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Performance Analysis of Outage-Limited Multi-access Cellular Systems with Macro-diversity

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***Abstract*—In this paper, we shall analysis the uplink system *goodput* (bits/sec/Hz successfully decoded) and *per-user packet outage* in a cellular network using multi-user detection with successive interference cancellation (MUD-SIC). Slow fading channels and interference cancellation error occurs due to *packet outage* in each decoding stage are considered. We are interested to study the roles of *macro-diversity*, decoding order and power allocation on the MUD-SIC performance where the effect of potential *error-propagation* during the SIC detection is taken into account. Based on the information theoretical framework, we derive the closed-form expressions on the total system goodput as well as the *per-user packet outage probability*. We show that the system goodput does not scale with SNR due to mutual interference in the SIC process and macro-diversity (MDiv) could alleviate the problem and benefit to the system goodput.**

I. Introduction

Multiuser detection (MUD) have been a topic of intense research interest over the past two decades. The optimal multi-user receiver for CDMA was first introduced by Verdu [1] in the mid 1980. Although MUD provides a promising result in multi-access channel, it has not found widespread acceptance in commercial systems because the major problem with multiuser detector is the maintenance of simplicity. In recent years, there has been a growing interest of low-complexity MUD schemes such as the linear MUD [2], [3]. Among these substitutes, successive interference cancellation (SIC) [4], [5] is a key technology at the base station to mitigate intra-cell interference and achieve corner points in the capacity region at reasonably low complexity. Furthermore, it has been shown that commercial CDMA one bit up/down power control command can be directly applied to SIC without any modification[6]. Therefore, SIC receivers have been considered as a candidate in the future communication system.

In view of prior work on MUD [7], [8], there exist some open issues concerning MUD-SIC in multicell systems. First, conventional performance analysis of multi-access fading channel is usually based on the ergodic capacity. However, the ergodic capacity is a meaning performance measure metric only for fast fading channels. When slow fading channel is considered, *packet outage* occurs if the data rate exceed the channel capacity even if powerful forward error correction coding is applied. On the other hand, because of perfect cancellation can be assumed in fast fading channel, all these works did not take into account of packet errors due to *packet outage* and the *error-propagation* effects of MUD-SIC. When error-propagation effect of the MUD-SIC is considered, the packet error events between the K users are coupled together and the outage event cannot be determined by whether the rate vector is inside the capacity region or not. For example, whether the packet of the user decoded in the 2-nd iteration is successful depends not only on the channel condition of that user but also on whether the 1-st decoded packet is successful. Furthermore, even if a rate vector is outside the multi-access capacity region, some user(s) may still be able to decode the packet successfully. Second, one of the consequence of the *per-user outage outage* and *error propagation effects* is that the system goodput cannot scale with SNR due to potential mutual interference between users. Therefore, optimization of transmit power and decoding order in MUD-SIC is needed. Third, in multi-cell systems, macro-diversity enhances signal detection by exploiting the inter-cell interference [9], [10]. Although *macro-diversity* is a well studied technique commonly used in CDMA networks to enhance the uplink performance, it is not clear how the macro-diversity can alleviate the error propagation effects in the MUD-SIC at each base station.

In this paper, we shall attempt to address the above issues. We consider the uplink of a multi-cell systems with n_B base stations (each has MUD-SIC) and K mobile users. Using information theoretical framework, we derive the closed-form expressions on the *system goodput* as well as the *per-user packet outage probability* of the MUD-SIC detection under *macro-diversity* and potential error-propagation in the SIC process. Besides, asymptotically optimal decoding order and optimal power transmission strategy are derived to give a direction in practical MUD-SIC implementation. Based on the results, we obtain insights on how *macrodiversity* could alleviates the error propagation and how the power allocation with optimal decoding order and enhance the system goodput of MUD-SIC in a multi-cell network.

