}{n_F} E_{\hat{\mathbf{H}}} \left\{ \sum_{k=1}^{K} r_k \Pr[r_k \le C_k | \hat{\mathbf{H}}] \right\}$$

where $\mathcal{R} = \{r_1, ..., r_K\}$ is the rate allocation policy, $\mathcal{P} = \{p_1, ..., p_K : \sum_k p_k \le P_0\}$ is the power allocation policy , $\{\mathcal{A}\}$ is the user selection policy with respect to the outdated CSIT $\hat{\mathbf{H}}$, $\{\mathcal{B}\}$ is the set of subband allocation policy with respect to N_d independent subbands and $E_{\hat{\mathbf{H}}}\{X\}$ denotes the expectation of the random variable X w.r.t $\hat{\mathbf{H}}$. These policies are formally defined in the next section.

III. Cross-Layer Design for OFDMA Systems

In this section, we shall formulate the cross-layer scheduling design as an optimization problem.

A. Cross-Layer Design Optimization Formulation

The cross-layer scheduling algorithm is responsible for the allocation of channel resource at every scheduling slot. The base station collects the delayed CSIT from the K mobile users at the beginning of the scheduling slot and deduces the user selection (*admitted set* $A(\widehat{\mathbf{H}})$), the subband allocation $\{B_k(\widehat{\mathbf{H}}), k \in A(\widehat{\mathbf{H}})\}$, the *power allocation* $\{p_k(\widehat{\mathbf{H}}) \ge 0, k \in A(\widehat{\mathbf{H}})\}$ and the *rate allocation* $\{r_k(\widehat{\mathbf{H}}), k \in A(\widehat{\mathbf{H}})\}$ so as to optimize the total average system goodput $U_{goodput}(\mathcal{A},\mathcal{R},\mathcal{P},\mathcal{B})$ at a target packet outage probability ϵ. This can be written into the following optimization problem.

Problem 1 (Cross-Layer Optimization Problem): The optimal power allocation policy $\mathcal{P}^*$, rate allocation policy $\mathcal{R}^*$, user selection policy $\mathcal{A}^*$ and subband allocation policy $\mathcal{B}^*$ are obtained by solving the following optimization problem:

$$\arg \max_{\mathcal{P},\mathcal{R},\mathcal{A},\mathcal{B}} U_{goodput}(\mathcal{A},\mathcal{R},\mathcal{P},\mathcal{B}) \text{ s.t.}$$

$$\Pr\{r_k > \sum_{n=0}^{L_s-1} \sum_{m\in B_k} \log_2\left(1 + \frac{n_F p_k}{L_s N_d} |H_{mL_s+n}^{(k)}|^2\right) |\hat{\mathbf{H}}\} = \epsilon$$

where L_s is the number of correlated subcarriers in one subband. The key to solve the above optimization problem is on the modeling of the conditional packet outage probability $P_{out}(k, \hat{\mathbf{H}})$. The cumulative distribution function (cdf) of the random variable $I_k = \sum_{n=0}^{L_s-1} \sum_{m\in B_k} \log_2\left(1 + \frac{n_F p_k}{N_d L_s} |H_{mL_s+n}^{(k)}|^2\right)$ (conditioned on the estimated CSIT $\hat{\mathbf{H}}$) is in general very tedious and it is virtually impossible to obtain closed-form rate and power solutions by brute force optimization on top of the complicated expression. To obtain first order design insight and simple closed-form solutions, we shall consider asymptotic $P_{out}(k, \hat{\mathbf{H}})$ for high and low SNR. Using similar approach as in [6], we shall summarizes the result as the following lemma:

Lemma 1: (Asymptotic Outage Probability for High and Low SNR :) ($P_0 \to \infty$ or $P_0 \to 0$), the asymptotic conditional packet outage probability $P_{out}(k, \hat{\mathbf{H}})$ is given by:

$$P_{out}(k, \hat{\mathbf{H}}) \doteq F_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}\left(\frac{(2^{\frac{r_k}{L_s N_d}} - 1) L_s N_d}{n_F p_k}\right) \quad (8)$$

where $F_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(x)$ is the cdf of non-central chi-square random variable $\chi_k^2 = \frac{1}{N_d}\sum_{m\in B_k} |H_{mL_s}^{(k)}|^2$ with $2N_d$ degrees of freedom, non-centrality parameter $s^2(B_k) = \frac{1}{N_d}\sum_{m\in B_k} |\hat{H}_{mL_s}^{(k)}|^2$ and variance σ_e^2/N_d.

