Power Control and Performance Analysis of Outage-Limited Cellular Network with MUD-SIC and Macro-Diversity

Derrick Wing Kwan Ng, *Student Member, IEEE,* and Vincent K. N. Lau, *Senior Member, IEEE*

***Abstract*—In this paper, we analyze the uplink *goodput* (bits/sec/Hz successfully decoded) and *per-user packet outage* in a cellular network using multi-user detection with successive interference cancellation (MUD-SIC). We are interested to study the role of *macro-diversity* (MDiv) between multiple base stations on the MUD-SIC performance where the effect of potential *error-propagation* during the SIC processing is taken into account. While the jointly optimal power and decoding order in the MUD-SIC are NP hard problem, we derive a simple on/off power control and asymptotically optimal decoding order with respect to the transmit power. Based on the information theoretical framework, we derive the closed-form expressions on the total *system goodput* as well as the *per-user packet outage probability*.**

***Index Terms*—Successive interference cancellation, error propagation, macro-diversity, optimal decoding order, order statistics.**

I. Introduction

THere are two important technologies that could substantially enhance the uplink performance of cellular systems, namely the multi-user detection (MUD) and the macro-diversity (MDiv). The MUD is effective to mitigate intra-cell interference while the MDiv is effective to exploit of inter-cell interference from adjacent base stations. It is well-known that jointly maximum likelihood multi-user detection (ML MUD) is optimal but with exponential order of complexity with respect to (w.r.t.) the number of users in the system. There are a lot of research works on low complexity MUD such as the linear MUD [1], [2] and the successive interference cancellation (SIC) [3], [4]. In [5], [6], the authors analyzed the system goodput (bit/s/Hz successfully delivered to mobile user) for multi-access channels with minimum mean square error (MMSE) detector. However, the MMSE MUD cannot achieve *Pareto optimality* in the capacity region. On the other hand, MUD-SIC is a promising technology at the base station to mitigate intra-cell interference at reasonably low complexity. In this paper, we study the uplink performance analysis of an outage-limited multi-cell system with both MUD-SIC at each base station and MDiv between adjacent base stations. While there are quite a number of works studying the MUD design and performance analysis on single cell systems [7], [8], there are still a number of open technical challenges to apply MUD-SIC in multi-cell systems with MDiv. They are elaborated in the following:

- **Per-user Outage and Error Propagation in MUD-SIC** Conventional performance analysis of multi-access fading channel is usually based on the ergodic capacity [9], [10]. Uplink power adaptation for multiaccess channel is addressed in [11]–[13] where the transmit power of mobile users are optimized with respect to a system objective function of user capacities. However, in all these works, they did not take into account of the potential packet errors (and the error propagation effects in the SIC process) due to channel outage. When error-propagation effect of the MUD-SIC is considered, the packet error events between the K users are coupled together and the outage event cannot be determined by whether the rate vector is inside the capacity region or not.
- **Power and Decoding Order Optimization** One of the consequence of the per-user outage and error propagation effects is that the system goodput cannot scale with SNR due to potential mutual interference between users. To alleviate this issue, optimization of transmit power and decoding order in MUD-SIC is needed. Yet, such optimization problem (taking into account of error propagation) is extremely complicated and has not been addressed in the literature.
- **Macro-Diversity** In multi-cell systems, macro-diversity (MDiv) enhances signal detection by exploiting the inter-cell interference [14], [15]. For instance, packet detection is terminated at each base station locally and the decoded packets from the base stations (in the active set) are delivered to a *base station controller* where packet selection is performed. *Macro-diversity* is a well studied technique in CDMA systems with single-user detection at the base station. However, it is not clear how the MDiv could alleviate the error propagation effects in the multi-cell network with MUD-SIC.

In this paper, we attempt to address the above issues. We consider an uplink of a multi-cell system with n_B base stations (each has MUD-SIC) and K mobile users. We derive the closed-form expressions on the *average system goodput* as well as the *per-user packet outage probability* of the MUD-SIC detection under *macro-diversity* and potential error-

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D. W. K. Ng is with the Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, British Columbia, V6T 1Z4 Canada (e-mail: wingn@ece.ubc.ca).

V. K. N. Lau is with the Hong Kong University of Science and Technology (e-mail: eeknlau@ee.ust.hk).

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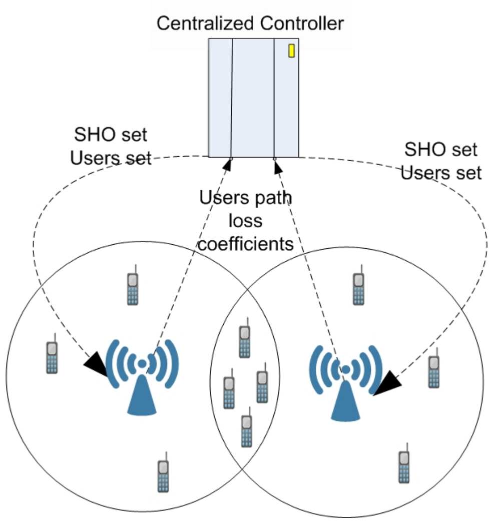

Fig. 1. Multi-cell system model with n_B base stations, K mobile users (each has single antenna), and a centralized controller.

propagation in the SIC process. While joint power and decoding order optimization is a $\mathcal{NP}$ hard problem, we derive a simple on/off power control and decoding ordering which is asymptotically optimal w.r.t. the transmit power. Based on the results, we found that power adaptation, decoding order and MDiv are important to enhance the system goodput of MUD-SIC in multi-cell network.

