Energy-Efficient Resource Allocation in Multi-Cell OFDMA Systems with Limited Backhaul Capacity

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Abstract— **In this paper, resource allocation for energy efficient communication in multi-cell orthogonal frequency division multiple access (OFDMA) downlink networks with cooperative base stations (BSs) is studied. The considered problem is formulated as a non-convex optimization problem which takes into account the circuit power consumption, the limited backhaul capacity, and the minimum required data rate for joint BS zero-forcing beamforming (ZFBF) transmission. By exploiting the properties of fractional programming, the considered non-convex optimization problem in fractional form is transformed into an equivalent optimization problem in subtractive form, which enables the derivation of an efficient iterative resource allocation algorithm. For each iteration, the optimal power allocation solution is derived with a low complexity suboptimal subcarrier allocation policy for maximization of the energy efficiency of data transmission (bit/Joule delivered to the users). Simulation results illustrate that the proposed iterative resource allocation algorithm converges in a small number of iterations, and unveil the trade-off between energy efficiency and network capacity.**

I. Introduction

Cooperative communication for wireless networks has received considerable interest in both industry and academia as it provides extra degrees of freedom in resource allocation. A particularly interesting approach is base station (BS) cooperation for mitigating strong multi-cell interference due to aggressive/universal frequency reuse in the network. In the past decade, a number of interference mitigation techniques have been proposed in the literature, including successive interference cancellation (SIC) and interference nulling through multiple antennas, for alleviating the negative side-effects of aggressive/universal frequency reuse. Unfortunately, those techniques may be too complex for low-power battery driven mobile receiver units. On the contrary, BS cooperation shifts the signal processing burden to the BSs and provides a promising system performance [1]-[3]. In [1], the sum-rate of multi-cell zero-forcing beamforming (ZFBF) systems was studied under the assumption of the Wyner interference model for a large number of users. In [2] and [3], the authors investigated the optimal block diagonalization precoding matrix and the optimal max-min beamformer in multi-cell environments, respectively. However, the results in [1]-[3] are based on the ideal backhaul assumption such that an unlimited amount of control signals, user channel information, and precoding data can be exchanged. Besides, if a multi-carrier system is considered, the results in [1]-[3] which are valid for single-carrier transmission, may no longer be applicable.

Recently, an increasing interest in power hungry services such as video conferencing and online high definition (HD) video streaming has led to a tremendous demand for high data rate communications. Multi-cell orthogonal frequency division multiple access (OFDMA) with BS cooperation is considered as a possible solution for fulfilling this demand [4], [5], [6]. In [4] and [5], user assignment and BS assignment in multi-cell OFDMA systems with limited backhaul capacity constraints were studied, respectively. In [6], the authors proposed a dynamic frequency allocation scheme with fractional frequency reuse with equal power allocation across all cooperating BSs. A substantial capacity gain and a better interference management can be achieved, compared to non-cooperative systems in all studies [1]-[6]. Yet, the advantages of BS cooperation do not come for free. They have significant financial implications for service providers due to the high power consumption in electronic circuitries, radio frequency (RF) transmission, and data exchange via backhaul links. These factors have been overlooked in the literature, e.g. [1]-[6]. In fact, *energy efficiency* (bit-per-Joule) may be a better performance metric compared to *system capacity* (bit-per-second-per-Hz) in evaluating the utilization of resources.

In this paper, we address the above issues. For this purpose, we formulate the resource allocation problem for energy efficient communication in multi-cell OFDMA systems with limited backhaul capacity as an optimization problem. By exploiting the properties of fractional programming, the considered non-convex optimization problem in fractional form is transformed into an equivalent optimization problem in subtractive form with a tractable solution, which can be found with an iterative algorithm. In each iteration, a closed-form power allocation solution and a low complexity user selection policy are computed for maximization of the network energy efficiency.

II. Multi-Cell OFDMA network model

A. Multi-Cell System Model

We consider a multi-cell OFDMA network which consists of a total of M BSs and K mobile users. All transceivers are equipped with a single antenna, cf. Figure 1. We assume universal frequency reuse and the M BSs share the total bandwidth $\mathcal{B}$. The channel state information (CSI) is assumed to be perfectly known at a central unit and all computations are performed in this unit. All BSs are cooperating with each other by sharing the CSI and data symbols of all selected users via capacity limited backhaul communication links. Note that the energy consumptions incurred by exchanging CSI and other overheads are not considered here since they are relatively insignificant, compared to the resources used for data exchange. On the other hand, intra-cell interference does not exist since each subcarrier is only occupied by one user in each cell.

