

# Energy-Efficient Resource Allocation in Multi-Cell OFDMA Systems with Limited Backhaul Capacity

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**Abstract**—We study resource allocation for energy efficient communication in multi-cell orthogonal frequency division multiple access (OFDMA) downlink networks with cooperative base stations (BSs). We formulate the resource allocation problem for joint BS transmission as a non-convex optimization problem which takes into account the circuit power consumption, the limited backhaul capacity, and the minimum required data rate. By using the concept of perturbation function, we show that the duality gap for the considered problem is zero under some general conditions, despite the non-convexity of the primal problem. Thus, dual decomposition can be used to derive an efficient closed-form power allocation solution for maximization of the energy efficiency of data transmission (bit/Joule delivered to the users).

## I. INTRODUCTION

Cooperative communication for wireless networks has received considerable interest in both industry and academia as it provides extra degrees of freedom in resource allocation. A particularly interesting approach is base station (BS) cooperation for mitigation of strong multi-cell interference due to aggressive/universal frequency reuse in the network. In particular, all BSs participating in BS cooperation share the channel state information (CSI) and the data of all users through backhaul communication links, which enables coordinated transmission. However, the results in the literature [1], [2] are mostly based on the ideal backhaul assumption such that an unlimited amount of control signals, user CSI, and precoding data can be exchanged. In practice, the backhaul capacity can be limited due to the deployment costs of the backhaul links. On the other hand, the advantages of BS cooperation do not come for free. They have significant financial implications for service providers due to the high power consumption in electronic circuitries, radio frequency (RF) transmission, and data exchange via backhaul links. Thus, *energy efficiency* (bit-per-Joule) may be a better performance metric compared to *system capacity* (bit-per-second-per-Hz) in evaluating the utilization of resources in such systems.

## II. MULTI-CELL OFDMA NETWORK MODEL

We consider a multi-cell orthogonal frequency division multiple access (OFDMA) network which consists of a total of  $M$  base stations (BSs) and  $K$  mobile users. All transceivers are equipped with a single antenna, cf. Figure 1. We assume universal frequency reuse and the  $M$  BSs share a total bandwidth of  $B$  Hertz. The global CSI is assumed to be perfectly known at a *central unit* and all computations are performed in this unit. Based on the available CSI, the central unit determines the resource allocation policy and broadcasts it to all BSs via backhaul connections which are dedicated to control signals only. On the other hand, all BSs are cooperating with each other by sharing the CSI and the data symbols of all selected users via capacity limited backhaul communication links.

### A. OFDMA Channel Model

We consider an OFDMA system with  $n_F$  subcarriers. The channel impulse response is assumed to be time-invariant within

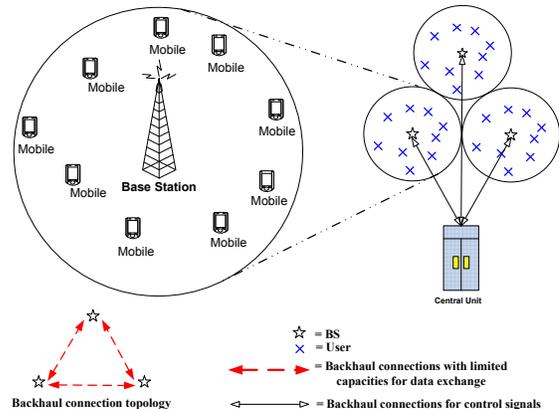


Fig. 1. A multi-cell system with  $M = 3$  cells and a fully connected backhaul link topology. There are in total  $K = 27$  users in the system. Each transceiver is equipped with a single antenna.

each frame. Suppose user  $k \in \{1, \dots, K\}$  is associated with BS  $m \in \{1, \dots, M\}$ . Let  $w_{B_m}^k(i)$  be the precoding coefficient used by BS  $m$  in subcarrier  $i \in \{1, \dots, n_F\}$  for user  $k$ . Given perfect CSI at the receiver, the maximum channel capacity between all the cooperating BSs and user  $k$  on subcarrier  $i$  with subcarrier bandwidth  $\frac{B}{n_F}$  is given by

$$C^k(i) = \frac{B}{n_F} \log_2 \left( 1 + \Gamma^k(i) \right), \quad (1)$$

$$\Gamma^k(i) = \frac{\left| \sum_{c=1}^M H_{B_c}^k(i) w_{B_c}^k(i) \sqrt{P_{B_c}^k(i) l_{B_c}^k} \right|^2}{\sigma_z^2 + I^k(i)}, \quad (2)$$