The paper is organized as follows. Section II outlines the multi-cell system and the base station MUD-SIC processing. Section III provides the analysis of the network goodput of the multi-cell system with MUD-SIC and MDiv. Section IV presents numerical results on the performance and verify with the analytical expression. Section V concludes with a summary of results.

II. System Model

A. Notation

Upper and lower case letters represent random variables and realizations of the variables, respectively. $\mathcal{E}[X]$ denotes the expectation of the random variable X. $X_{k:n}$ represents the k-th order statistic ($X_{1:n} < X_{2:n}, \ldots, < X_{n:n}$) of n ordered random variables. Matrix $\mathbf{\Pi}$ contains vectors $\{\pi_1, \pi_2, \ldots, \pi_b\}$, where π_b represents a particular decoding order for base station b. $\pi_b(i)$ gives the user index of users k in the i-th decoding iteration at the b-th base station and $\pi_b^{-1}(k)$ returns the decoding iteration index of user k at the b-th base station.

B. Multi-user Multi-cell Channel Model

We consider a wireless communication system which consists of n_B base stations, K mobile users, and a centralized controller as shown in Figure 1. The base stations and mobile terminals all have single antenna. The signal received by the b-th base station is given by

$$Y_b = \sum_{i=1}^{K} \sqrt{P_i g_{i,b}} H_{i,b} X_i + Z_b, \tag{1}$$

where X_i is the transmitted signal from the i-th mobile station, P_i is the transmitted power of the i-th mobile station which has range $[0, P_{max}]$, and Z_b is complex Gaussian noise with zero mean and unit variance at the b-th base station, i.e., $CN(0,1)$. The path loss and shadowing effect, i.e., $g_{i,b}$, between the b-th base station and the i-th mobile station can be expressed as

$$g_{i,b}(\mathrm{dB}) = \overline{PL_b}(d_0) + 10\psi_b \log_{10}\left(\frac{d_i}{d_o}\right) + \omega_\sigma, \tag{2}$$

where $\overline{PL_b}(d_0)$ is the average path loss at the reference point d_o meters away from the b-th base station, ψ_b is the path loss exponent in the b-th cell, d_i is the distance in meters away the b-th BS, and ω_σ denotes the shadowing effect which is modeled as a zero mean Gaussian distributed random variable with standard deviation σ. In order words, $g_{i,b}$ is log-normal distributed (in dB) with mean $\overline{PL_b}(d_0)+10\psi_b \log_{10}\left(\frac{d_i}{d_o}\right)$ and standard deviation σ dB. We model the channel coefficient $H_{i,b}$ between the $i-$th mobile station and the b-th base station as circularly complex Gaussian random variable with zero mean and unit variance. The proposed system model is a generalized model of CDMA, since the constant spreading factor/ processing gain can be treated as a multiplicative factor and absorbed in the path loss variables.

In this paper we assume that the power and data rates of the K users are adaptive w.r.t. long-term fading (path loss and shadowing).

C. Centralized Controller Processing

The centralized controller is responsible for determining a user assignment set of each base station and a set of users who need MDiv to enhance the performance. The b-th base station should pass the estimated macroscopic fading coefficients (average path loss and shadowing) from all K users to the centralized controller. After collecting all the macroscopic fading information from the n_B base stations, the centralized controller compares the differences of average path loss and shadowing effect, i.e., $g_{i,b}$, between each mobile user and all the base stations with a predefine threshold $\triangle_{\text{threshold}}$, and then sends out the MDiv users list to all base stations. Furthermore, for those mobile users who require MDiv, the decoded messages are passed to the centralized controller from the corresponding base stations. Then the controller selects a successfully decoded packet based on the Cyclic Redundancy Check (CRC) field. Since multiple base stations are decoding the same message for a user who demands MDiv and only the correct decoding messages are selected, a form of selection diversity protection is achieved.

D. MUD-SIC Processing and Per-User Packet Error Model

In this paper, we assume that the base stations are equipped with synchronous multi-user detector with successive interference cancellation. Furthermore, we assume that the base stations have knowledge of the channel statistic of multipath fading, average path loss and shadowing for all mobile users by long term measurement. On the other hand, the mobile stations do not have channel state information (CSI) and power allocation in the uplink are calculated at the base station and fed forward to the mobile stations. The received signal at the b-th base station is given by

$$Y_b = \underbrace{\sum_{i \in A_b} P_i g_{i,b} H_{i,b} X_i}_{\text{Intra-cell interference}} + \underbrace{\sum_{i \notin A_b} P_i g_{i,b} H_{i,b} X_i}_{\text{Inter-cell interference}} + \underbrace{Z_b}_{\text{Gaussian noise}}, \quad (3)$$

where A_b is a user set (including the users which perform MDiv) that are associated with the b-th base station.