The paper is organized as follows. Section II outlines the multi-cell system and the base station MUD-SIC processing. Section III provides the analysis of the network goodput of the multi-cell system with MUD-SIC and macro-diversity . Section IV presents numerical results on the performance and verify with the analytical expression. Section V concludes with

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a summary of results.

II. System Model

A. Notation

Upper and lower case letters represent random variables and realizations of the variables, respectively. $\mathcal{E}[X]$ denotes the expectation of the random variable X. $X_{k:n}$ represents the k-th order statistic ($X_{1:n} < X_{2:n}, ... < X_{n:n}$) of n ordered random variable. π represents a particular decoding order where $\pi(i)$ gives the user index in the i-th decoding iteration and $\pi^{-1}(i)$ gives the decoding order of the k-th user.

B. Multi-user Multicell Channel Model

We consider a multicell system that consists of n_B base stations, K mobile users and a centralized controller. The base stations and mobile terminals all have single antenna. The signal received by the b-th base station is given by:

$$Y_b = \sum_{i=1}^{K} P_i g_{i,b} H_{i,b} X_i + Z_b \tag{1}$$

where X_i is the transmitted signal from the i-th mobile station, P_i is the transmitted power of the i-th mobile station which has range $[0, P_{max}]$, and Z_b is complex Gaussian noise with zero mean and unit variance at the b-th basestation. The path loss ($g_{i,b}$) between the b-th base station and the i-th mobile station is given by:

$$g_{i,b}(dB) = PL_b(d_o) + 10\psi_b \log_{10}\left(\frac{d_i}{d_o}\right) \tag{2}$$

where $PL_b(d_o)$ is the path loss at the reference point d_o meters away from the b-th BS, ψ_b is the path loss exponent in the b-th cell, and d_i is the distance in meters away the b-th BS. We assume slow fading where the channel coefficient $H_{i,b}$ between the $i-$th mobile station and the b-th BS is modeled as i.i.d. $CN(0,1)$. Assuming that the channel is quasi-static and remains constant for several transmission frame, the base stations (BSs) calculate the uplink power and rate for each users based on the path loss information to avoid per frame adaptation overhead.

C. Centralized Controller Processing

The centralized controller is responsible for determining a user assignment set of each base station and a set of users who should perform MDiv. The b-th base station should pass the estimated macroscopic fading coefficients (path loss) from all K users to the centralized controller. After collecting all the information from the n_B base stations, the centralized controller compares the path loss difference between each mobile user and all the BSs with a predefine threshold $\triangle_{\text{threshold}}$, and then sends out the MDiv information to all n_B base stations. Furthermore, for those mobile users performing MDiv, the decoded message are passed to the centralized controller from the corresponding base stations. Finally, the controller should selects a successfully decoded packet based on the Cyclic Redundancy Check (CRC) field.

D. MUD-SIC Processing and Per-User Packet Error Model

The base stations are equipped with synchronous SIC receiver. We assume that only the base stations have knowledge of the channel statistic and average path loss for all mobile users, so power and rate in the uplink are calculated at the BS and fed forward to the mobile station. Define A_b as a user set (including the users which perform MDiv) that are associated with the b-th base station. The received signal at the b-th base station can be expressed as:

$$Y_b = \sum_{i \in A_b} P_i g_{i,b} H_{i,b} X_i + \sum_{i \notin A_b} P_i g_{i,b} H_{i,b} X_i + Z_b \tag{3}$$

where the first term represents the intra-cell interference and the second term represents the inter-cell interference. The instantaneous mutual information between the b-th base station and the k-th user is given by the maximum mutual information[1] between the channel input X and channel output Y. Hence, for a given decoding order $\pi_b = \{\pi(1), \pi(2), ..., \pi(n)\}$ and user assignment set A_b with cardinality n , the instantaneous mutual information between the b-th base station and the j-th user is given by:

$$\mathcal{C}_b(\mathbf{H{,}G}, \pi_b, j) = \log_2\left(1 + \frac{P_j |H_{\pi_b(j),b}|^2 g_{\pi_b(j),b}}{1 + I_j}\right) \tag{4}$$

where $I_j = \sum_{i \in A_b, i=1}^{j-1} P_i |H_{\pi_b(i),b}|^2 g_{\pi_b(i),b} + \sum_{i \in A_b, i=j+1}^{n} P_i |H_{\pi_b(i),b}|^2 g_{\pi_b(i),b} + \sum_{i \notin A_b} P_i g_{i,b} |H_{i,b}|^2$, $\mathbf{H}$ is the CSIR matrix and $\mathbf{G}$ is the path loss matrix. The first term of I_j represents the cancellation errors in previous decoding stage, the second term is undetected intra-cell signal and the last term is the inter-cell interference. In general, packet error is contributed by the channel noise (finite block length) and/or the channel outage. However, with strong channel coding such as turbo code or LDPC code and 4K bytes block size, it is shown in [11] that we can approach the Shannon sense capacity within 0.0045 dB for a target frame error rate (FER) of 10^{-6}. Hence, packet errors due to finite block-length effect is negligible we have strong coding with reasonable block size. On the other hand, packet error due to channel outage is a systematic error which cannot be avoided even when capacity achieving coding is applied to protect the packet. This is because the instantaneous mutual information depends on the actual channel state information which is not available to the transmitters. Accordingly, we assume FER is due to channel outage. Traditional system performance measure using ergodic capacity may not be a good choice in this situation since it fails to account for the penalty of channel outage. In order to model the effect of packet errors, we consider the performance in terms of the system goodput (effective throughput). We model the per-user goodput as follows. For user k, let B_k denote the MDiv BS assignment list and the instantaneous goodput of a packet transmission

[1] The maximum mutual information can be achieved if we assume Gaussian random codebook is used.

(bit/s/Hz successfully delivered) to the b-th BS is given by:

$$\rho_k = r_k\{\mathcal{I}[\bigcup_{b\in\mathcal{B}_k} r_k < \mathcal{C}_b(\mathbf{H},\mathbf{G},\pi_b,k)]\} \quad (5)$$

where r_k is data rate of user k which is assumed to be fixed within a given path loss realization, $\mathcal{I}[.]$ is an indicator function that evaluates to 1 when the event is true and 0 otherwise. If strong error correction code is applied to the packet, the conditional average packet error rate (PER) of the k-th user (conditioned on the path loss realization) is given by:

$$\begin{aligned}&\overline{\mathrm{PER}_k}(r_k, P_k; \mathbf{G}) \approx \overline{P_{out_k}}(r_k, P_k; \mathbf{G}) \quad (6)\\ &= \sum_{\pi_b\in\mathcal{B}_k}\prod_{b\in\mathcal{B}_k}\{\Pr\left[r_k > \mathcal{C}_b(\mathbf{H},\mathbf{G},\pi_b,k)|\pi_b,\mathbf{G}\right]\Pr(\pi_b)\}\end{aligned}$$

Therefore, the *average system goodput* conditioned on all users' path loss matrix $\mathbf{G}$ is given by:

$$\begin{aligned}U_{goodput}(P,R,\pi;\mathbf{G}) &= \mathcal{E}_H\left[\sum_{k=1}^{K}\rho_k|\mathbf{G}\right] \quad (7)\\ &= \mathcal{E}_H\left\{\sum_{k=1}^{K} r_k\left(1-\overline{P_{out_k}}(r_k,P_k;\mathbf{G})\right)|\mathbf{G}\right\}\end{aligned}$$

Note that the average system goodput and PER are both functions of the transmission power of users and the decoding order. In the next section, we shall derive the optimal transmit power of each user and the asymptotically optimal decoding order such that the system goodput is maximized.