B. OFDMA Channel Model

We consider an OFDMA system with n_F subcarriers. The channel impulse response is assumed to be time-invariant within each frame. Suppose user $k \in \{1, \dots, K\}$ is associated[1]

[1]The data symbols for user k are first available at BS m only and are forwarded to other BSs via backhaul links.

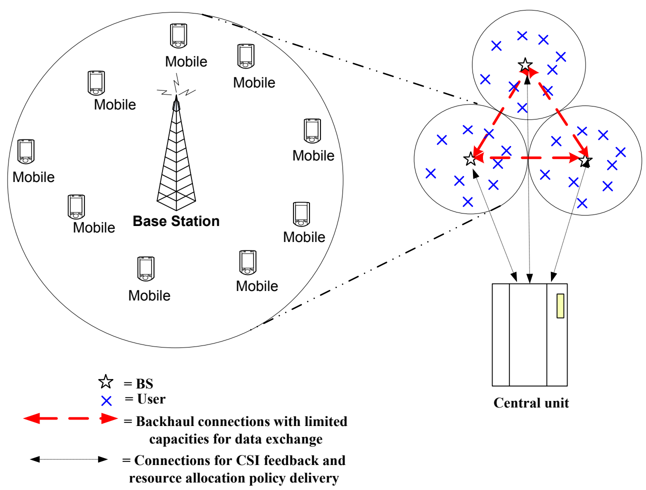

Fig. 1. A multi-cell system with $M = 3$ cells. There are in total $K = 27$ users in the system. Each transceiver is equipped with a single antenna.

with BS $m \in \{1, \dots, M\}$. Let $w^j_{B_m,B_c}(i)$ be the precoding coefficient used by BS m to suppress the inter-cell interference caused by BS $c \in \{1, \dots, M\}$ in subcarrier $i \in \{1, \dots, n_F\}$ for user $j \in \{1, \dots, K\}$. Then, the transmitted signal from BS m to all selected users on subcarrier i is given by

$$\sum_{k\in\mathcal{S}(i)} x^k_m(i) \;=\; \sum_{c=1}^{M} \sum_{k\in\mathcal{S}(i)} w^k_{B_m,B_c}(i)\sqrt{P^k_{B_m}(i)}u^k(i) \quad (1)$$

where $x^k_m(i) = \sum_{c=1}^{M} w^k_{B_{m,c}}(i)\sqrt{P^k_{B_m}(i)}u^k(i)$ is the pre-coded signal transmitted from BS m for user k on subcarrier i, $P^k_{B_m}(i)$ is the transmit power for the link between BS m and user k in subcarrier i, $u^k(i)$ is the transmitted information symbol for user k on subcarrier i, and $\mathcal{S}(i)$ is a user set selected for using subcarrier i and the cardinality of the set is $|\mathcal{S}(i)| \le M, \forall i$.

The received signal at user k in subcarrier i is given by

$$Y^k(i) = \Big(\sum_{c=1}^{M} H^k_{B_c}(i)w^k_{B_c,B_m}(i)\sqrt{P^k_{B_c}(i)l^k_{B_c}}\Big)u^k(i) \quad (2)$$
$$+\sum_{m=1}^{M}\sum_{\substack{c=1\\c\neq m}}^{M}\sum_{\substack{j\in\mathcal{S}(i)\\j\neq k}}\sqrt{P^j_{B_m}(i)l^k_{B_m}}H^k_{B_m}(i)w^j_{B_m,B_c}(i)u^j(i)+z^k(i),$$

where $l^k_{B_m}$ represents the path loss between BS m and user k. $z^k(i)$ is the additive white Gaussian noise (AWGN) in subcarrier i at user k with zero mean and variance σ_z^2. $H^k_{B_m}(i)$ is the small scale fading coefficient between the BS m and user k in subcarrier i.