The instantaneous channel capacity between the b-th base station and the k-th user is given by the maximum mutual information between the channel input X and channel output Y. Hence, for a given decoding order $\pi_b = \{\pi_b(1), \pi_b(2), \dots, \pi_b(u_b)\}$ and user assignment set A_b with cardinality u_b, the instantaneous channel capacity between the b-th base station and the user j in the $\pi_b^{-1}(j)$-th decoding iteration is $\mathcal{C}_b(\mathbf{H},\mathbf{G}, \pi_b, j) =$

$$\log_2\left(1+\frac{P_j |H_{\pi_b^{-1}(j),b}|^2 g_{\pi_b^{-1}(j),b}}{1+\widetilde{W^{\pi_b}_{\pi_b^{-1}(j)}} + \Phi_b(\mathbf{H},\mathbf{G}, \pi_b, j) + \Omega_b(\mathbf{H},\mathbf{G})}\right) \quad (4)$$

where $\mathbf{H}$ is the channel state information at the receiver (CSIR) matrix, $\mathbf{G}$ is the average path loss and shadowing matrix, $\Phi_b(\mathbf{H},\mathbf{G}, \pi_b, j) = \sum_{i \in A_b, \, i=\pi_b^{-1}(j)+1}^{u_b} P_i |H_{\pi_b(i),b}|^2 g_{\pi_b(i),b}$ is the *undetected signal*, $\Omega_b(\mathbf{H},\mathbf{G}) = \sum_{i \notin A_b} P_i g_{i,b} |H_{i,b}|^2$ is the *inter-cell interference*, and $\widetilde{W^{\pi_b}_{\pi_b^{-1}(j)}}$ denotes the accumulated *undecodable interference* after $\pi_b^{-1}(j)-1$ decoding iterations.

In this paper, we assume packet errors are contributed by channel outage which is a systematic error and cannot be avoided even when a capacity achieving coding is applied to protect the packet. As a result, traditional system performance measure using ergodic capacity may not be a good choice in this situation since it fails to account for the penalty of packet errors. In order to model the effect of packet errors, we consider the performance in terms of the system goodput (bit/s/Hz successfully received).

We model the *undecodable interference* and *per-user goodput* as follows. The *undecodable interference* at the b-th base station of user j in the $\pi_b^{-1}(j)$-th decoding iteration is

$$\widetilde{W^{\pi_b}_{\pi_b^{-1}(j)}} = \sum_{i=1}^{\pi_b^{-1}(j)-1} P_{\pi_b^{-1}(i)} |H_{\pi_b^{-1}(i),b}|^2 g_{\pi_b^{-1}(i),b} \times \mathcal{I}\left\{ r_{\pi_b^{-1}(i)} > \mathcal{C}_b(\mathbf{H},\mathbf{G}, \pi_b, \pi_b^{-1}(i)) \right\}. \quad (5)$$

For the per-user goodput of user k, let B_k denotes the MDiv base station assignment list and the instantaneous goodput of a packet transmission (bit/s/Hz successfully delivered) to the b-th base station is given by

$$\rho_k = r_k \times \left[1 - \prod_{b \in B_k} \mathcal{I}\{r_k > \mathcal{C}_b(\mathbf{H},\mathbf{G}, \pi_b, k)\}\right], \quad (6)$$

where r_k is the transmitted data rate of user k, which is a function of the average path loss and shadowing realization only. $\mathcal{I}\{\cdot\}$ is an indicator function that evaluates to 1 when the event is true and 0 otherwise. In (6), we can see that the goodput of user k depends on a set of base stations B_k[1] if the user is performing MDiv, otherwise the goodput of this user only depends on one base station. If strong error correction code is applied to the packet, the conditional average packet error rate (PER) of the user k (conditioned on the path loss and shadowing realization) can be expressed as

$$\overline{\mathrm{PER}_k}(r_k, P_k; \mathbf{G}) \approx \overline{P_{out_k}}(r_k, P_k; \mathbf{G}) = \sum_{\pi_b \in B_k} \prod_{b \in B_k} \left\{ \Pr\left[r_k > \mathcal{C}_b(\mathbf{H},\mathbf{G}, \pi_b, k) | \pi_b, \mathbf{G}\right] \Pr(\pi_b) \right\}, \quad (7)$$

where the first summation accounts for all the possible combinations of decoding order in $|B_k|$ number of MDiv stations. Therefore, the average system goodput (conditioned on the path loss and shadowing matrix $\mathbf{G}$) is given by

$$U_{gp}(P, R, \mathbf{\Pi}; \mathbf{G}) = \mathcal{E}_H\left[\sum_{k=1}^{K} \rho_k | \mathbf{G}\right] = \mathcal{E}_H\left[\sum_{k=1}^{K} r_k \left(1 - \overline{P_{out_k}}(r_k, P_k; \mathbf{G})\right) | \mathbf{G}\right]. \quad (8)$$

Note that the average system goodput and PER are both functions of the transmission power of users and the decoding order. In the next section, we shall derive the optimal transmit power of each user and the asymptotically optimal decoding order w.r.t. the transmit power.

