Energy-Efficient Power Allocation for M2M Communications with Energy Harvesting Transmitter

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***Abstract*—In this paper, power allocation for energy-efficient point-to-point machine-to-machine (M2M) communication systems with multiple energy harvesting sources is studied. Under a deterministic system setting, we formulate the power allocation problem as a non-convex optimization problem over a finite horizon taking into account the circuit energy consumption, finite battery storage capacities, and a minimum required data rate. The considered non-convex optimization problem is transformed into a convex optimization problem by exploiting the properties of fractional programming which results in an efficient optimal off-line iterative power allocation algorithm. In each iteration, the transformed problem is solved by using dual decomposition and a recursive power allocation solution is obtained for maximization of the energy efficiency of data transmission (bit/Joule delivered to the receiver).**

I. Introduction

Recently, a large amount of work has been devoted to machine-to-machine (M2M) communication due to its wide spread applications in e-health care, smart city, and remote monitoring, etc. In practice, M2M sensor type devices are usually small and inexpensive which puts stringent constraints (i.e., bandwidth and energy consumption) on the system design [1]. On the other hand, green communication has received much attention in recent years driven by environmental concerns [2], [3]. As a result, M2M communication systems are not only envisioned to be energy-efficient, but also to be self-sustainable. In the literature, a tremendous number of green technologies/methods have been proposed for maximizing the energy efficiency (bit-per-Joule) of wireless communication systems [3]-[7]. Among these technologies, energy harvesting is particularly appealing and suitable for M2M communication since each M2M device can harvest energy from natural renewable energy sources such as solar, wind, and vibration, etc, thereby reducing substantially the operating cost of the service providers.

The introduction of energy harvesting capabilities into M2M systems poses many interesting new challenges for the transmission design due to the time varying availability of renewable energy sources. In [4] and [5], optimal packet scheduling and power allocation algorithms were proposed for energy harvesting systems to minimize the transmission completion time, respectively. However, these works assumed an infinite battery capacity in the energy harvester and the obtained results may not be applicable to the case of finite battery storage. Besides, they did not take into account the circuit energy consumption and the maximum energy efficiency of these systems is still unknown even for the case of point-to-point communication. In [6] and [7], the authors proposed an optimal power control time sequence for maximizing the throughput by a deadline with a single energy harvester for different channel scenarios. Yet, the intermittent nature of energy harvesting of a single energy source will cause the power availability at the M2M device to be highly random. In other words, such single energy harvester design may not be able to guarantee the demanding quality of service requirements of M2M applications such as a minimum data rate requirement.

In this paper, we address the above issues. We study the structure of the optimal *off–line* power allocation solution where we assume that non-causal information of channel state information (CSI) and energy arrivals is available at the transmitter. The derived *off–line* solution constitutes a performance upper bound which sheds some light on the design of efficient *on–line* solutions in future research. We formulate the power allocation problem for energy-efficient M2M communication with multiple energy harvesters as an optimization problem. By using nonlinear fractional programming, the considered non-convex optimization problem in fractional form is transformed into an equivalent optimization problem in subtractive form with a tractable solution, which can be found with an iterative algorithm. In each iteration, dual decomposition is used and a recursive closed-form power allocation solution is computed for maximization of the system energy efficiency.

II. System Model

We consider a single link continuous time narrowband M2M communication system. The transmission time is T seconds. We assume that the transmitter adapts the power allocation L times for a given value of T. Note that the optimal value of L and the time instant of each adaption operation will be found in the next section. The data symbol received at the user at time instant t, $0 \le t \le T$, is given by

$$y(t) \quad = \quad \sqrt{P(t)g(t)}h(t)x(t) + z(t), \tag{1}$$

where $P(t)$ and $x(t)$ are the transmitted power and the transmitted symbol at time t, respectively. $h(t)$ and $g(t)$ are the small scale fading coefficient and the path loss between transmitter and receiver at time t, respectively. $z(t)$ is the additive white Gaussian noise (AWGN) at time t with zero

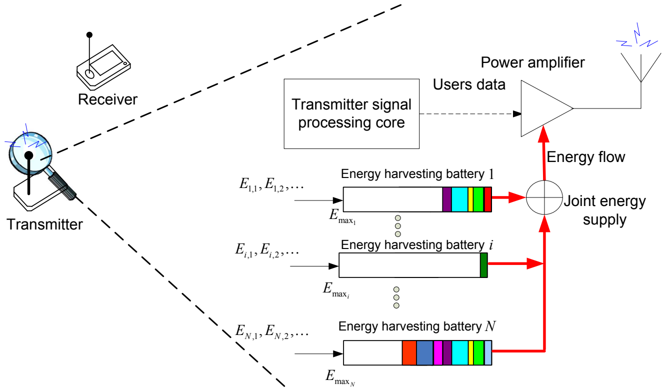

Fig. 1. A transmitter with multiple energy harvesting sources.