C. Backhaul Model

To facilitate the implementation of an efficient resource allocation algorithm, we assume orthogonal frequency division multiplexing (OFDM) wireline backhaul connections[2] with n_F subcarriers. There are M such low cost backhaul connections between each pair of BSs for multiplexing the data symbols of a maximum of M users. In other words, multiuser interference does not exist in the backhaul links. Since user k is associated with BS m, BS m has to forward the data symbols of user k to the other $M-1$ BSs for joint cooperation. The data symbol of user k received at BS c from BS m is given by

$$Y^k_{B_m,B_c}(i) = P^k_{B_m,B_c}(i)G_{B_m,B_c}(i)u^k(i) + n^c(i), \quad (3)$$

where $P^k_{B_m,B_c}(i)$ and $G_{B_m,B_c}(i)$ are the allocated power and power attenuation factor in the backhaul connection between BS m and BS c in subcarrier i, respectively. $n^c(i)$ is the AWGN in subcarrier i at BS c with zero mean and variance $\sigma^2_{B_c}$.

III. Resource Allocation and Scheduling

A. Instantaneous Channel Capacity

In this subsection, we define the adopted system performance measure. Given perfect CSI at the receiver, the maximum channel capacity between all the cooperating BSs and user k on subcarrier i with subcarrier bandwidth $\frac{\mathcal{B}}{n_F}$ is given by

$$C^k(i) = \frac{\mathcal{B}}{n_F}\log_2\Big(1+\Gamma^k(i)\Big), \quad (4)$$

$$\Gamma^k(i) = \frac{\Big|\sum_{c=1}^{M} H^k_{B_c}(i)w^k_{B_c,B_m}(i)\sqrt{P^k_{B_c}(i)l^k_{B_c}}\Big|^2}{\sigma_z^2 + I^k(i)}, \quad (5)$$

$$I^k(i) = \Big|\sum_{m=1}^{M}\sum_{\substack{c=1\\c\neq m}}^{M}\sum_{\substack{j\in\mathcal{S}(i)\\j\neq k}}\sqrt{P^j_{B_m}(i)l^k_{B_m}}H^k_{B_m}(i)w^j_{B_m,B_c}(i)u^j(i)\Big|^2, \quad (6)$$

where $\Gamma^k(i)$ and $I^k(i)$ are the received signal-to-interference-plus-noise ratio (SINR) and the received interference power at user k on subcarrier i, respectively. On the other hand, we assume the total bandwidth of each backhaul link is also $\mathcal{B}$, thus the channel capacity between BS m and BS c in the backhaul link for the data of user k in subcarrier i is given by

$$C^k_{B_m,B_c}(i) = \frac{\mathcal{B}}{n_F}\log_2\Big(1+\frac{P^k_{B_m,B_c}(i)|G_{B_m,B_c}(i)|^2}{\sigma^2_{B_c}}\Big). \quad (7)$$

To simplify the subsequent mathematical expressions and without loss of generality, we assume in the following identical noise variances at all BSs and all subcarriers, i.e., $\sigma^2_{B_c} = \sigma^2_B$. The instantaneous capacity (bit/s/Hz successfully delivered to user k) of user k in subcarrier i is given by

$$\rho^k(i)=\min\Big\{C^k(i),\underbrace{C^k_{B_m,B_1}(i),C^k_{B_m,B_2}(i),\dots,C^k_{B_m,B_M}(i)}_{M-1\text{ backhaul connections}}\Big\}. \quad (8)$$

The *average weighted system capacity* is defined as the total average number of bits successfully delivered to the K mobile users and is given by

$$U(\mathcal{P},\mathcal{W},\mathcal{S}) = \sum_{m=1}^{M}\sum_{k\in\mathcal{A}_m}\alpha_k\sum_{i=1}^{n_F}s^k(i)\times\rho^k(i), \quad (9)$$

where $\mathcal{P}$, $\mathcal{W}$, and $\mathcal{S}$ are the power, precoding coefficient, and subcarrier allocation policies, respectively. $\mathcal{A}_m$ is the user admission set of BS m and $s^k(i) \in \{0,1\}$ is the subcarrier allocation indicator. $0 < \alpha_k \le 1$ is a positive constant provided by the upper layers, which allows the resource allocator to give different priorities to different users and to enforce certain notions of fairness. On the other hand, for designing a resource allocation algorithm for energy efficient communication, the total power consumption should be included in the optimization objective function. Thus, we model the power dissipation

[2]OFDM has been widely used in low cost backhaul wireline communication systems such as digital subscriber line (DSL) [7].