III. Performance Analysis

In this section, we shall analyzes the average system goodput and per-user outage probability of the MUD-SIC system taking into account of transmission power, potential error propagation and macro-diversity.

A. Optimal Power Transmission Level with MUD-SIC under Macro-Diversity

Traditionally, power control is employed to outage-limited MUD-SIC with potential error-propagation, it is not obvious if all the users should transmit at their maximum power due to potential interference in the SIC process. In the following lemma, we prove that a simple on/off power control is asymptotically optimal with respect to high transmit power in the outage limited case.

Lemma 1 (Optimal Power Allocation): With the same peak power constraint $0 \le P_k \le P_{\max}$ for all users, the optimal power allocation that maximizes the instantaneous mutual information in the outage-limited MUD-SIC system (with potential error propagation) is given by the simple on/off rule:

$$P_k = \{0, P_{\max}\}, \quad \forall k \quad (9)$$

This lemma suggests that a user either transmits at full power or does not transmit at all.

Proof: Please refer to the full paper [16]. ■

[1]The cardinality of B_k is one if user k is assigned to one base station and no MDiv will be performed.

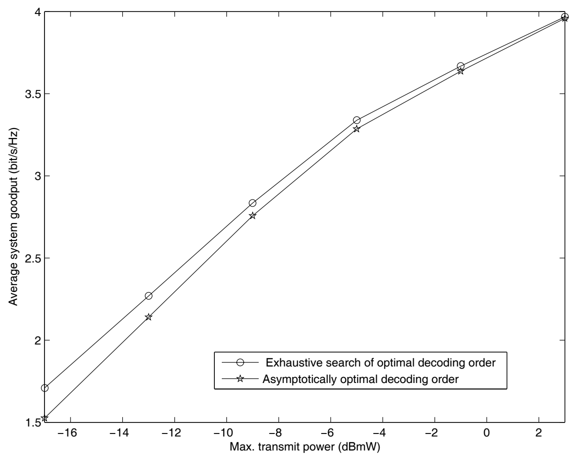

Fig. 2. A comparison of asymptotical optimal decoding order and exhaustive search of optimal decoding order in a two cells system. Average system goodput versus max. transmit power with K=10. Each user is on/off power controlled and with outage requirement 5%.

B. *Asymptotically Optimal Decoding Order with MUD-SIC under Macro-diversity*

In the existing literature, the decoding order of successive interference cancellation is usually designed to either minimize the transmit power subject to performance requirement constraints or to maximize system capacity with power constraint. Due to the mutual coupling of the outage events in the MUD-SIC processing, the optimal decoding order, which is given by $\mathbf{\Pi}^* = \arg\max_{\mathbf{\Pi}} U_{goodput}(P, R, \mathbf{\Pi}; \mathbf{G})$, is very complicated and requires exhaustive search in general. Yet, we shall show in Lemma 2 that a simple decoding ordering would be asymptotically optimal for large transmit power.

Lemma 2 (Asymptotically Optimal Decoding Order): For a given path loss realization $\mathbf{G}$, let $A_b(\mathbf{G}) = \{1, 2, \ldots, \mu_b\}$ be the set of active users (users with non-zero transmit power). Suppose all the users have the same conditional average PER requirement, i.e., $\overline{\text{PER}}_k(r_k, P_k; \mathbf{G}) = \epsilon$, then the following decoding order is asymptotically optimal for sufficiently large P_{max}.

$$\pi_b^*(j) = \arg \max_{k \in [1,K] \setminus \{\pi_b(1), \pi_b(2), \ldots, \pi_b(j-1)\}} \gamma_k \qquad (10)$$

where $\gamma_k = P_{max}|H_{k,b}|^2 g_{k,b}$ is the instantaneous receive SNR of all active users.

Proof: Please refer to the full paper [16]. ■

While the decoding rule in (10) is only asymptotically optimal, we show in Figure 2 that the decoding rule in (10) achieves close-to-optimal performance even in moderate SNR.

In order to characterize the per-user outage probability, let's define $S_i = \{0, 1\}$ as the i-th stage iteration decoding event with $S_i = 1$ denotes successful decoding and $S_i = 0$ denotes decoding failure. Given the asymptotically optimal decoding order policy in (10), we assume that for user $\pi_b^*(i)$ fails in the i-th decoding iteration, then we can declare packet error for all the remaining users in the same base station. This assumption cause a neglectable sub-optimality to the system performance which can be verified by numerical simulation, however, it can provide a tractable analysis expression and provide some important insights regarding the system performance.