The optimization Problem III-A consists of a mixture of combinatorial variables $(A, \{B_k\})$ and real variables $(\{r_k\}, \{p_k\})$. We shall first obtain closed-form solution for rate and power allocation for a given admitted user set A and subcarrier allocation $\{B_k\}$.

B. Closed-form Solutions for Power and Rate Allocation Policies

In this section, we shall focus on deriving the asymptotically optimal power and rate allocation solution that optimize the system goodput for a given admitted user set A and subcarrier allocation $\{B_k\}$. Using Lemma 1, the target packet outage constraint in (8) for high and low SNR is equivalent to the following:

$$r_k = L_s N_d \log_2\left(1 + \frac{n_F p_k}{N_d L_s} F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon)\right) \quad (9)$$

Taking into consideration of the total transmit power constraint P_0, the Lagrangian function $L(\{p_k\}, \lambda)$ of the optimization problem in (8) is given by:

$$\frac{(1-\epsilon) L_s N_d}{n_F} \sum_{k\in A} \log_2\left(1 + \frac{n_F p_k}{N_d L_s} F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon)\right) - \lambda p_k$$

where $\lambda > 0$ is the Lagrange multiplier with respect to the total transmit power constraint. Using standard optimization techniques, the optimal power and rate allocation are given by:

$$p_k^* = \frac{L_s N_d}{n_F}\left(\frac{1-\epsilon}{\lambda} - \frac{1}{F^{-1}_{\chi_k^2; s^2(B_k); \sigma_e^2/N_d}(\epsilon)}\right)^+ \quad \forall k \in A(\hat{\mathbf{H}})$$

$$r_k^* = \left(L_s N_d \log_2\left((1-\epsilon) F^{-1}_{\chi_k^2; s^2(B_k); \frac{\sigma_e^2}{N_d}}(\epsilon) \frac{1}{\lambda}\right)\right)^+ \quad \forall k \in A(\hat{\mathbf{H}}) \quad (10)$$

C. Low Complexity User Selection and Subcarrier Allocation Policies

In this section, we focus on the combinatorial algorithm for user selection and subcarrier allocation given a imperfect CSIT $\hat{\mathbf{H}}$. Using the optimal power allocation solution in (10) and for sufficiently large average SNR constraint P_0, the Lagrange multiplier λ is given by:

[2]The utility function can incorporate fairness, we can modify the system utility to be another function of average goodputs such as $U_{PF}(\overline{\rho_1}, \overline{\rho_2}, ..., \overline{\rho_K}) = \sum_{i=1}^{K} log(\overline{\rho_i})$ or $U_{weight}(\overline{\rho_1}, \overline{\rho_2}, ..., \overline{\rho_K}) = \sum_{i=1}^{K} \alpha_i \overline{\rho_i}$. Then we can follow the same procedure of this paper to derive a scheduling algorithm which consider fairness.

$$\lambda = \frac{|A|(1-\varepsilon)}{n_F P_0/N_d L_s + \sum_{k\in A} \frac{1}{F^{-1}_{\chi_k^2;s^2(B_k);\sigma_e^2/N_d}(\epsilon)}} \tag{11}$$

Substituting into the rate allocation solution in (10), for large average SNR P_0, the conditional system goodput $G^*_{goodput}(A, \{B_k\})$ can be approximated by:

$$\frac{(1-\epsilon)L_s}{n_F/N_d} \sum_{k\in A} \log_2 \left(\frac{F^{-1}_{\chi_k^2;s^2(B_k);\frac{\sigma_e^2}{N_d}}(\epsilon)}{N_d L_s |A|/(P_0 n_F)} \right) \tag{12}$$