$U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})$ in the system as the sum of two dynamic terms and one static term:

$$U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S}) = \sum_{m=1}^{M}\sum_{\substack{c=1\\c\neq m}}^{M}\sum_{k\in\mathcal{A}_c}\sum_{i=1}^{n_F}\varepsilon P_{B_m,B_c}^{k}(i)s^k(i)$$
$$+P_C\times M+\sum_{m=1}^{M}\sum_{c=1}^{M}\sum_{k=1}^{K}\sum_{i=1}^{n_F}\varepsilon P_{B_m}^{k}(i)|w_{B_m,B_c}^{k}(i)|^2 s^k(i), \quad (10)$$

where $P_C > 0$ is a constant signal processing power in each BS. The first term in (10) represents the power consumption for data exchange via the limited capacity backhaul connections between the BSs. The last two terms represent the constant total circuit power consumption and the total power dissipation in the power amplifiers of the M BSs, respectively. $\varepsilon \geq 1$ is a constant which accounts for the inefficiency of the power amplifier. For example, if $\varepsilon = 5$, it means that for every 10 Watts of power radiated in the radio frequency (RF), 50 Watts are consumed in the power amplifier and the power efficiency is $\frac{1}{\varepsilon} = \frac{1}{5} = 20\%$. Hence, the *energy efficiency* of the considered system is defined as the total average number of bits/Joule

$$U_{eff}(\mathcal{P},\mathcal{W},\mathcal{S}) \quad = \quad \frac{U(\mathcal{P},\mathcal{W},\mathcal{S})}{U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})}. \quad (11)$$

B. Optimization Problem Formulation

The optimal power allocation policy, $\mathcal{P}^*$, precoding policy, $\mathcal{W}^*$, and subcarrier allocation policy, $\mathcal{S}^*$, can be obtained by solving

$$\begin{aligned}
&\max_{\mathcal{P},\mathcal{W},\mathcal{S}} \; U_{eff}(\mathcal{P},\mathcal{W},\mathcal{S})\\
\text{s.t.}\;\; &\text{C1:}\sum_{c=1}^{M}\sum_{k=1}^{K}\sum_{i=1}^{n_F}|w_{B_m,B_c}^{k}(i)|^2 P_{B_m}^{k}(i)s^k(i)\\
&+\sum_{\substack{c=1\\c\neq m}}^{M}\sum_{\substack{k=1\\k\in\mathcal{A}_c}}^{K}\sum_{i=1}^{n_F}P_{B_m,B_c}^{k}(i)s^k(i)\leq P_T, \quad \forall m\\
&\text{C2:}\sum_{m=1}^{M}\sum_{k\in\mathcal{A}_m}^{K}\sum_{i=1}^{n_F}s^k(i)\rho^k(i)\geq R,\\
&\text{C3:}\sum_{k=1}^{K}s^k(i)\leq M, \quad \forall i, \quad \text{C4: } s^k(i)=\{0,1\},\ \forall i,k,\\
&\text{C5: } P_{B_m}^{k}(i), P_{B_m,B_c}^{k}(i)\geq 0,\ \forall i,k,m,c, \qquad (12)
\end{aligned}$$

where C1 is a joint power constraint[3] of RF transmission and backhaul transmission for each BS. C2 specifies the minimum system data rate requirement R. Note that although variable R in C2 is not an optimization variable in this paper, a balance between energy efficiency and aggregate system capacity can be struck by varying R. C3 is the subcarrier reuse constraint. C3 and C4 are imposed to guarantee that each subcarrier can be shared by M users, but each user can only use a subcarrier once. In other words, selected users are not allowed to multiplex different messages on the same subcarrier, since a sophisticated receiver would be required at each user, such as a SIC receiver, to recover more than one messages.

[3] We assume that the power amplifiers in both the backhaul transmission and the RF transmission at each BS share a single power source.

TABLE I
ITERATIVE RESOURCE ALLOCATION ALGORITHM.