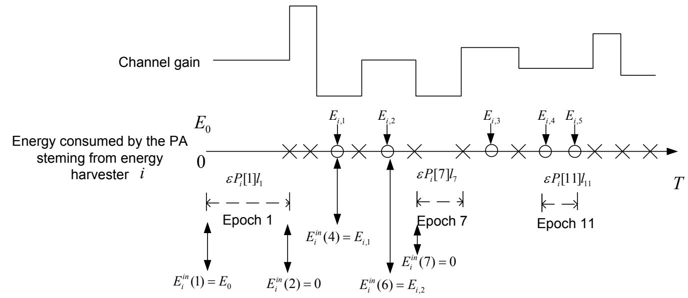

Fig. 2. An illustration of epoches and the meaning of $E_i^{in}(\cdot)$ for different events and different arrival time for energy harvester i. Fading changes and energies are harvested at time instants denoted by $\times$ and $\circ$, respectively.

mean and variance $N_0 W$, where N_0 is the noise power spectral density and W is the signal bandwidth.

A. Time Varying Fading Model and Energy Supply Model

We adopt a time varying system model similar to the one in [6]. At the transmitter, there are N different types of energy harvesters for supplying the energy required for transmission by the power amplifier (PA), cf. Figure 1. For instance, the M2M sensor can extract wind energy or solar energy with the help of a wind harvester and a solar energy harvester, respectively. As a result, the instantaneous total radio frequency (RF) transmit power at the PA in time instant t can be written as

$$P(t) = \sum_{i=1}^{N} P_i(t), \quad 0 \le t \le T, \tag{2}$$

where $P_i(t)$ is the instantaneous power transmitted by the power amplifier, which is fueled by energy harvester i, $i \in \{1, \ldots, N\}$.

We assume that the changes in the channel gains and energy arrivals in energy harvester i are stochastic processes in time which can be modeled as Poisson counting processes with rates λ_F and λ_{E_i}, respectively [6], [7]. Therefore, changes in the channel gains and the energy arrivals, respectively, occur in a countable number of time instants, which are indexed as $t_1^F, t_2^F \ldots$ and $t_1^{E_i}, t_2^{E_i} \ldots$, respectively. The inter-occurrence times $t_a^F - t_{a-1}^F, a \in \{1, 2, \ldots\}$, and $t_b^{E_i} - t_{b-1}^{E_i}, b \in \{1, 2, \ldots\}$, are exponentially distributed with means $1/\lambda_F$ and $1/\lambda_{E_i}$, respectively. Note that we set $t_0^{E_i} = t_0^F = 0$ for convenience. A block fading time varying communication channel model is considered. In other words, the fading level in $0 < t \le t_1^F$ is constant but changes to an independent value in the next time interval $t_1^F < t \le t_2^F$, and so on. Similarly, $E_{i,a}$ units of energy arrive (to be harvested) at time $t_a^{E_i}$ in energy harvester i, cf. Figure 2. The incoming energies are collected by the N energy harvesters and buffered in the battery before they are used in data transmission. On the other hand, we assume that E_0 units of energy arrive/(are available) in the battery of energy harvester i at $t_0^{E_i} = 0$ and the maximum amount of energy storage in the battery is denoted by $E_{\max_i}$. In the following, we refer to a change in the channel gains or in the energy level in any one of the batteries as an *event* and to the time interval between two consecutive events as an *epoch*. Specifically, *epoch* a is defined as the time interval $[t_{a-1}, t_a)$, where t_{a-1} and t_a are the times at which successive events happen, cf. Figure 2.

B. Physical Constraints on the Energy Harvesters

There are two inherent constraints on each energy harvester:

$$\text{C1:} \int_0^{t_b^E - \delta} \varepsilon P_i(u)\, du \le \sum_{j=0}^{b-1} E_{i,j}, \ \forall b \in \{1, 2, \ldots\}, \forall i \tag{3}$$

$$\text{C2:} \sum_{j=0}^{d_i(t)} E_{i,j} - \int_0^t \varepsilon P_i(u)\, du \le E_{\max_i}, \quad 0 \le t \le T, \forall i, \tag{4}$$

where $E_{i,j}$ is the amount of energy harvested by energy harvester i at energy arrival j at time t_j. $\delta \to 0$ is an infinitesimal positive constant for modeling purpose. $d_i(t) = \arg\max_a \{t_a^{E_i} : t_a^{E_i} \le t\}$ and $\varepsilon \ge 1$ is a constant which accounts for the inefficiency of the PA. For instance, if $\varepsilon = 5$, 50 Watts of power are consumed in the PA for every 10 Watts of power radiated in the RF and the power efficiency is $\frac{1}{\varepsilon} = \frac{1}{5} = 20\%$. Constraint C1 implies that in every time instant, if the transmitter draws energy from energy harvester i to cover the energy required at the PA, it is constrained to use at most the amount of stored energy currently available in energy harvester i (causality), although there will be possibly more energy arrivals in the future. Constraint C2 states that the energy level in energy harvester i never exceeds $E_{\max_i}$ to prevent the occurrence of an energy overflow in the battery.

III. Power Optimization Problem Formulation

In the following, we design the power allocation algorithm based on an information theoretic approach which inherently assumes that the data buffer at the transmitter is always full[1].