Observe that $F^{-1}_{\chi_k^2;s^2;\sigma_e^2/N_d}(x)$ is a increasing function of s^2 for a given x. Hence, the equivalent combinatorial search problem for A and $\{B_k\}$ is given by:

$$(A^*, \{B_k^*\}) = \arg \max_{\substack{A,\{B_k\} \\ |B_k|=N_d}} \prod_{k\in A} \left[\sum_{m\in B_k} |\widehat{\mathbf{H}}^{(k)}_{mL_s}|^2 \right] \tag{13}$$

However, even with the simplified searching objective in (13), the search for A and $\{B_k\}$ are still coupled among the n_F subcarriers due to the constraint that each B_k should contain N_d independent subbands. To address the complexity issue, we shall propose a low complexity *greedy* combinatorial search algorithm to obtain the admitted user set A^* and the subcarrier allocation sets $\{B_k^*\}$. The proposed algorithm is shown to achieve close-to-optimal performance by numerical simulation. (Result is omitted in this paper due to page limitation) The *greedy* algorithm is summarized below. Greedy Algorithm for A and $\{B_k\}$ at high SNR.

- Step 1:Initialize $A^* = \emptyset, B_k^* = \emptyset$, a user selection list $A_{selection}$ which include all user indices and a subband selection list $B_{selection}$ which include all independent subband indices.
- Step 2:Initialize a temporary list T_k for all user in $A_{selection}$ to store subband indices.
  $T_k = \arg \max_{|T_k|=N_d} \sum_{m\in B_{selection}} |\widehat{\mathbf{H}}^{(k)}_{mL_s}|^2$
- Step 3:Select user $k = \arg \max_{k\in A_{selection}} \sum_{m\in T_k} |\widehat{\mathbf{H}}^{(k)}_{mL_s}|^2$.
- Step 4:Put the selected users into set A^* and the corresponding subbands into set B_k^*.
- Step 5:Remove the selected users and the selected subbands from $A_{selection}$ and $B_{selection}$ and repeated step 2 until all the independent subbands are allocated to users.

On the other hand, the water-filling solution in (10) for low SNR ($P_0 \to 0$) will give only one non-zero term for p_k^*. In other words, for low SNR, we have $|A| = 1$ only and the $p_k^* = P_0$ for some $k \in A$. The corresponding system goodput $G^*_{goodput}(A, B_k)$ for low SNR is given by:

$$\frac{(1-\epsilon)N_d L_s}{n_F} \log_2 \left(1 + \frac{F^{-1}_{\chi_k^2;s^2(B_k);\sigma_e^2/N_d}(\epsilon) P_0 n_F}{N_d L_s} \right) \quad \text{for } k \in A$$

Observe that $F^{-1}_{\chi_k^2;s^2(B_k);\sigma_e^2/N_d}(x)$ is a increasing function of s^2 for a given x. Hence, the equivalent combinatorial search problem for A and B_k is given by:

$$(A^*, B_k^*) = \arg \max_{\substack{k,B_k \\ |B_k|=N_d}} \left[\sum_{m\in B_k} |\widehat{\mathbf{H}}^{(k)}_{mL_s}|^2 \right] \tag{14}$$

In this case, the optimal combinatorial search algorithm for A and B_k in low SNR is similar to the one in high SNR, except that we only select one user with the corresponding subbbands and stop the algorithm after the first iteration.

IV. Asymptotic Performance Analysis for Cross-Layer Design

We shall first introduce the following important lemma based on *extreme value theorem.*

Lemma 2 (Extreme Value Theorem): Let $\{X_1, ..., X_K\}$ be a set of K i.i.d. central chi-square random variables with $2n$ degrees of freedom and variance σ_X^2 and $X^* = \max_k X_k, \phi = \sigma_X^2 \log K$. For large K, we have

$$\Pr(\phi + \sigma_X^2 (n-2) \log\log K \le X^* \le \phi + \sigma_X^2 n \log\log K) \\ \ge 1 - \mathcal{O}\left(\frac{1}{\log K}\right) \tag{15}$$

In other words, $X^* \approx \mathcal{O}\left(\sigma_X^2 \log K + \sigma_X^2 n \log\log K\right)$ with probability one for sufficiently large K.