Next, we define the event S_i which is given by

$$S_i = \mathcal{I}\left\{r_{\pi_b^*(i)} < \log_2\left(1 + \text{SINR}_{\pi_b^*(i)}\right)\right\} = \{0, 1\}, \qquad (11)$$

where $\text{SINR}_{\pi_b^*(i)} = \frac{\gamma_{\pi_b^*(i)}}{1 + \sum_{j<i} \gamma_{\pi_b^*(j)}(1 - S_j) + \sum_{j>i} \gamma_{\pi_b^*(j)}}$ is the signal-to-interference plus noise ratio and $\mathcal{I}(\cdot)$ is the *indicator function* which is 1 when the event is true and 0 otherwise. Since we assume that any packet error before the i-th stage will result decoding error in the remaining stages, we can define the following event

$$\mathcal{O}_i = \mathcal{I}\left\{r_{\pi_b^*(i)} < \log_2\left(1 + \frac{\gamma_{\pi_b^*(i)}}{1 + \sum_{j>i} \gamma_{\pi_b^*(j)}}\right)\right\}. \qquad (12)$$

Based on the above, we can duce that:

$$S_i = 0 \Rightarrow \mathcal{O}_1 \cup \mathcal{O}_2 \cup \ldots. \cup \mathcal{O}_i. \qquad (13)$$

Therefore, the packet outage probability of user k is given by

$$\begin{aligned} &\overline{P_{out_k}}(r_k, P, k; \mathbf{G}) \\ &\sum_{\pi_b^* \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \Pr\left[\mathcal{O}_1 \cup \mathcal{O}_2 \cup \ldots. \cup \mathcal{O}_i = 0 | \pi_b^*\right] \Pr(\pi_b^*) \\ &\leq \sum_{\pi_b^* \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \Pr\left[\mathcal{O}_j = 0 | \pi_b^*\right] \Pr(\pi_b^*) \qquad (14) \end{aligned}$$

By substituting (14) into (8), the average system goodput under the asymptotically optimal decoding order is given by

$$\begin{aligned} U_{gp}(P, R, \mathbf{\Pi}^*; \mathbf{G}) &= \sum_{k=1}^{K} r_k (1 - \overline{P_{out_k}}(r_k, P, k; \mathbf{G})) \quad (15) \\ &\geq \sum_{k=1}^{K} r_k \big(1 - \sum_{\pi_b^* \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \Pr\left[\mathcal{O}_j = 0 | \pi_b^*\right] \Pr(\pi_b^*)\big). \end{aligned}$$

C. *Per-user PER and Average System Goodput*

Under the asymptotically optimal decoding order in Lemma 2, the average system goodput and per-user outage probability can be expressed in term of the conditional outage probability. In order to solve the per-user outage probability and average system goodput, we should obtain the closed form expression of the conditional outage probability. For a given asymptotically optimal decoding order π_b^*, the conditional outage probability of user k in the j-th iteration can be expressed as:

$$\begin{aligned} \Pr\left[\mathcal{O}_j = 0 | \pi_b^*\right] &= \Pr(r_k > C_{\pi_b^*(k)}(\mathbf{H}, \mathbf{G}, \pi_b^*, k) | \pi_b^*) \\ &= \Pr\left\{\gamma_{\pi_b^*(j)} - \vartheta_{\pi_b^*(j)} \sum_{l=j+1}^{u_b} \gamma_{\pi_b^*(l)} < \vartheta_k\right\} \qquad (16) \end{aligned}$$

where $\vartheta_k = 2^{r_k} - 1$. In general, the conditional outage probability involve μ_b dimensions nested integration which is complicated and non-traceable when the dimension of integration grows. However, by taking the advantage of the additive Markov chain property from the exponential random variable order statistics, the conditional outage probability can be calculated by a one dimensional integration. We first introduce the following lemma.

Lemma 3 (Closed-form Expression of Conditional PER): The conditional outage probability of user k in the j-th iteration in (16) can be written in a summation of exponential functions which is given by

$$\Pr\left[\mathcal{O}_j = 0|\pi_b^*\right] \;=\; 1 - \sum_{l=j,\upsilon_l>0}^{\mu_b} \Psi_l \frac{\beta_l}{\upsilon_l} \exp(-\frac{\vartheta_k \beta_l}{\upsilon_l}) \quad (17)$$

where $\Psi_l = \prod_{i=j,i\neq l}^{\mu_b} \frac{\upsilon_l}{\upsilon_l - \frac{\beta_l}{\beta_i}\upsilon_i}$, $\upsilon_l = \frac{1-l\times\vartheta_k + j\times\vartheta_k}{l}$, $\beta_l = \frac{\sum_{u=1}^{l}\frac{1}{g_{\pi^*(u),b}}}{P_{\max} l}$, and $\vartheta_k = 2^{r_k} - 1$. ■

Proof: Please refer to the full paper [16].