III. Performance Analysis For the Outage-Limited MUD-SIC System with Macro-diversity

In this section, we shall analyzes the average system goodput and per-user outage probability of the of the MUD-SIC system taking into account of transmission power, potential error propagation and macro-diversity.

A. Optimal power transmission level with MUD-SIC under macro-diversity

Traditional power control is employed to eliminate the near/far problem by maintaining equal received SINR among all mobile users when BSs are configured to perform single user detection. However, in our case of outage-limited MUD-SIC with potential error-propagation, it is not clear how the power should be adjusted to achieve the system objective. In the following lemma, we prove that a simple on/off power control is optimal in the outage limited case.

Lemma 1 (Optimal power allocation): With the same peak power constraint $0 \le p_k \le P_{\max}$ for all users, the optimal power allocation that maximizes the average system goodput in the outage-limited MUD-SIC system (with potential error propagation) is given by the simple on/off rule:

$$p_k = \{0, P_{\max}\}, \quad \forall k \quad (8)$$

This lemma suggests that a user either transmits at full power or does not transmit at all.

Proof 1: Please refer to Appendix A.

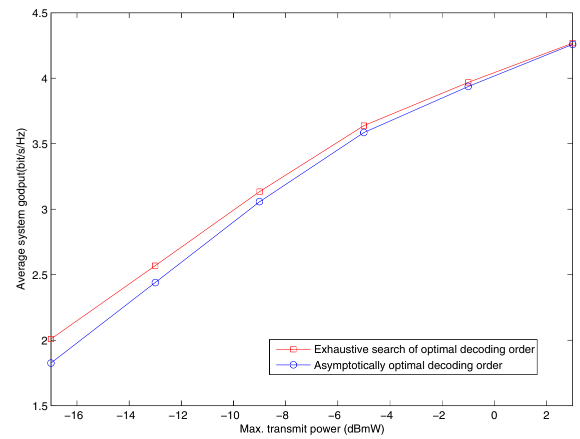

Fig. 1. A comparison of asymptotical optimal decoding order and exhaustive search of optimal decoding order in a two cells system. Average system goodput versus max. transmit power with K=10. Each user is on/off power controlled and with outage requirement 5%, $\triangle_{\text{threshold}} = \infty$(no MDiv).

B. Asymptotically Optimal Decoding Order with MUD-SIC under Macro-diversity

In the existing literature, the decoding order of successive interference cancellation is usually designed to either minimize the transmit power subject to performance requirement constraints [12] or to maximize system ergodic capacity with power constraint in fast fading channel [13]. However, these results failed to account for the packet errors in slow fading channels. Furthermore, due to the mutual coupling of the outage events in the MUD-SIC processing, the optimal decoding order, which is given by $\pi^* = \arg\max_{\pi} U_{goodput}(P,R,\pi;\mathbf{G})$, is very complicated and requires exhaustive search in general. Yet, we shall show in Lemma 2 that a simple decoding ordering would be asymptotically optimal for large transmit power.

Lemma 2 (Asymptotically Optimal Decoding Order): Given a user assignment set $A_b = \{1,2,...,n\}$ to the b-th basestation, the instantaneous receive SNR from all active users (intra-cell and inter-cell users) are $\{\gamma_1,..,\gamma_n,...,\gamma_K\}$, where $\gamma_k = P_{max}|H_{k,b}|^2 g_{k,b}$. For all users have the same outage requirement ϵ, the following decoding order is asymptotically optimal for sufficiently large transit power of users. It is given by:

$$\pi_b^*(j) = \arg\max_{k\in[1,K]\setminus\{\pi_b(1),\pi_b(2),...,\pi_b(j-1)\}} \gamma_k \quad (9)$$

Proof 2: Please refer to Appendix B.

While the decoding rule in (9) is optimal only if the transmit power of users are large. In Figure 1, the decoding rule illustrates close-to-optimal performance when compare with optimal exhaustive search even for moderate transmit power.