Proof 1: Please refer to appendix A.

By using Lemma 2 and equation (12), the average system goodput is given by:

Theorem 1: Asymptotic System Goodput for High and Low SNR:

$$\overline{\rho}^* = E_{\hat{\mathbf{H}}}[G^{**}_{goodput}(\hat{\mathbf{H}})] \tag{16}$$
$$= \begin{cases} \mathcal{O}\left[(1-\epsilon)\log\left(F^{-1}_{\chi_{k^*}^2;\tilde{s}^2;\sigma_e^2/N_d}(\epsilon)P_0\right)\right] & \text{for high SNR} \\ \mathcal{O}\left[(1-\epsilon)P_0 F^{-1}_{\chi_{k^*}^2;\tilde{s}^2;\sigma_e^2/N_d}(\epsilon)\right] & \text{for low SNR.} \end{cases}$$

for sufficiently large K where $\tilde{s}^2 = \left(\frac{1-\sigma_e^2}{N_d}(\log K + N_d \log\log K)\right)$.

Hence, the order of growth in the cross-layer goodput gain is contained entirely in the inverse non-central chi-square cdf via the non-centrality parameter s^2. Yet, there is no closed form for $F^{-1}_{\chi_k^2;s^2;\sigma_e^2/N_d}(x)$ in general case. We shall discuss the asymptotic tradeoff between cross-layer goodput gain and the packet outage diversity N_d in the following asymptotic cases.In addition to the asymptotic analysis, we shall also simulate the system performance in term of average system goodput and compare the result with asymptotic performance in different scenarios. In our simulation, frequency selective fading channel is considered with uniform power-delay profile for simplicity. The number of subcarriers N_f is 1024 and the total number of independent taps $L = 16$. Hence, the 1024 subcarriers are grouped into 16 subbands, each containing $L_s = 64$ correlated subcarriers. The target packet error probability ϵ is set to 0.01. Each point in the figure is obtained by 5000 realizations.

A. Frequency Diversity at Small Target Packet Outage Probability ϵ

We shall first introduce the following lemma about $F^{-1}_{\chi^2_k;s^2;\sigma_e^2/N_d}(x)$ for small x.

Lemma 3 (Order of Growth for small ϵ): Let X be a non-central chi r.v. with $2n$ degrees of freedom, noncentral parameter s^2 and variance σ_X^2. For a given s^2, the inverse cdf of X can be expressed as below for asymptotically small ϵ.

$$F_X^{-1}(\epsilon) \doteq \epsilon^{1/n}\sigma_X^2(n!)^{1/n}\exp\left(\frac{s^2}{n\sigma_X^2}\right) \tag{17}$$

Thus, the average outage probability $\overline{P_{out}(k)}$ is given by the following theorem:

Theorem 2 (Frequency Diversity at Small ϵ): For sufficiently small ϵ, the average packet outage probability $\overline{P_{out}(k)}$ scales with the SNR P_0 (at a given average goodput) in the order of:

$$\overline{P_{out}(k)} = E_{\hat{\mathbf{H}}}\left[P_{out}(k,\hat{\mathbf{H}})\right] = \mathcal{O}\left(P_0^{-N_d}\right) \tag{18}$$

Hence, N_d is the order of frequency diversity protection against packet outage.

B. Cross-Layer Goodput Gains at Large K and fixed N_d

We have the following lemma about the order of growth of inverse non-central chi-square cdf $F^{-1}_{\chi^2_k;s^2;\sigma^2_X}(x)$ with respect to s^2 for large s^2.