After obtaining the closed-form of the conditional outage probability, we need to calculate the probability of a particular decoding order which is summarized in the following:

Lemma 4 (Probability of a Decoding Order Policy π_b): Consider a set of independent non-identical distributed (i.ni.d.) exponential random variables $X_1, X_2, X_3, \ldots, X_{\mu_b}$. From [17], the probability of a particular order $X_{i_1:\mu_b} < X_{i_2:\mu_b} < ... < X_{i_{\mu_b}:\mu_b}$ is given by

$$\Pr(X_{i_1:\mu_b} < X_{i_2:\mu_b} < \ldots < X_{i_{\mu_b}:\mu_b}) \quad (18)$$
$$= \frac{\beta_{i_1}\beta_{i_2}\beta_{i_3}\ldots\beta_{i_{\mu_b}}}{(\beta_{i_1}+\beta_{i_2}+\ldots+\beta_{i_{\mu_b}})(\beta_{i_2}+\beta_{i_3}+\ldots+\beta_{i_{\mu_b}})\ldots\beta_{i_{\mu_b}}}$$

As a result, the per-user outage probability is given by the following lemma.

Lemma 5 (Per-User Conditional PER with Macro-diversity): The average packet error probability of user k under the asymptotically optimal decoding order policy $\mathbf{\Pi}^*$ is given by:

$$\begin{aligned}
&\overline{P_{out_k}}(r_k, P_{max}; \mathbf{G}) \\
&\leq \sum_{\pi_b^* \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \Pr\left[\mathcal{O}_j = 0|\pi_b^*\right] \Pr(\pi_b^*) \\
&= \sum_{\pi_b^* \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \big(1 - \sum_{l=j,\upsilon_l>0}^{\mu_b} \Psi_l \frac{\beta_l}{\upsilon_l} \exp(-\frac{\vartheta_k\beta_l}{\upsilon_l})\big) \Pr(\pi_b^*)
\end{aligned} \quad (19)$$

where $\Pr(\pi_b^*)$ is given in equation (19) and $\vartheta_k = 2^{r_k} - 1$.

Therefore, the average system goodput can be summarized by the following theorem:

Theorem 1 (Lower Bound for the Average System Goodput):

$$\begin{aligned}
&U_{gp}(P, R, \mathbf{\Pi}^*; \mathbf{G}) = \sum_{k=1}^{K} r_k (1 - \overline{P_{out}}(r_k, P_k; \mathbf{G})) \\
&\geq \sum_{k=1}^{K} r_k \left\{ 1 - \sum_{\pi_b^* \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \Pr\left[\mathcal{O}_j = 0|\pi_b^*\right] \Pr(\pi_b^*) \right\} \\
&= \sum_{k=1}^{K} r_k \times \Bigg\{ 1 - \\
&\underbrace{\sum_{\pi_b^* \in B_k} \prod_{b \in B_k} \sum_{i=1}^{\pi_b^{*-1}(k)} \sum_{j=1}^{i} \sum_{l=j,\upsilon_l>0}^{\mu_b} \Psi_l \frac{\beta_l}{\upsilon_l} \exp(-\frac{\vartheta_k\beta_l}{\upsilon_l}) \Pr(\pi_b^*)}_{\text{Selection diversity protection}} \Bigg\}
\end{aligned} \quad (20)$$

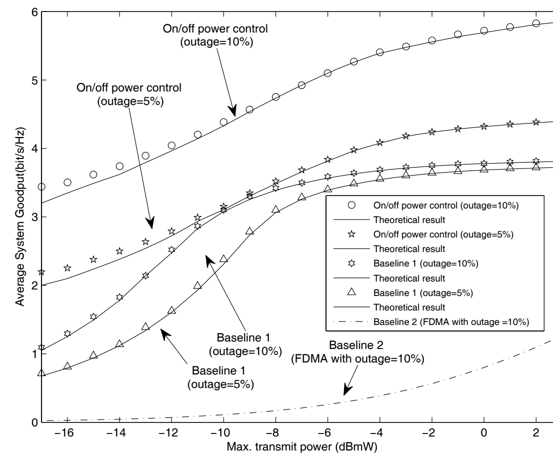

Fig. 3. Average system goodput versus max transmit power in a two cells system with K=10, outage requirement 5% or 10% for $\triangle_{\text{threshold}} = \infty$ (no MDiv) and different power control schemes.

From the above expression, the second summation represents the system goodput corresponds to each decoding permutation of the decoding rule in equation (10). The product term in (20) offers MDiv protection as a packet has to fail in all the base stations to declare packet error.

Remark 1: The data rate $r_k(\mathbf{G})$ can be determined by solving the per-user conditional packet error requirement $\overline{P_{out_k}}(r_k, P_{max}; \mathbf{G}) = \epsilon$.