$$I^k(i) = \sum_{\substack{j \in \mathcal{S}(i) \\ j \neq k}} \left| \sum_{m=1}^M \sqrt{P_{B_m}^j(i) w_{B_m}^j(i) \sqrt{l_{B_m}^j} H_{B_m}^k(i)} \right|^2, \quad (3)$$

where  $\Gamma^k(i)$  and  $I^k(i)$  are the received signal-to-interference-plus-noise ratio (SINR) and the received interference power at user  $k$  on subcarrier  $i$ , respectively.  $P_{B_m}^k(i)$  is the transmit power for the link between BS  $m$  and user  $k$  in subcarrier  $i$  and  $\mathcal{S}(i)$  is the user set selected for using subcarrier  $i$ .  $l_{B_m}^k$  represents the path loss between BS  $m$  and user  $k$ ,  $\sigma_z^2$  is the variance of the additive white Gaussian noise (AWGN) in subcarrier  $i$  at user  $k$ , and  $H_{B_m}^k(i)$  is the small scale fading coefficient between BS  $m$  and user  $k$  in subcarrier  $i$ .

The *weighted system capacity* is defined as the total number of bits successfully delivered to the  $K$  mobile users and is given by  $U(\mathcal{P}, \mathcal{W}, \mathcal{S}) = \sum_{m=1}^M \sum_{k \in \mathcal{A}_m} \alpha_k \sum_{i=1}^{n_F} s^k(i) C^k(i)$  where  $\mathcal{P}$ ,  $\mathcal{W}$ , and  $\mathcal{S}$  are the power, precoding coefficient, and subcarrier allocation policies, respectively.  $\mathcal{A}_m$  is the user admission set of BS  $m$  and  $s^k(i) \in \{0, 1\}$  is the subcarrier allocation indicator.  $0 < \alpha_k \leq 1$  is a positive constant provided by the upper layers, which allows the resource allocator to give different priorities to different users. On the other hand, we model the power dissipation in the system as  $U_{TP}(\mathcal{P}, \mathcal{W}, \mathcal{S}) = P_C \times M + \delta \times P_{BH} + \sum_{m=1}^M \sum_{k=1}^K \sum_{i=1}^{n_F} \varepsilon P_{B_m}^k(i) |w_{B_m}^k(i)|^2 s^k(i)$ , where  $P_C$  is the constant signal processing power required at each BS which includes the power dissipations in the transmit filter,

mixer, frequency synthesizer, and digital-to-analog converter, etc.  $P_{BH}$  is a fixed power consumption of each backhaul connection.  $P_C \times M$  represents the total signal processing power consumed by the  $M$  BSs.  $\delta \times P_{BH}$  denotes the total power dissipation in the backhaul links where  $\delta$  is an integer variable which indicates the number of backhaul links in the system.  $\sum_{m=1}^M \sum_{k=1}^K \sum_{i=1}^{n_F} \varepsilon P_{B_m}^k(i) |w_{B_m}^k(i)|^2 s^k(i)$  represents the total power consumption in the power amplifiers of the  $M$  BSs.  $\varepsilon \geq 1$  is a constant which accounts for the inefficiency of the power amplifier.

Hence, the *weighted energy efficiency* of the considered system is defined as the total average number of bits/Joule

$$U_{eff}(\mathcal{P}, \mathcal{W}, \mathcal{S}) = \frac{U(\mathcal{P}, \mathcal{W}, \mathcal{S})}{U_{TP}(\mathcal{P}, \mathcal{W}, \mathcal{S})}. \quad (4)$$

### B. Optimization Problem Formulation

The optimal resource allocation policy can be obtained by solving

$$\begin{aligned} & \max_{\mathcal{P}, \mathcal{W}, \mathcal{S}} U_{eff}(\mathcal{P}, \mathcal{W}, \mathcal{S}) \\ \text{s.t. C1: } & \sum_{k=1}^K \sum_{i=1}^{n_F} |w_{B_m}^k(i)|^2 P_{B_m}^k(i) s^k(i) \leq P_{T_m}, \quad \forall m, \\ \text{C2: } & \sum_{m=1}^M \sum_{k \in \mathcal{A}_m} \sum_{i=1}^{n_F} s^k(i) C^k(i) \geq R_{\min}, \\ \text{C3: } & \sum_{k \in \mathcal{A}_m} \sum_{i=1}^{n_F} s^k(i) C^k(i) \leq R_{\max_m}, \quad \forall m, \\ \text{C4: } & \sum_{k=1}^K s^k(i) \leq M, \quad \forall i, \quad \text{C5: } s^k(i) = \{0, 1\}, \quad \forall i, k, \\ \text{C6: } & P_{B_m}^k(i) \geq 0, \quad \forall i, k, m, \end{aligned} \quad (5)$$

where C1 is an individual power constraint of BS  $m$ . C2 specifies the minimum system data rate requirement  $R_{\min}$ . Note that although variable  $R_{\min}$  in C2 is not an optimization variable in this paper, a balance between energy efficiency and aggregate system capacity can be struck by varying  $R_{\min}$ .  $R_{\max_m}$  in C3 puts a limit on the maximum transmission rate at each BS due to the limited backhaul capacities. C4 and C5 are imposed to guarantee that each subcarrier can be shared by  $M$  users, but each user can only use a subcarrier once.