In order to characterize the per-user outage probability, let's define $S_i = \{0,1\}$ as the i-th stage iteration decoding event with $S_i = 1$ denotes successful decoding and $S_i = 0$ denotes decoding failure. The event S_i is given by:

$$\mathcal{I}\left\{r_{\pi_b^*(i)} < \log\left(1 + \frac{\gamma_{\pi_b^*(i)}}{1 + \sum\limits_{j<i}\gamma_{\pi_b^*(j)}(1-S_j) + \sum\limits_{j>i}\gamma_{\pi_b^*(j)}}\right)\right\} \tag{10}$$

where $\mathcal{I}(A)$ is the *indicator function* which is 1 when the event is true and 0 otherwise. If the interferences in the previous decoding stages are completely cancellated, then we can define the following event:

$$\mathcal{O}_i = \mathcal{I}\left\{r_{\pi_b^*(i)} < \log\left(1 + \frac{\gamma_{\pi_b^*(i)}}{1 + \sum_{j>i}\gamma_{\pi_b^*(j)}}\right)\right\}. \tag{11}$$

Given the asymptotically optimal decoding order policy in (9), we further assume that for user $\pi_b(i)$ fail in the i-th decoding iteration, then we can declare packet error for all the remaining users in the same basestation. This assumption cause a neglectable sub-optimality to the system performance which can be verified by numerical simulation, however, it can provide a traceable analysis expression and provide some important insight regards to the system performance. Based on the above assumption and event definition, we can duce that:

$$S_i = 0 \Rightarrow \mathcal{O}_1 \cup \mathcal{O}_2 \cup .... \cup \mathcal{O}_i \tag{12}$$

Therefore, the packet outage probability of user k with n intra-cell users can be written as:

$$\begin{aligned}
&\overline{P_{out_k}}(r_k, P, k; \mathbf{G}) \\
&= \sum_{\pi_b^* \in \mathcal{B}_k} \prod_{b \in \mathcal{B}_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \Pr\left[S_i = 0 | \pi_b^*\right] \Pr(\pi_b^*) \\
&\leq \sum_{\pi_b^* \in \mathcal{B}_k} \prod_{b \in \mathcal{B}_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \Pr\left[\mathcal{O}_j = 0 | \pi_b^*\right] \Pr(\pi_b^*)
\end{aligned} \tag{13}$$

Therefore, the average system goodput under the asymptotically optical decoding order is given by the following:

$$\begin{aligned}
&U_{goodput}(P, R, \pi^*; \mathbf{G}) \geq \sum_{k=1}^{K} r_k \times \\
&\{1 - [\sum_{\pi_b^* \in \mathcal{B}_k} \prod_{b \in \mathcal{B}_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \Pr\left[\mathcal{O}_j = 0 | \pi_b^*\right] \Pr(\pi_b^*)]\}
\end{aligned} \tag{14}$$

C. Per-user outage probability and average system goodput

Under the asymptotically optimal decoding order in Lemma 2, the average system goodput and per-user outage probability can be expressed in term of the conditional outage probability. For a given asymptotically optimal decoding order π_b^*, the conditional outage probability of user-k in the j-th iteration at the b-th base station can be expressed as:

$$\begin{aligned}
\Pr\left[\mathcal{O}_j = 0 | \pi_b^*\right] &= \Pr(r_k > C_{\pi_b^*(k)}(\mathbf{H}, \mathbf{G}, \pi_b^*, k) | \pi_b^*) \\
&= \Pr[\gamma_{\pi_b^*(j)} - \vartheta_{\pi_b^*(j)} \sum_{l=j+1}^{n} \gamma_{\pi_b^*(l)} < \vartheta_{\pi_b^*(j)}]
\end{aligned} \tag{15}$$

where $\vartheta_{\pi_b^*(j)} = 2^{r_k} - 1$. In general, the conditional outage probability involve n dimensions nested integration which is complicated and non-traceable when the dimension of integration grows. However, by taking the advantage of the additive Markov chain property from the exponential random variable order statistics, the conditional outage probability can be calculated by a one dimensional integration. We first introduce the following lemma:

Lemma 3 (Closed form of conditional outage probability): The conditional outage probability can be written in a summation of exponential functions which is given by:

$$\Pr\left[\mathcal{O}_j = 0 | \pi_b^*\right] = 1 - \sum_{l=j, \upsilon_l > 0}^{n} \Psi_l \frac{\beta_l}{\upsilon_l} \exp(-\frac{\vartheta_k \beta_l}{\upsilon_l}) \tag{16}$$

where $\Psi_l = \prod_{i=j, i \neq l}^{n} \frac{\upsilon_l}{\upsilon_l - \frac{\beta_l}{\beta_i}\upsilon_i}$, $\upsilon_l = \frac{1 - l \times \vartheta_k + j \times \vartheta_k}{l}$ and $\beta_l = \frac{\sum_{u=1}^{l} \frac{1}{g_{\pi(u),b}}}{P_{\max} l}$.

Proof 3: Proof is omitted due to page limitation.

After obtaining the closed-form of the conditional outage probability, we have to obtain the probability of a particular decoding order which is summarized in the following:

Lemma 4 (Probability of a particular decoding order policy π): Consider a set of i.ni.d exponential random variables $X_1, X_2, X_3, ..., X_n$ which has a p.d.f given by

$$f_{X_i}(x) = \beta_i \exp(-x\beta_i), \qquad \forall x, \beta_i \geq 0 \tag{17}$$

Then the probability of a particular order $\Pr(X_{i_1:n} < X_{i_2:n} < ... < X_{i_n:n})$ is given by:

$$\frac{\beta_{i_1}\beta_{i_2}\beta_{i_3}...\beta_{i_n}}{(\beta_{i_1} + \beta_{i_2} + ... + \beta_{i_n})(\beta_{i_2} + \beta_{i_3} + ... + \beta_{i_n})...\beta_{i_n}} \tag{18}$$

As a result, the per-user outage probability is given by the following Lemma:

Lemma 5 (Per-user packet outage probability with MDiv.): The average packet error probability of user k under the optimal decoding order policy π^* is given by:

$$\begin{aligned}
&\overline{P_{out_k}}(r_k, P_{max}; \mathbf{G}) \\
&\leq \sum_{\pi_b^* \in \mathcal{B}_k} \prod_{b \in \mathcal{B}_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} [1 - \sum_{l=j, \upsilon_l > 0}^{n} \Psi_l \frac{\beta_l}{\upsilon_l} \exp(\frac{-\vartheta_k \beta_l}{\upsilon_l})] \Pr(\pi_b^*)
\end{aligned} \tag{19}$$

where $\Pr(\pi_b^*)$ is given in equation (18). The data rate $r_k(\mathbf{G})$ can be determined from the per-user conditional packet error requirement $\overline{P_{out,k;\mathbf{G}}}(r_k, P_{max}; \mathbf{G}) = \epsilon$. Therefore, the average system goodput can be summarized by the Theorem 1 in equation (19).

From the expression in Theorem 1, the second summation represents the system goodput corresponds to each decoding permutation of the decoding rule in equation (9). The product term in (19) offers MDiv protection as a packet has to fail in all the BSs to declare packet error.