IV. Results and Discussions

We consider a multi-cell system with 2 base stations. Every cell has radius of 1 km and path loss exponent 3.6. There are K active users uniformly distributed in the cells and the distance the mobile and b-th base station and the k-th mobile user is $d_{k,b}$. All the channel fading coefficients $\{H_1, H_2, ..., H_K\}$ are generated as i.i.d. complex Gaussian random realizations with zero mean and unit variance. Average system goodput is obtained by counting the number of packets which are successfully decoded by the base station for all users and average the result over both macroscopic and microscopic fading. In the simulation, each point is obtained by averaging 100000 macroscopic and microscopic realizations.

A. Average System Goodput

Figure 3 illustrates the average system goodput versus the transmit power (dBmW) of mobile user for $K = 10$ with asymptotical optimal decoding order. The optimal data rate of each user is obtained by numerical method such as Newton method in solving equation (20) for $\overline{P_{out_k}}(r_k, P_{max}; \mathbf{G}) = \epsilon$, $\forall k$. For the system goodput grows with SNR at small SNR[2] but quickly saturated at moderate SNR. This is because the performance is always limited by the weakest users. On the other hand, the goodput performance of the proposed on/off power control scheme does not saturate even at high SNR regime. It can be explained that in the proposed on/off power

[2]Because all active users are transmitting at their max power, therefore the SNR of each user is directly proportional to the max power.

control, strong users do not required to decrease the transmission power to maintain the same SINR as those weak users, this factor contribute significantly to the system goodput. Furthermore, we compare the proposed design with a baseline 2 (FDMA system) where each user transmits at its peak power. Although multiple access interference does not exist in the FDMA system due to orthogonal transmission, it has a very low spectral efficiency. On the contrary, the proposed design provides a substantial performance gain compared with the FDMA system in the interference limited environment.

Figure 4 shows the average system goodput versus the transmit power with different MDiv threshold ($\triangle_{\text{threshold}}$). Each user is power controlled by the on/off scheme and there is 5% outage probability requirement. We compare the performance of the proposed design with a system that does not perform MDiv in which all the inter-cell users are treated as interference. For the system without MDiv, the average system goodput saturated at high SNR because strong interference from inter-cell becomes a dominate factor in the system performance. On the contrary, the average system goodput of the proposed design increase with the transmit power when MDiv is performed in the base station. The reason is that strong interference is regarded as desired user signal and it will be decoded by corresponding base station. Furthermore, the optimal power control (either full power transmission or completely silent) create a high disparities[3] received power at the base station and strong enough interference environment for MDiv to exploit. Therefore, the system goodput has a significant gain when MDiv is performed in multi-cell environment. Furthermore, the goodput of the joint ML detection (which consider common outage[4]) is plotted for comparison. In low SNR regime, the SIC outperforms the joint ML detection since the outage performance of the joint ML detection is always limited by the weakest users. On the other hand, the SIC approach consider per-user outage and packets for some users may be decoded correctly even though the rate vector lies outside the capacity region. In high SNR region, the performance of SIC is limited by strong interference from both intra-cell and inter-cell interference. Nevertheless, using MDiv, the performance of the SIC scheme can be improved at high SNR regime. On the other hand, the joint ML detection does not suffer from multi-user interference and hence the performance is able to scale with SNR.

Similarly, Figure 5 shows the average system goodput versus different value of path loss exponent for K=10. Along all the curves, the same user transmit power is fixed at -10 dBmW and -3 dBmW respectively. It is very interesting that the average system goodput first increase with the value of path loss exponent and then decrease when the path loss exponent is beyond certain value. This counter intuitive result is due to the fact that when the path loss exponent increases, both desired signal and interference signal received by base station decreases. However, the attenuation of interference occurs to be larger than desired signal because interference users are usually located far away from desired home base station. As path loss exponent increases, the operating region of the system is shifting from interference limited region to noise limited region, and the desired users signal strength attenuate to a level that high data rate communication is impossible, and it results in a decreasing trend of average system goodput.

Figure 6 depicts that average number of system goodput versus the number users in a two cells system. Similarly, the transmit power of an active user is again fixed at -10 dBmW and -3 dBmW respectively. It can be observed that the system goodput gain due to MDiv is not significant when the number of user is small, especially users are transmitting at low power (-10 dBmW). When the transmit power is low, signal strength of interference can not satisfy the MDiv threshold requirement, so there is nearly no MDiv performed in the base stations. However, when the number of users increases, it is more likely that there exists a user who locates near the cell boundary, creates large interference to neighboring cells. Therefore, base

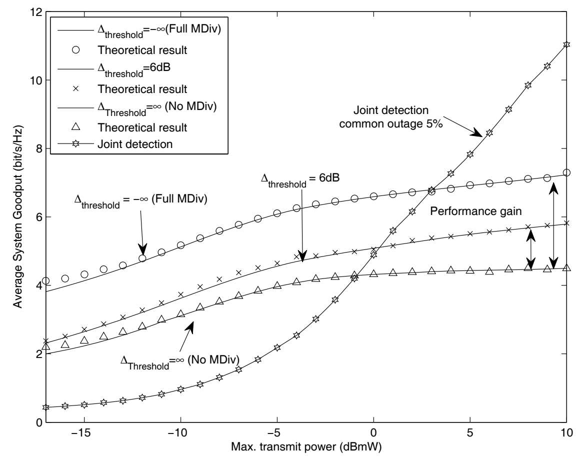

Fig. 4. Average system goodput versus max transmit power with different MDiv threshold in a two cells system, $K = 10$. Each user has an outage requirement $= 5\%$ and is power controlled by on/off transmission. The double sided arrow represents the performance gain due to MDiv.