Algorithm 1 Iterative Resource Allocation Algorithm

Initialize the maximum number of iterations L_{max} and the maximum tolerance ϵ
Set maximum energy efficiency $q = 0$ and iteration index $n = 0$
repeat {Main Loop}
  Solve the inner loop problem in (15) for a given q and obtain resource allocation policies $\{\mathcal{P}',\mathcal{W}',\mathcal{S}'\}$
  if $U(\mathcal{P}',\mathcal{W}',\mathcal{S}') - qU_{TP}(\mathcal{P}',\mathcal{W}',\mathcal{S}') < \epsilon$ **then**
    Convergence = **true**
    return $\{\mathcal{P}^*,\mathcal{W}^*,\mathcal{S}^*\} = \{\mathcal{P}',\mathcal{W}',\mathcal{S}'\}$ and $q^* = \frac{U(\mathcal{P}',\mathcal{W}',\mathcal{S}')}{U_{TP}(\mathcal{P}',\mathcal{W}',\mathcal{S}')}$
  else
    Set $q = \frac{U(\mathcal{P}',\mathcal{W}',\mathcal{S}')}{U_{TP}(\mathcal{P}',\mathcal{W}',\mathcal{S}')}$ and $n = n+1$
    Convergence = **false**
  end if
until Convergence = **true** or $n = L_{max}$

IV. SOLUTION OF THE OPTIMIZATION PROBLEM

The objective function in (12) is a ratio of two functions which is generally a non-convex function. As a result, a brute force approach is required for obtaining a global optimal solution. However, such a method has exponential complexity with respect to the number of subcarriers and the number of users which is computationally infeasible even for small size systems. In order to derive an efficient resource allocation algorithm, we introduce the following transformation.

A. Transformation of the Objective Function

The objective function in (12) can be classified as nonlinear fractional program [8]. For the sake of notational simplicity, we define $\mathcal{F}$ as the set of feasible solutions of the optimization problem in (12) and $\{\mathcal{P},\mathcal{W},\mathcal{S}\} \in \mathcal{F}$. Without loss of generality, we define the maximum energy efficiency q^* of the considered system as

$$q^* = \frac{U(\mathcal{P}^*,\mathcal{W}^*,\mathcal{S}^*)}{U_{TP}(\mathcal{P}^*,\mathcal{W}^*,\mathcal{S}^*)} = \max_{\mathcal{P},\mathcal{W},\mathcal{S}} \frac{U(\mathcal{P},\mathcal{W},\mathcal{S})}{U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})}. \quad (13)$$

We are now ready to introduce the following Theorem.

Theorem 1: The maximum energy efficiency q^* is achieved if and only if

$$\begin{aligned}\max_{\mathcal{P},\mathcal{W},\mathcal{S}} \quad & U(\mathcal{P},\mathcal{W},\mathcal{S}) - q^*U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})\\ = \quad & U(\mathcal{P}^*,\mathcal{W}^*,\mathcal{S}^*) - q^*U_{TP}(\mathcal{P}^*,\mathcal{W}^*,\mathcal{S}^*) = 0, \quad (14)\end{aligned}$$

for $U(\mathcal{P},\mathcal{W},\mathcal{S}) \geq 0$ and $U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S}) > 0$.

for $U(\mathcal{P},\mathcal{W},\mathcal{S}) \geq 0$ and $U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S}) > 0$.

Proof: Please refer to Appendix A for a proof of Theorem 1.

By Theorem 1, for any optimization problem with an objective function in fractional form, there exists an equivalent[4] objective function in subtractive form, e.g. $U(\mathcal{P},\mathcal{W},\mathcal{S}) - q^*U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})$, in the considered case. As a result, we can focus on the equivalent objective function in the rest of the paper.

[4] Here, "equivalent" means that both problem formulations will lead to the same resource allocation policies.

B. Iterative Algorithm for Energy Efficiency Maximization

In this section, we propose an iterative algorithm (known as the Dinkelbach method [8]) for solving (12) with an equivalent objective function. The proposed algorithm is summarized in Table I and the convergence to the optimal energy efficiency is guaranteed.

Proof: Please refer to Appendix B for the proof of convergence.