Theorem 1 (Lower bound for the average system goodput):

$$U_{goodput}(P,R,\pi^*;\mathbf{G}) = \sum_{k=1}^{K} r_k(1-\overline{P_{out}}(r_k,P_k)) \geq \sum_{k=1}^{K} r_k\{1-[\sum_{\pi_b^*\in\mathcal{B}_k}\prod_{b\in\mathcal{B}_k}\sum_{i=1}^{\pi_b^{*-1}(k)}\sum_{j=1}^{i}\Pr[\mathcal{O}_j=0|\pi_b^*]\Pr(\pi_b^*)]\}$$

$$= \sum_{k=1}^{K} r_k\{1-\sum_{\pi_b^*\in\mathcal{B}_k}\underbrace{\prod_{b\in\mathcal{B}_k}\sum_{i=1}^{\pi_b^{*-1}(k)}\sum_{j=1}^{i}[1-\sum_{l=j,\upsilon_l>0}^{n}\Psi_l\frac{\beta_l}{\upsilon_l}\exp(-\frac{\vartheta_k\beta_l}{\upsilon_l})]}_{\text{Selection diversity protection}}\Pr(\pi_b^*)\} \quad (19)$$

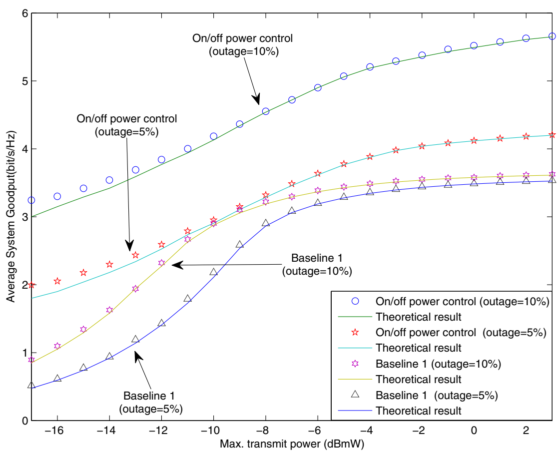

Fig. 2. A comparison of on off power control and traditional perfect power control. Average system goodput versus max transmit power in a two cells system with K=10, outage requirement 5% or 10%, $\triangle_{\text{threshold}} = \infty$(no MDiv).

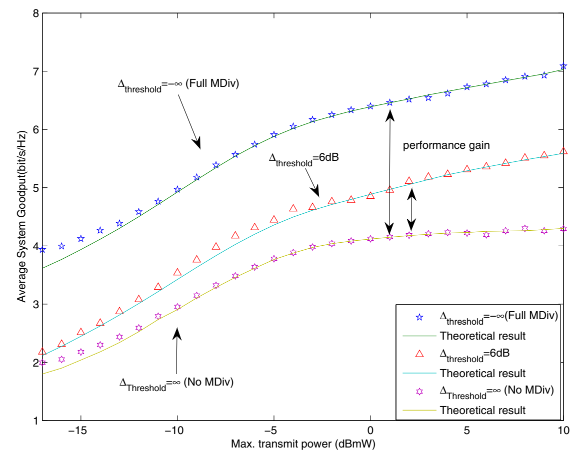

Fig. 3. Average System goodput versus max transmit power with different MDiv threshold in a two cells system, K=10. Each user has outage a requirement=5% and is power controlled by on/off transmission. The double sided arrow represents the performance gain due to MDiv.

IV. Results and Discussions

In this section, we evaluate the theoretical results using simulations. We consider a multi-cell system with 2 base stations with K active users. Every cell has radius of 1 km and path loss exponent 3.6. Assume that the minimum distance from a mobile station to the home base station be 30m. Average system goodput is obtained by counting the number of packets which are successfully decoded by the base station for all users and average the result over both macroscopic and microscopic fading. In the simulation, each point is obtained by averaging 100000 macroscopic and microscopic realizations.

A. Average System Goodput

Figure 2 illustrates the average system goodput versus the transmit power (dBmW) of mobile user for K=10 with asymptotical optimal decoding order. Each curve in the graph represents different type of power control with same target outage probability for all user (5% or 10%). We compare the performance of the proposed design with a conventional baseline CDMA power control algorithm in which the transmit powers of all users are adjusted such that the received SINR of them are the same at base station. For the baseline, the system goodput grows with SNR[2] at small SNR but quickly saturated at moderate SNR. This is because the performance is always limited by the weakest users. On the other hand, the goodput performance of the proposed on/off power control scheme does not saturate even at high SNR regime. It can be explained that in the proposed on/off power control, strong users do not required to decrease the transmission power to maintain the same SINR as those weak users, this factor contribute significantly to the system goodput.