[1]In practice, if there is no data in the buffer, the transmitter can simply shut off the PA and store all the harvested energy if possible.

A. Instantaneous Channel Capacity

In this subsection, we define the adopted system performance measure. Given perfect CSI at the receiver, the channel capacity between the transmitter and receiver over a transmission period of T second(s) with bandwidth W is given by

$$\begin{aligned} C(\mathcal{P}) &= \int_0^T W \log_2 \Big(1 + P(t)\Gamma(t)\Big) dt \text{ and} \\ \Gamma(t) &= \frac{g(t)|h(t)|^2}{N_0 W}, \end{aligned} \tag{5}$$

where $\Gamma(t)$ is the received channel gain-to-noise ratio (CNR) at the receiver at time t and $\mathcal{P} = \{P_i(t) \geq 0, \forall i, 0 \leq t \leq T\}$ is the power allocation policy. On the other hand, we take into account the total energy consumption of the system by including it in the optimization objective function. For this purpose, we model the weighted *energy dissipation* in the system as the sum of two terms which can be expressed as

$$U_{TP}(\mathcal{P}) = \int_0^T \varepsilon \sum_{i=1}^{N} \phi_i P_i(t)\, dt + P_C T, \tag{6}$$

where $\phi_i > 0$ is a non-negative constant weight imposed on the use of energy harvester i and $\phi_i \neq \phi_k, \forall i \neq k$. In particular, the value of ϕ_i can be interpreted as the cost or preference in using energy harvester i. For instance, if energy harvester i exploits solar energy, the transmitter may prefer to use battery i on sunny days for transmission by setting $\phi_i \rightarrow 0$. On the other hand, P_C in (6) is the constant required signal processing power[2] at each time instant which includes the power dissipations in the mixer, transmit filter, frequency synthesizer, and digital-to-analog converter (DAC), etc. Hence, the *weighted energy efficiency* of the considered system over a time period of T seconds is defined as the total average number of received bits/Joule

$$U_{eff}(\mathcal{P}) = \frac{C(\mathcal{P})}{U_{TP}(\mathcal{P})}. \tag{7}$$

B. Optimization Problem Formulation

The optimal power allocation policy, $\mathcal{P}^*$, can be obtained by solving

$$\begin{aligned} &\max_{\mathcal{P}} \; U_{eff}(\mathcal{P}) \\ &\text{s.t.} \quad \text{C1, C2,} \\ &\text{C3: } C(\mathcal{P}) \geq R_{\min}, \quad \text{C4: } \sum_{i=1}^{N} P_i(t) \leq P_{\max},\, 0 \leq t \leq T, \\ &\text{C5: } P_i(t) \geq 0, \quad \forall i, 0 \leq t \leq T, \end{aligned} \tag{8}$$

where C3 specifies the minimum system data rate requirement $R_{\min}$. C3 can also be interpreted as a delay constraint for data transmission since at least $R_{\min}$ amount of data has to be transmitted by the end of time T. In particicle, such constraint is needed for real time M2M communication services such as vehicle and asset tracking. Note that although variable $R_{\min}$ in C3 is not an optimization variable in this paper, a balance between energy efficiency and system capacity can be struck by varying $R_{\min}$. C4 is a constraint on the maximum transmit power of the transmitter. For instance, if Zigbee is used for M2M communication, the maximum transmit power is $P_{\max} = 1$ W in the US. C5 is the non-negative constraint on the power allocation variables.

[2]We assume that there is a constant energy supply from a non-renewable energy source (e.g. from power grid) for supplying the energy required in signal processing. Note that we can incorporate the constant energy supply into (6) by treating it as the N-th energy harvester which has the highest value of weight ϕ_N.

IV. Solution of the Optimization Problem

The optimization problem in (8) is non-convex due to the fractional form of the objective function. We note that there is no standard approach for solving non-convex optimization problems. In order to derive an efficient power allocation algorithm for the considered problem, we introduce the following transformation.

A. Transformation of the Objective Function

The objective function in (8) can be classified as nonlinear fractional program [8] and has some interesting properties that will be introduced in the following. Without loss of generality, we define the maximum energy efficiency q^* of the considered system as

$$q^* = \frac{C(\mathcal{P}^*)}{U_{TP}(\mathcal{P}^*)} = \max_{\mathcal{P}} \frac{C(\mathcal{P})}{U_{TP}(\mathcal{P})}. \tag{9}$$

Then, we can establish the following theorem.

Theorem 1: The maximum energy efficiency q^* is achieved *if and only if* the optimal power allocation policy satisfies the following condition:

$$\begin{aligned} &\max_{\mathcal{P}} \quad C(\mathcal{P}) - q^* U_{TP}(\mathcal{P}) \\ &= \quad C(\mathcal{P}^*) - q^* U_{TP}(\mathcal{P}^*) = 0, \end{aligned} \tag{10}$$

for $C(\mathcal{P}) \geq 0$ and $U_{TP}(\mathcal{P}) > 0$.

Proof: We can follow a similar approach as in [9] to prove Theorem 1. The detailed proof is omitted here because of space constraints.