Lemma 4 (Order of Growth for large s): Let X be a non-central random variable with $2n$ degrees of freedom, noncentrality parameter $s^2 > 0$ and variance σ_X^2. For a given ϵ, the inverse cdf of X can be expressed as $F_X^{-1}(\epsilon) \doteq \mathcal{O}(s^2\sigma_X^2)$ asymptotically for large s^2.

Using the results of Lemma 2 and Lemma 4 for large K and $\sigma_e^2 < 1$, we have the following Theorem:

Theorem 3: [3]*(Asymptotic System Goodput at Large K for High and Low SNR at fixed N_d and $\sigma_e^2 < 1$):*

$$\begin{aligned}\overline{\rho}^* &= E_{\hat{\mathbf{H}}}[G^{**}_{goodput}(\hat{\mathbf{H}})] \\ &= \begin{cases}\mathcal{O}\left\{(1-\epsilon)\log\left[P_0\left(1-\sigma_e^2\right)(\log K)\right]\right\} & \text{for high SNR,} \\ \mathcal{O}\left\{(1-\epsilon)P_0(1-\sigma_e^2)(\log K)\right\} & \text{for low SNR.}\end{cases}\end{aligned} \tag{19}$$

Figure 1 depicts the average system goodput performance(bit/s/Hz) of the proposed scheduling schemes as a function of the number of users in high SNR (20 dB) and frequency diversity order $N_d = 2$. It can be seen that when the number of user K increases, the system goodput grows as $\mathcal{O}\left\{\log\left[(1-\sigma_e^2)\log K\right]\right\}$ due to multi-user diversity.

[3]Theorem 3 is valid for estimation error $\sigma_e^2 \in [0,1)$. When going from equation (17) to (19), we used Lemma 4: $F^{-1}_{\chi^2_{k*};\tilde{s^2};\sigma_e^2/N_d}(\epsilon) \doteq \mathcal{O}(s^2\sigma_e^2)$, but this holds only for non-zero and sufficiently large non central parameter s^2. Hence, the results in equation (19) holds only for $\sigma_e^2 < 1$. For the case when $\sigma_e^2 = 1$ and $s^2 = 0$,the $F^{-1}_{\chi^2_{k*};\tilde{s^2};\sigma_e^2/N_d}(\epsilon)$ in Theorem 1 becomes inverse cdf of central chi square. In that case, the average goodput is given by equation (17). As a result, the average goodput does not growth with the number of users as illustrated in Figure 1.

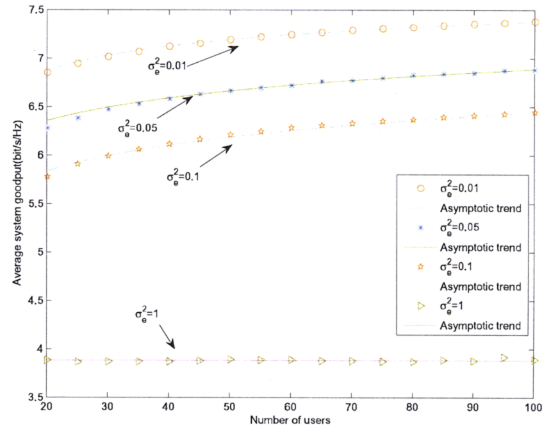

Fig. 1. Average system goodput versus number of users with N_d=2, different CSIT error (σ_e^2=0.01,0.05,0.1,1) at high SNR(20dB).

C. Asymptotic System Goodput at Large N_d and fixed K

From equation (15) in Lemma 2, there exists $K_0 > 0$ such that for $K_0 > 0$, the non-central parameter $\left[\frac{1-\sigma_e^2}{N_d}(\log K + (N_d-2)\log\log K)\right] \leq \tilde{s}^2(\widehat{H}) \leq \left[\frac{1-\sigma_e^2}{N_d}(\log K + N_d\log\log K)\right]$ with probability one for all N_d. As a result, consider the case for large N_d and fixed $K > K_0$[4]. From equation (17), the first term in the equation $\left(\frac{1-\sigma_e^2}{N_d}(\log K + N_d \log\log K)\right)$ will trend to zero as N_d increases faster than $\log K$ while the second term will be bounded by $\log\log K$. In this case, we have the non central parameter $\tilde{s^2}$ which is bounded by:

$$\tilde{s^2} = \mathcal{O}\left\{\left[(1-\sigma_e^2)(\log\log K)\right]\right\} \tag{20}$$

for some $K > K_0 > 0$ such that $\frac{N_d}{\log K} \to \infty$.