Figure 3 shows the average system goodput versus the transmit power with different MDiv threshold ($\triangle_{\text{threshold}}$). Each user is power controlled by the on/off scheme and there is 5% outage probability requirement. We compare the performance of the proposed design with a system that does not perform MDiv in which all the inter-cell users are treated as interference. For the system without MDiv, the average system goodput saturated at high SNR because strong interference from inter-cell becomes a dominate factor in the system performance. On the contrary, the average system goodput of the proposed design increase with the transmit power when

[2]Because all active users are transmitting at their max power, therefore the SNR of each user is directly proportional to the max power.

MDiv is performed in the base station. The reason is that strong interference is regraded as desired user signal and it will be decoded by corresponding base station. Therefore, the system goodput has a significant gain when MDiv is performed in multi-cell environment.

V. Conclusion

In this paper, a generic multicell system with K client users, n_B base stations and a centralized controller is considered. Based on the asymptotic optimal decoding order, we incorporate the mathematical tool of order statistics to obtain the closed-form solution of system performance. Numerical simulations result are obtained to verify the analytical expressions. The closed form solutions allow efficient numerical evaluations to find out how the system performance is affected by the system parameter such as number of users and path loss exponent. From the results, we see that in interference limited region (users transmit at high power), MDiv improve the system goodput significantly by introducing macrodiversity protection to alleviate the consequences of error propagation. Furthermore, system with MDiv allows more users to be served at the same time through taking advantage of strong interference.

Appendix

A. Proof of Lemma 1

Note that since our power constraint is instantaneous, average system goodput maximization is the same as maximize the instantaneous mutual information for each fade vector. The total mutual information in the system can be expressed as $\sum_k \mathcal{C}_b(\mathbf{H},\mathbf{G},\pi_b,k)$. To find the optimal power allocation that maximizes the system goodput, it is equivalent to find P^* in the following optimization problem:

$$\arg\max_{\{P_1,P_2,\ldots,P_K\}} \mathcal{Q} = \arg\max_{\{P_1,P_2,\ldots,P_K\}} \sum_j \mathcal{C}_b(\mathbf{H},\mathbf{G},\pi_b,j) \quad (21)$$

where $\mathcal{Q}$ is equal to $\sum_{i=1}^{K} 2^{r_i-1}$ and r_i is the data that satisfy the outage requirement. Differentiating the equivalent objective function $\mathcal{Q}$ twice with respect to P_k. We obtain that $\frac{\partial^2 \mathcal{Q}}{\partial {p_k}^2}$ is always non-negative, so the equivalent problem is a strictly convex function of p_k. Therefore, the maximum value must always lie at the boundary (satisfy with equality) for a given particular decoding order. i.e. $P^* = \{0, P_{max}\}$

B. Proof of Lemma 2

Due to page limitation, only the direction of proof is provided. From equations (4),(6) and (8), it can be observed that the optimal decoding order in maximizing the system goodput is equivalent to a decoding order, which maximize the instantaneous mutual information in each decoding iteration at each base station. And it can be shown that for asymptotically large transmit power for all users, the mutual information is independent of transit power. Without loss of generality, let's assume the mutual information of the first iteration at base station b be $\mathcal{C}_b(\mathbf{H},\mathbf{G},\pi_b,1)$. In order to maximize this term, the strongest users should be decoded first, because it can contribute the largest mutual information and the lowest potentially accumulated undecoded interference. Similar argument can be applied in the second iteration. By induction, the optimal decoding order is to decode the users sequentially in decreasing receive SNR as in (9).

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