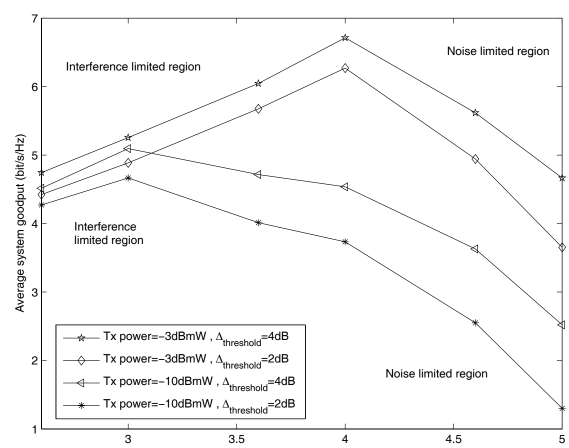

Fig. 5. Average system goodput versus path loss exponent in a two cells system and K=10. Each user has a outage requirement 10%. Transmit power of users are on/off power controlled and the max power are fixed at -3 dBmW and -10 dBmW. respectively.

[3]A high disparities received power can significantly increase the system capacity for MUD-SIC receiver [18].

[4]Common outage is declared as rate vector is outside the instantaneous capacity region.

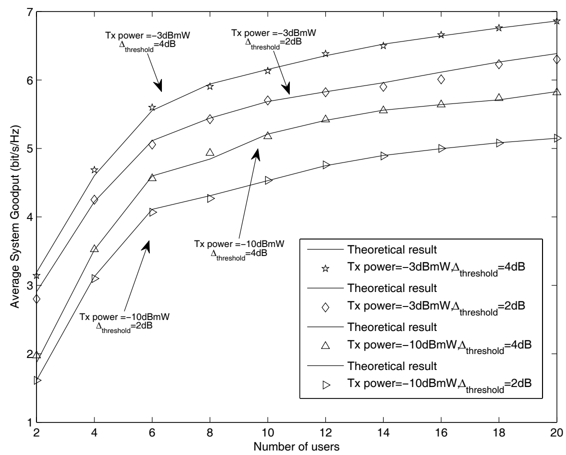

Fig. 6. Average system goodput versus number of users in a two cells system. Each user has a outage requirement 10%. Transmit power of users are on/off power controlled and the max power is fixed at -3 dBmW and -10 dBmW, respectively.

stations can take advantage of the strong interference and perform MDiv to improve the system goodput. On the other hand, there is a diminishing return in the system goodput when the number of users increases, particular in high transmit power with small MDiv threshold value (2 dB). This is due to the fact that base stations do not fully utilize the benefit of strong interference by setting a small threshold value, therefore interference can not be decoded and causes the degradation in system performance.

V. CONCLUSION

In this paper, a generic multi-cell system with K client users, n_B base stations and a centralized controller is considered. Based on the asymptotic optimal decoding order with respect to the transmit power, we incorporate the mathematical tool of order statistics to obtain the closed-form solution of system performance. Numerical simulations result are obtained to verify the analytical expressions. The closed form solutions allow efficient numerical evaluations to find out how the system performance is affected by the system parameter such as number of users and path loss exponent. From the results, we see that in interference limited region (users transmit at high power), MDiv improves the system goodput significantly by introducing macro-diversity protection to alleviate the consequences of error propagation. Furthermore, system with MDiv allows more users to be served at the same time through taking advantage of strong interference.

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Derrick Wing Kwan Ng (S'06) received the bachelor degree with First class honor and Master of Philosophy (MPhil) degree in electronic engineering from the Hong Kong University of Science and Technology (HKUST) in 2006 and 2008, respectively. He is currently working toward the Ph.D. degree in the University of British Columbia (UBC).

Vincent K.N.Lau (M'98-SM'01) obtained a B.Eng (Distinction 1st Hons) from the University of Hong Kong (1989-1992) and a Ph.D. from Cambridge University (1995-1997). He was with HK Telecom (PCCW) as system engineer from 1992-1995 and Bell Labs - Lucent Technologies as a member of the technical staff from 1997-2003. He joined the Department of ECE, Hong Kong University of Science and Technology (HKUST) as an Associate Professor and current a professor. At the same time, he is a technology advisor of HKASTRI and currently as a professor, leading the Advanced Technology Team on Wireless Access Systems. His current research focus is on the robust cross layer scheduling for MIMO/OFDM wireless systems with imperfect channel state information, communication theory with limited feedback as well as cross layer scheduling for users with heterogeneous delay requirements.