By Theorem 1, for any optimization problem with an objective function in fractional form, there exists an equivalent[3] objective function in subtractive form, e.g. $C(\mathcal{P}) - q^* U_{TP}(\mathcal{P})$ in the considered case. As a result, we can focus on this equivalent objective function in the rest of the paper.

B. Iterative Algorithm for Energy Efficiency Maximization

In this section, we adopt an iterative algorithm (known as the Dinkelbach method) for solving (8) with an equivalent objective function. The proposed algorithm is summarized in Table I and the convergence to the optimal energy efficiency is guaranteed if we are able to solve the inner problem (11) in each iteration.

Proof: Please refer to [9] for a proof of convergence.

[3]Here, "equivalent" means that both problem formulations lead to the same optimal power allocation policy.

TABLE I
ITERATIVE POWER ALLOCATION ALGORITHM.

Algorithm 1 Iterative Power Allocation Algorithm

Initialize the maximum number of iterations L_{max} and the maximum tolerance ϵ
Set maximum energy efficiency $q = 0$ and iteration index $n = 0$
repeat {Main Loop}
  Solve the inner loop problem in (11) for a given q and obtain power allocation policy $\{\mathcal{P}'\}$
  if $C(\mathcal{P}') - qU_{TP}(\mathcal{P}') < \epsilon$ **then**
    Convergence $=$ **true**
    return $\{\mathcal{P}^*\} = \{\mathcal{P}'\}$ and $q^* = \frac{C(\mathcal{P}')}{U_{TP}(\mathcal{P}')}$
  else
    Set $q = \frac{C(\mathcal{P}')}{U_{TP}(\mathcal{P}')}$ and $n = n + 1$
    Convergence $=$ **false**
  end if
until Convergence $=$ **true** or $n = L_{max}$

As shown in Table I, in each iteration of the main loop, we solve the following optimization problem for a given parameter q:

$$\begin{aligned} \max_{\mathcal{P}} \quad & C(\mathcal{P}) - qU_{TP}(\mathcal{P}) \\ \text{s.t. } & \text{C1, C2, C3, C4, C5.} \end{aligned} \tag{11}$$

Solution of the Main Loop Problem: Although the objective function is now in a subtractive form which is easier to handle, there is still an obstacle in solving the above problem. The optimal power allocation policy is expected to be time varying in the considered duration of T seconds. However, it is unclear how often the transmitter should update the power allocation policy which is a hurdle for designing a practical power allocation algorithm. In order to strike a balance between solution tractability and computational complexity, we introduce the following lemma which provides valuable insight into the time varying dynamic of the optimal power allocation policy.

Lemma 1: The optimal power allocation policy maximizing the system energy efficiency does not change within an epoch.

Proof: Please refer to the Appendix for a proof of Lemma 1.

As revealed by Lemma 1, the optimal power allocation policy must be kept constant in each epoch for maximizing the system energy efficiency. As a result, we can discretize the integrals and continuous variables involved in (11). In other words, the number of constraints in (11) reduce to countable quantities. Without loss of generality, we assume that the channel states change $M \geq 0$ times and energy arrives $K \geq 0$ times in the N energy sources in the duration of $[0, T]$. Specifically, we have $L = M + K$ epoch(s) for the considered duration of T seconds which includes the epoch caused by E_0 at $t = 0$ for all energy harvesters. Besides, time instant T is treated as an additional fading epoch with zero channel gain to terminate the process. We define the length of each epoch as $l_j = t_j - t_{j-1}$ where epoch $j \in \{1, 2, \ldots, L\}$ is defined as the time interval $[t_{j-1}, t_j)$, cf. Figure 2. Note that t_0 is defined as $t_0 = 0$. For the sake of notational simplicity and clarity, we replace the continuous time variables with the corresponding discrete time variables, i.e., $P(t) \to P[j]$, $P_i(t) \to P_i[j]$, and $\Gamma(t) \to \Gamma[j]$. Then, the *weighted average system throughput* and the *total weighted energy consumption* can be re-written as

$$\begin{aligned} C(\mathcal{P}) &= \sum_{j=1}^{L} l_j C[j] \quad \text{and} \\ U_{TP}(\mathcal{P}) &= \sum_{j=1}^{L} l_j P_C + \sum_{j=1}^{L} l_j \varepsilon \sum_{i=1}^{N} P_i[j]\phi_i, \end{aligned} \tag{12}$$

respectively, where $C[j] = W \log_2 \Big(1 + (\sum_{i=1}^{N} P_i[j])\Gamma[j]\Big)$ is the channel capacity between the transmitter and the receiver in epoch l. As a result, the optimization problem in (11) is transformed into the following convex optimization problem:

$$\begin{aligned} & \max_{\mathcal{P}} \; C(\mathcal{P}) - qU_{TP}(\mathcal{P}) \\ & \text{C1:} \sum_{j=1}^{e} l_j \varepsilon P_i[j] \leq \sum_{j=1}^{e} E_i^{in}[j], \;\; \forall e, \forall i \\ & \text{C2:} \sum_{j=1}^{r} E_i^{in}[j] - \sum_{j=1}^{r-1} \varepsilon l_j P_i[j] \leq E_{\max_i}, \, \forall r, \forall i \\ & \text{C3:} \sum_{j=1}^{L} l_j C[j] \geq R_{\min}, \qquad \text{C4: } l_e P[e] \leq l_e P_{\max}, \, \forall e, \\ & \text{C5: } P_i[e] \geq 0, \, \forall i, e, \end{aligned} \tag{13}$$