The asymptotic goodput at Large N_d for High and Low SNR for $K > K_0$ is given by :

$$\overline{\rho}^* = E_{\hat{\mathbf{H}}}[G^{**}_{goodput}(\hat{\mathbf{H}})] = \begin{cases}\mathcal{O}\left[(1-\epsilon)\log\left(F^{-1}_{\chi^2_{k*};\tilde{s^2};\sigma_e^2/N_d}(\epsilon)P_0\right)\right] & \text{for high SNR,} \\ \mathcal{O}\left[(1-\epsilon)P_0F^{-1}_{\chi^2_{k*};\tilde{s^2};\sigma_e^2/N_d}(\epsilon)\right] & \text{for low SNR.}\end{cases} \tag{21}$$

There is a factor $(1-\sigma_e^2)$ in $\tilde{s}^2$ outside the $\log\log K$ in equation (20) and $F^{-1}_{\chi^2_k;s^2;\sigma^2_X}(x)$ in equation (21) is an increasing function of $\tilde{s}^2$. Hence, we need *double exponentially* more users K to compensate the penalty due to $(1-\sigma_e^2)$ in the system goodput (via $\tilde{s}^2$).

Figure 2 illustrates the average system goodput performance versus N_d in high SNR (20dB) at different CSIT errors $\sigma_e^2 = 0, 0.05, 0.1, 0.15, 1$. The system goodput is shown to be a decreasing function of N_d. For large N_d, the cross-layer

[4]In general, the results will hold if we allow K to grow as N_d increase as long as $N_d/\log K \to \infty$.

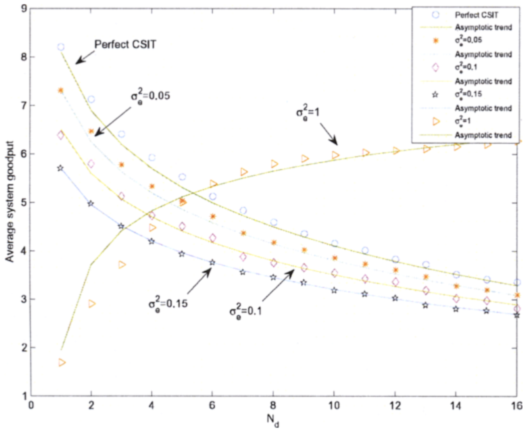

Fig. 2. Average system goodput versus packet diversity order (N_d) with different CSIT error σ_e^2 at high SNR(20dB) and K=20.

goodput gain is decreased substantially. On the other hand, the average packet outage probability scales in the order of $\mathcal{O}(P_0^{-N_d})$. From these results, we can deduce that there is a natural tradeoff between packet outage diversity order N_d and the cross-layer goodput gain. Comparing with the well-known cross-layer throughput gain of $\mathcal{O}(\log\log K)$ when we have perfect CSIT, we observe that the efficiency of the multiuser selection diversity (goodput) is reduced to $\log\log\log K$ for large N_d. Low SNR simulation results is omitted due to page limitation.

V. Conclusion

In this paper, we explore the asymptotic trade-off between cross-layer goodput gain and packet outage in OFDMA downlink system, with delayed CSIT in quasi static fading frequency selective channel. We formulate the cross-layer design as a mixed convex and combinational optimization problem. Due to the delayed CSIT, it is critical to account for potential packet errors (due to channel outage) and we consider total system goodput as our optimization objective. By allocating N_d independent subbands to a user, the packet outage probability drops in the order of SNR^{-N_d}. On the other hand, the *system goodput* scales in the order of $\mathcal{O}[(1-\epsilon)\log(F^{-1}_{\chi^2_{k*};\tilde{s}^2;\sigma_e^2/N_d}(\epsilon)P_0)]$ at high SNR where $\tilde{s}^2$ $= \mathcal{O}\left\{(1-\sigma_e^2)\log\log K\right\}$ and $\mathcal{O}\left\{(1-\sigma_e^2)(\log K)\right\}$ for large N_d $[K > K_0]$ and large K [fixed N_d] respectively.