### Solution of the Optimization Problem

Please refer to the related technical report [3] for the solution of the considered non-convex optimization problem.

## III. RESULTS

A multi-cell system with 3 cells is considered. The inter-site distance between each pair of BSs is 500 meters as suggested in the 3GPP specification. The number of subcarriers is  $n_F = 128$  with carrier center frequency 2.5 GHz, system bandwidth  $\mathcal{B} = 1.25$  MHz, and  $\alpha_k = 1, \forall k$ . Each subcarrier for RF transmission has a bandwidth of 9.7656 kHz and the noise variance is  $\sigma_z^2 = -134$  dBm. The 3GPP path loss model is used. The small scale fading coefficients of the BS-to-user links are generated as independent and identically distributed (i.i.d.) Rayleigh random variables with zero means and unit variances. We assume that all BSs have the same maximum transmit power, i.e.,  $P_{T_m} = P_T, \forall m$ . Besides, a

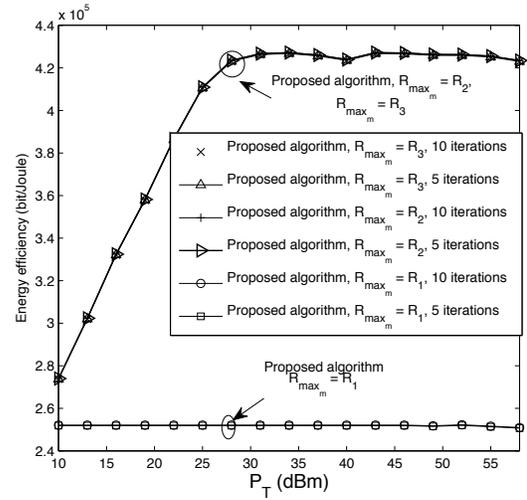


Fig. 2. Energy efficiency (bit-per-Joule) versus the maximum transmit power allowance at each BS,  $P_T$ , for different resource allocation algorithms and different backhaul capacities with  $K = 45$  users.

fully connected backhaul connection topology is considered for simulation purpose, i.e., there are  $\delta = 6$  connections, cf. Figure 1. For the backhaul connections, we adopt the specifications of a commercial optical fiber modem which supports three types of data rates for backhaul within a distance of 2.5 km:  $R_1 = 11.184$  Mbit/s,  $R_2 = 34.368$  Mbit/s, and  $R_3 = 44.736$  Mbit/s. The maximum power consumption of each backhaul link is  $P_{BH} = 15$  Watts. We assume a static circuit power consumption of  $P_C = 40$  dBm, and a data rate requirement of  $R_{\min} = 4$  bit/s/Hz/cell. Furthermore, we assume a power efficiency of 20% for the power amplifiers used in the RF, i.e.,  $\varepsilon = \frac{1}{0.2} = 5$ .

Figure 2 illustrates the energy efficiency versus the maximum transmit power allowance at each BS,  $P_T$ , for  $K = 45$  users. The number of iterations for the proposed iterative resource allocation algorithm is 5 and 10. It can be seen that the performance difference between 5 iterations and 10 iterations is negligible which confirms the practicality of our proposed iterative resource allocation algorithm. On the other hand, when both the maximum transmit power allowance at the power amplifier and the capacities of the backhaul links are large enough, e.g.,  $P_T \geq 30$  dBm and  $R_{\max_m} \geq R_2 \forall m$ , the energy efficiency of the proposed algorithm approaches a constant value since the resource allocator is not willing to consume more power, when the maximum energy efficiency is achieved. Besides, further increasing the backhaul capacities from  $R_{\max_m} = R_2 \forall m$  to  $R_{\max_m} = R_3 \forall m$  is not beneficial for energy efficiency as the system performance is now confined by the capacity of the radio links. However, for the case of backhaul capacity  $R_{\max_m} = R_1 \forall m$ , the energy efficiency is quickly saturated even if the transmit powers at the BSs are low since the system capacity is always limited by the bottleneck of the backhaul connections.

## REFERENCES

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