where $e \in \{1, 2, \ldots, L\}$ and $r \in \{2, \ldots, L+1\}$. In (13), $E_i^{in}[j]$ is defined as the energy which arrives in epoch j in battery i. Hence, $E_i^{in}[j] = E_{i,a}$ for some a if event j is an energy arrival and $E_i^{in}[j] = 0$ if event j is a channel gain change, cf. Figure 2. Now, the transformed problem is jointly concave with respect to all optimization variables[4], and under some mild conditions [10], solving the dual problem is equivalent to solving the primal problem.

C. Dual Problem Formulation

In this subsection, we solve the power allocation and scheduling optimization problem by solving its dual. For this purpose, we first need the Lagrangian function of the primal problem which can be written as

$$\begin{aligned} \mathcal{L}(\boldsymbol{\gamma}, \boldsymbol{\beta}, \rho, \boldsymbol{\mu}, \mathcal{P}) &= \sum_{j=1}^{L} l_j (1+\rho) C[j] - \rho R_{\min} \\ & - \sum_{i=1}^{N} \sum_{j=1}^{L} \gamma_{i,j} \Big(\sum_{m=1}^{j} l_m \varepsilon P_i[m] - \sum_{m=1}^{j} E_i^{in}[m] \Big) \\ & - q \Big(\sum_{j=1}^{L} l_j P_C + \sum_{j=1}^{L} l_j \varepsilon \sum_{i=1}^{N} P_i[j] \phi_i \Big) \end{aligned}$$

[4]We can follow a similar approach as in Appendix A to prove the convexity of the above problem for the discrete time model.

$$-\sum_{i=1}^{N}\sum_{j=2}^{L+1}\beta_{i,j}\Big(\sum_{m=1}^{j}E_i^{in}[m]-\sum_{m=1}^{j-1}\varepsilon l_m P_i[m]-E_{\max_i}\Big)$$
$$-\sum_{j=1}^{L}\mu_j\Big(l_j\sum_{i=1}^{N}P_i[j]-l_jP_{\max}\Big), \tag{14}$$

where $\boldsymbol{\gamma}$ is the Lagrange multiplier vector associated with the causality constraint C1 in drawing energy from each energy harvester with elements $\gamma_{i,j}$, $i \in \{1,\ldots,N\}, j \in \{1,\ldots,L\}$. $\boldsymbol{\beta}$ is the Lagrange multiplier vector corresponding to the maximum energy level constraint C2 in the battery of the energy harvester with elements $\beta_{i,j}$ where $\beta_{i,1}=0, \forall i$. ρ is the Lagrange multiplier corresponding to the minimum data rate requirement $R_{\min}$ in C5. $\boldsymbol{\mu}$ is the Lagrange multiplier vector for constraint C4 on the maximum power with elements μ_j. Note that the boundary constraints C5 are absorbed into the Karush-Kuhn-Tucker (KKT) conditions when deriving the optimal solution in Section IV-D.

Thus, the dual problem is given by

$$\min_{\boldsymbol{\gamma},\boldsymbol{\beta},\rho,\boldsymbol{\mu}\geq 0}\ \max_{\mathcal{P}}\ \mathcal{L}(\boldsymbol{\gamma},\boldsymbol{\beta},\rho,\boldsymbol{\mu},\mathcal{P}). \tag{15}$$

D. Dual Decomposition and Sub-Problem Solution

By Lagrange dual decomposition, the dual problem is decomposed into two parts (nested loops): the first part (inner loop) is known as sub-problem; the second part (outer loop) is the master problem [9]. Then, the dual problem can be solved iteratively, where in each iteration the transmitter solves the sub-problem (inner loop) by using KKT conditions for a fixed set of Lagrange multipliers, and the master problem (outer loop) is solved using gradient method.

Let $P_i^*[j]$ denotes the optimal power allocation solution of the subproblem for energy harvester i in epoch j. Without loss of generality, we assume $\phi_1 < \phi_2 < \ldots < \phi_N$ for the sake of notational simplicity. Using standard optimization techniques and the KKT conditions, the optimal power allocations for the N energy sources in epoch j are given by the following recursive equation:

$$P_1^*[j]=\left[\frac{W(1+\rho)}{(\ln(2)A_1[j])}-\frac{1}{\Gamma[j]}\right]^+ \text{ and} \tag{16}$$
$$P_{i+1}^*[j]=\left[\frac{W(1+\rho)}{(\ln(2)A_{i+1}[j])}-\frac{1}{\Gamma[j]}-\sum_{d=1}^{i}P_d^*[j]\right]^+, \tag{17}$$
$$\text{where } A_i[j]=\sum_{e=j}^{L}\gamma_{i,e}\varepsilon-\sum_{e=j}^{L}\beta_{i,e+1}\varepsilon+q\phi_i\varepsilon+\mu_j. \tag{18}$$