Appendix

A. Proof of Lemma 2

Consider a sequence of i.i.d. random variable x_k, having central chi-square distribution with degree of freedom $2n$. Formally, x_k is characterized by the CDF of $F(x) = 1 - e^{-\frac{x}{\sigma_X^2}} \sum_{m=0}^{n-1} \frac{1}{m!}\left(\frac{x}{\sigma_X^2}\right)^m$; the PDF of $f(x) = \frac{1}{\sigma_X^{2n}\Gamma(n)} x^{n-1} e^{-\frac{x}{\sigma_X^n}}, x \geq 0$, where σ_X^2 is the variance of the underlying complex Gaussian random variables. Define the growth function $g(x) = \frac{1-F(x)}{f(x)}$. It is obvious that

$$\lim_{x\to\infty} g(x) = 1 \tag{22}$$

From [7] and [8], we have the following expression

$$\begin{aligned}
&\log[-\log F^K(b_K + yg(b_K))] \\
&= -y + \tfrac{y^2}{2!} g'(b_K) + \tfrac{y^3}{3!}\left[g(b_K)g^{(2)}(b_K) - 2g'^2(b_K)\right] \ldots + \ldots \\
&+ \tfrac{e^{-y}+\ldots}{2K} + \tfrac{5e^{-2y}+\ldots}{2K} + \ldots - \tfrac{e^{-3y}}{8K^3} + \ldots + \ldots
\end{aligned}$$

where b_K is given by $F(b_K) = 1 - \frac{1}{K}$, i.e. $e^{-\frac{b_K}{\sigma_X^2}} \sum_{m=0}^{n-1} \frac{1}{m!}\left(\frac{b_K}{\sigma_X^2}\right)^m = \frac{1}{K}$.In the other words, b_K is the solution of $\frac{b_K}{\sigma_X^2} - \log \sum_{m=0}^{n-1} \frac{1}{m!}\left(\frac{b_K}{\sigma_X^2}\right)^m = \log K$. So $\frac{b_K}{\sigma_X^2} - \log\left(\frac{1}{(n-1)!}\left(\frac{b_K}{\sigma_X^2}\right)^{n-1}\right) - \mathcal{O}\left(\log\left(\frac{1}{(n-2)!}\left(\frac{b_K}{\sigma_X^2}\right)^{n-2}\right)\right) \doteq \log K$ and $\frac{b_K}{\sigma_X^2} - (n-1)\log\left(\frac{b_K}{\sigma_X^2}\right) - (n-2)\,\mathcal{O}\left(\log\left(\frac{b_K}{\sigma_X^2}\right)\right) \doteq \log K$.

Thus, $b_K = \sigma_X^2(\log K + (n-1)\log\log K)$ satisfies the above equation for large K. Note that the CDF of $\tilde{x} = \max_{1\leq k\leq K} x_k$ is given by $F^K(\tilde{x})$ substituting y as $\pm\log\log K$ in equation (23) and from equation (22),

$$\Pr\{-\log\log K \leq \max_{1\leq k\leq K} x_k - b_K \leq \log\log K\} \geq 1 - \mathcal{O}\left(\frac{1}{\log K}\right)$$

Therefore,

$$\begin{aligned}
&\Pr\{\sigma_X^2 \log K + \sigma_X^2(n-2)\log\log K \leq \max_{1\leq k\leq K} x_k \\
&\leq \sigma_X^2 \log K + \sigma_X^2 n \log\log K\} \\
&\geq 1 - \mathcal{O}\left(\frac{1}{\log K}\right)
\end{aligned} \tag{23}$$

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