The power allocation solutions in (16) and (17) can be interpreted as a form of water-filling. In particular, variable ρ forces the transmitter to assign more power for transmission if the data rate requirement $R_{\min}$ becomes stringent. Interestingly, the optimal values of $P_i^*[j], \forall i$, have a unidirectional dependence with each other according to the weights ϕ_i, i.e., the power drawn from an energy source with a higher weight depends on the power drawn from the energy sources with lesser weights, but not vice versa. Specifically, as can be seen in (17), $P_1^*[j]$ decreases the water-level in calculating the value of $P_{i+1}^*[j]$. In other words, $P_1^*[j]$ reduces the amount of energy drawn from the less preferable energy sources (higher values of ϕ_i) for maximization of energy efficiency.

E. Solution of the Master Dual Problem

To solve the master minimization problem in (15), i.e., to find $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, ρ, and $\boldsymbol{\mu}$ for a given $\mathcal{P}$, the gradient method can be used since the dual function is differentiable. The gradient update equations are given by:

$$\gamma_{i,j}(\varsigma+1)=\Big[\gamma_{i,j}(\varsigma)-\xi_1(\varsigma)\times\Big(\sum_{m=1}^{j}E_i^{in}[m]-\varepsilon P_i[m]l_m\Big)\Big]^+, \forall i,j, \tag{19}$$
$$\beta_{i,r}(\varsigma+1)=\Big[\beta_{i,r}(\varsigma)-\xi_2(\varsigma)\times\Big(E_{\max_i}-\sum_{m=1}^{r}E_i^{in}[m]+\sum_{m=1}^{r}\varepsilon l_m P_i[m]\Big)\Big]^+, \forall i,r, \tag{20}$$
$$\rho(\varsigma+1)=\Big[\rho(\varsigma)-\xi_3(\varsigma)\times\Big(\sum_{j=1}^{L}l_jC[j]-R_{\min}\Big)\Big]^+, \tag{21}$$
$$\mu_j(\varsigma+1)=\Big[\mu_j(\varsigma)-\xi_4(\varsigma)\times\Big(P_{\max}-\sum_{i=1}^{N}P_i[j]\Big)\Big]^+, \forall j, \tag{22}$$

where $j \in \{1,\ldots L\}$, $r \in \{2,\ldots L\}$, index $\varsigma \geq 0$ is the iteration index, and $\xi_u(\varsigma)$, $u \in \{1,\ldots,4\}$, are positive step sizes. Then, the updated Lagrange multipliers in (19)-(22) are used for solving the subproblem in (15) via updating the power allocation solution according to (16)-(18). Since the transformed problem in (13) is convex, the duality gap between dual optimum and primal optimum is zero and it is guaranteed that the iteration between the master problem and the subproblem converges to the optimal solution of (11) in the main loop, if the chosen step sizes satisfy the infinite travel conditions [10].

V. Results and Discussions

In this section, we evaluate the system performance using simulations. We assume a transmission duration of $T = 10$ seconds, a carrier center frequency of 2.4 GHz, a signal bandwidth of $W = 10$ kHz, a noise power of $N_0W = -134$ dBm, and the distance between transmitter and receiver is 50 meters. The small scale fading coefficients of the transmitter and receiver are generated as Rayleigh random variables with unit variances. The static circuit power consumption is set to $P_C = 23$ dBm [11], the minimum data rate requirement of the system is $R_{\min} = 200$ kbits/s, and the maximum transmit power is 1 W. The number of energy sources will be specified in each case study and each energy harvester has a maximum energy storage of $E_{\max_i} = 10$ J, $\forall i$ and an initial energy $E_0 = 0.05$ J in the battery. The amount of energy that can be harvested by each energy harvester in each energy epoch is assumed to be uniformly distributed in $[0,1]$ J [7]. The channel changes with rate $\lambda_f = 200$ ms. On the other hand, we assume a power efficiency of 35% in the PA, i.e., $\varepsilon = \frac{1}{0.35} = 2.8571$.

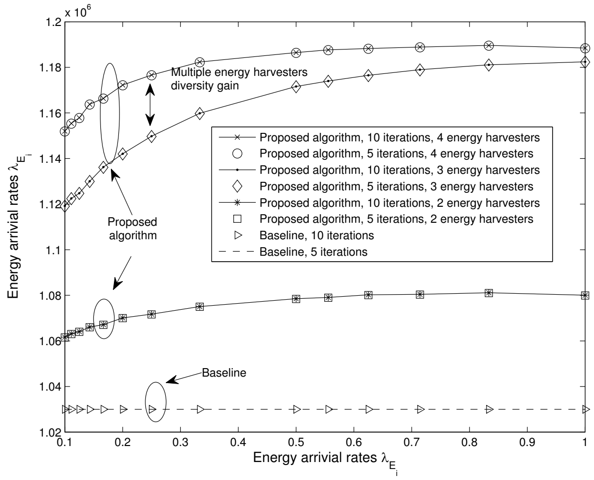

Fig. 3. Energy efficiency (bit-per-Joule) versus energy arrival rate, λ_{E_i}, for the proposed algorithm and the baseline with different numbers of energy harvesters.

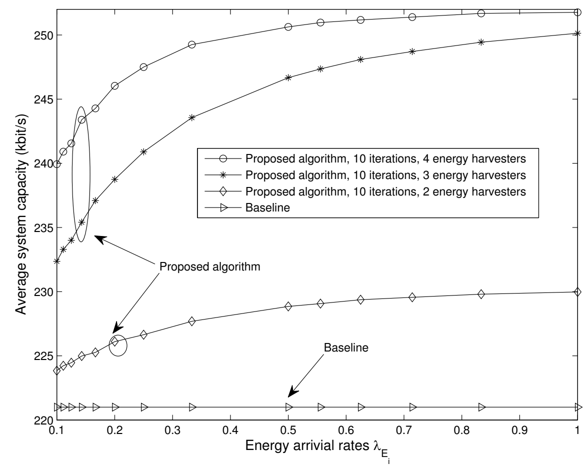

Fig. 4. Average system capacity (kbit/s) versus energy arrival rate, λ_{E_i}, for the proposed algorithm and the baseline.

Note that if the resource allocator is unable to guarantee the minimum data rate $R_{\min}$ in T, we set the energy efficiency and the average system throughput for that channel realization to zero to account for the corresponding failure. The average system energy efficiency is obtained by counting the number of bits which are successfully decoded by the receiver over the total energy consumption averaged over the microscopic fading. Unless further specified, in the following results, the "number of iterations" refers to the number of iterations of Algorithm 1 in Table I.

A. Energy Efficiency versus Energy Arrival Rates

Figure 3 illustrates the average energy efficiency versus the energy arrival rates, λ_{E_i}, for different numbers of energy harvesters. We define a vector $\vec{\phi} = [\phi_1 \dots \phi_i \dots \phi_N]$. For the case study of 1, 2, 3, and 4 energy harvester(s), the weight(s) of ϕ_i is/are set to $\vec{\phi_1} = [1]$, $\vec{\phi_2} = [0.5 \ \ 1]$, $\vec{\phi_3} = [0.1 \ \ 0.5 \ \ 1]$, and $\vec{\phi_4} = [0.1{-}\Delta \ \ 0.1{+}\Delta \ 0.5 \ 1]$, respectively, where $\Delta \to 0$ is a small positive constant for studying the effect of multiple energy harvester diversity. For an energy harvester with weight $\phi_i = 1$, a traditional continuous constant energy supply with an instantaneous power of 1 W is assumed. The case of $\vec{\phi_1} = [1]$ is treated as a baseline scheme for comparison. The energy harvesters with weights $\phi_i < 1$, represent some forms of clean energy such as solar energy and wind energy, etc. The number of iterations for the proposed iterative resource allocation algorithm is 5 and 10. It can be observed that the performance difference between 5 and 10 iterations is negligible which confirms the practicality of the proposed algorithm. On the other hand, the growth of energy efficiency has a diminishing return for high energy arrival rates. Indeed, when the energy arrival rate increases from a small value, the transmitter has a higher energy level in each battery for performing power allocation and thus the system energy efficiency is enhanced. However, when the arrival rates of energy become exceedingly large, the transmitter is forced to discharge the batteries in order to prevent a battery overflow, cf. C2 in (4). As a result, the transmitter has to transmit an excess amount of energy for discharging the batteries which decreases the system energy efficiency gain due to a higher energy arrival rate. It can be observed that there is an energy efficiency gain if we switch the case from $\vec{\phi_2}$ to $\vec{\phi_3}$. This is because in $\vec{\phi_3}$, a more energy efficient source is available for transmission compared to $\vec{\phi_2}$. Besides, a form of multiple energy harvester diversity can be observed in the energy efficiency when we switch from $\vec{\phi_3}$ to $\vec{\phi_4}$. Since $\Delta \to 0$, the performance gain is coming from the transmitter in exploiting energy from different energy sources which changes the intermittent nature of energy availability compared to the case of single energy source. On the other hand, the proposed algorithm provides a significant performance gain compared to the baseline scheme. This is because the baseline scheme can only drawn energy from a less energy-efficient source.

B. Average System Capacity versus Energy Arrival Rates

Figure 4 shows the average system capacity versus the energy arrival rates, λ_{E_i}, for different numbers of energy harvesters. We compare the system performance of the proposed algorithm again with the baseline scheme. The number of iterations in the proposed algorithm is set to 10. It can be observed that the average system capacity of the proposed algorithm increases with the energy arrival rates. This is because more energy is available for data transmission which results in a capacity gain. We note that, as expected, the baseline scheme achieves a smaller average system capacity than the proposed algorithm since the proposed algorithm is able to exploit energy from different energy sources in T seconds.

VI. Conclusion

In this paper, we formulated the power allocation algorithm design for a point-to-point M2M communication systems with multiple energy sources as a non-convex optimization problem, in which the circuit energy consumption, the finite battery storage capacity, and the system data rate requirement were taken into consideration. By exploiting the properties of nonlinear fractional programming, the considered problem was transformed into an equivalent convex optimization problem with a tractable solution. An efficient iterative off-line power allocation algorithm with recursive closed-form power allocation was derived for maximization of the energy efficiency. Simulation results did not only show that the proposed algorithm converges to the optimal solution within a small number of iterations, but unveiled also the achievable maximum energy efficiency. Interesting topics for future work include studying the optimal on-line solution in multi-channel M2M systems.

APPENDIX - PROOF OF LEMMA 1

The proof of Lemma 1 is divided into two parts. In the first part, we prove the convexity of the optimization problem in (11). Then, in the second part, we prove a necessary condition for the optimal power allocation policy based on the result in part one.

1) Proof of the Convexity of the Transformed Problem in (11): We first consider the concavity of the objective function on a per subcarrier basis with respect to all optimization variables. For the sake of notational simplicity, we define the channel capacity between the transmitter and the receiver at time instant t as $C(t) = W\log_2(1+P(t)\Gamma(t))$, respectively. Let the objective function in (11) at time instant t be $f(t,\mathcal{P}) = C(t) - q(\varepsilon\sum_{i=1}^{N}\phi_i P_i(t) + P_C t)$. Then, we denote the Hessian matrix of function $f(t,\mathcal{P})$ by $\mathbf{H}(f(t,\mathcal{P}))$ and the eigenvalues of $\mathbf{H}(f(t,\mathcal{P}))$ by φ_1, φ_2, $\ldots$, and φ_N, respectively. After some algebraic manipulation, the eigenvalues of $\mathbf{H}(f(t,\mathcal{P}))$ are given by

$$\varphi_1 = \varphi_2 = \ldots = \varphi_{N-1} = 0, \tag{23}$$

$$\varphi_N = \frac{-\Gamma^2(t) N \sum_{i=1}^{N} P_i(t)}{\ln(2)(\sum_{i=1}^{N} P_i(t)\Gamma(t)+1)^2} \leq 0. \tag{24}$$

Hence, $\mathbf{H}(f(t,\mathcal{P}))$ is a negative semi-definite matrix since $\varphi_i \leq 0$. Therefore, $f(t,\mathcal{P})$ is jointly concave with respect to (w.r.t.) optimization variables $P_i(t)$ at time instant t. Then, the integration of $f(t,\mathcal{P})$ over t preserves the concavity of the objective function in (11) [10]. On the other hand, the constraints C1-C5 in (11) span a convex feasible set and thus the transformed problem is a concave optimization problem.

2) Optimality of a Constant Power Allocation Policy in Each Epoch: Without loss of generality, we consider a time interval $[t_1, t_2)$ of *epoch 1* and a time instant τ_1, where $t_1 \leq \tau_1 < t_2$. Suppose an adaptive power allocation policy is adopted in $t_1 \leq \tau_1 < t_2$ such that two constant power allocation policies, $\{\mathcal{P}_1\}$ and $\{\mathcal{P}_2\}$, are applied in $t_1 \leq t < \tau_1$ and $\tau_1 \leq t < t_2$, respectively. We assume that $\{\mathcal{P}_1\}$ and $\{\mathcal{P}_2\}$ are feasible solutions to (11) while $\mathcal{P}_1 \neq \mathcal{P}_2$. Now, we define a third power allocation policy $\{\mathcal{P}_3\}$ such that $\mathcal{P}_3 = \frac{\mathcal{P}_1(\tau_1-t_1)+\mathcal{P}_2(t_2-\tau_1)}{t_2-t_1}$. Note that arithmetic operations between any two power allocation policies are defined element-wise. Then, we apply power allocation policy[5] $\{\mathcal{P}_3\}$ to the entire *epoch 1* and integrate $f(t,\mathcal{P})$ over time interval $[t_1, t_2)$ which yields:

$$\begin{aligned}\int_{t_1}^{t_2} f(t,\mathcal{P}_3)\,dt &\overset{(a)}{\geq} \int_{t_1}^{t_2} \frac{\tau_1-t_1}{t_2-t_1} f(t,\mathcal{P}_1) + \frac{t_2-\tau_1}{t_2-t_1} f(t,\mathcal{P}_2)\,dt \\ &= (\tau_1-t_1) f(t,\mathcal{P}_1) + (t_2-\tau_1) f(t,\mathcal{P}_2) \\ &= \int_{t_1}^{\tau_1} f(t,\mathcal{P}_1)\,dt + \int_{\tau_1}^{t_2} f(t,\mathcal{P}_2)\,dt,\end{aligned} \tag{25}$$

where (a) is due to the concavity of $f(t,\mathcal{P})$. In other words, for any adaptive power allocation policy within an epoch, there always exists at least one constant power allocation policy which outperforms the adaptive approach. As a result, the optimal power allocation policy is non-adaptive within each epoch.

[5] Power allocation policy $\{\mathcal{P}_3\}$ is also a feasible solution to (11) by the convexity of the feasible solution set.

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