As shown in Table I, in each iteration in the main loop, we solve the following optimization problem for a given parameter q:

$$\max_{\mathcal{P},\mathcal{W},\mathcal{S}} \quad U(\mathcal{P},\mathcal{W},\mathcal{S}) - qU_{TP}(\mathcal{P},\mathcal{W},\mathcal{S}) \quad \text{s.t. C1, C2, C3, C4, C5.} \tag{15}$$

1) Solution of the Main Loop Problem: The transformed problem is a mixed combinatorial and non-convex optimization problem. The non-convex nature comes from the power allocation variables and precoding coefficients. The multiuser interference appears in the denominator of the capacity equation in (4) which couples the power allocation variables. On the other hand, the combinatorial nature comes from the integer constraint for subcarrier allocation. To obtain an optimal solution, an exhaustive search is needed with complexity $n_F^{\sum_{g=1}^{M}\binom{K}{g}}$, which is computational infeasible for $n_F \gg K \gg M$. In order to derive an efficient resource allocation algorithm, we solve the above problem in two steps by fixing resource allocation policies $\{\mathcal{W},\mathcal{S}\}$. In the first step, we propose a low complexity sub-optimal user selection scheme. Then, in the second step, we derive the closed-form power allocation for a given selected user set with ZFBF precoding. Note that by fixing resource allocation policies $\{\mathcal{W},\mathcal{S}\}$, Algorithm 1 in Table I converges to a sub-optimal solution since only the power allocation is optimized for energy efficiency maximization.

Step 1 (Near Orthogonal User Selection): We propose an efficient user selection algorithm. Without loss of generality, we define a row vector $\vec{H}_{BS}^{k}(i) = \Big[\,H_{B_1}^k(i)\sqrt{l_{B_1}^k}\ H_{B_2}^k(i)\sqrt{l_{B_2}^k}\ \dots\ H_{B_M}^k(i)\sqrt{l_{B_M}^k}\,\Big]$ which represents the super-channel between all BSs and user k with elements $H_{B_m}^k(i)\sqrt{l_{B_m}^k}$, $k \in \{1,\dots,K\}$, $m \in \{1,\dots,M\}$, representing the channel coefficient between BS m and user k on subcarrier i. Let $\Delta\Big(\vec{H}_{BS}^k(i),\vec{H}_{BS}^j(i)\Big) = \frac{|\vec{H}_{BS}^k(i)(\vec{H}_{BS}^j(i))^\dagger|}{\|\vec{H}_{BS}^k(i)\|\|\vec{H}_{BS}^j(i)\|}$ where $\|\cdot\|$ and $[\cdot]^\dagger$ denote the Euclidean norm of a vector and the conjugate transpose operation, respectively. Then, a near orthogonal user set for subcarrier i, i.e., $\mathcal{S}_\perp(i)$, is given by

$$\mathcal{S}_\perp(i) = \Big\{k,j\Big|k,j=\{1,\dots,K\}, k = \arg\max_t \|\vec{H}_{BS}^t(i)\|^2, \quad \Delta\Big(\vec{H}_{BS}^k(i),\vec{H}_{BS}^j(i)\Big) \le \delta\times\alpha_j, \forall j\neq k\Big\}, \tag{16}$$

where δ represent a threshold for measuring orthogonality. $k = \arg\max_t \|\vec{H}_{BS}^t(i)\|^2$ represents the user who has the largest channel gain for joint BS transmission and is able to tolerate strong interference due to subcarrier reuse. Note that a user with higher value of α_k (priority) has a higher chance to be selected. On the other hand, as $\delta \to 0$, each selected user in the set is increasingly orthogonal to user k, i.e., the strongest user. In other words, users associated with the set cause less interference to the user with the strongest channel gain. Hence, we can first select the strongest user and then perform user selection on subcarrier i by selecting at most the $M-1$ smallest elements of $\Delta\Big(\vec{H}_{BS}^k(i),\vec{H}_{BS}^j(i)\Big)$ in set $S_\perp(i)$, since those users introduce less interference to the strongest user. Note that the search space of each subcarrier decreases from $\sum_{g=1}^{M}\binom{K}{g}$ to $2K-1$ and $\frac{2K-1}{\sum_{g=1}^{M}\binom{K}{g}} \ll 1$ for $K \gg M$. Note that although the proposed algorithm can only guarantee near orthogonality between the strongest user and each other selected user, it has been shown that the proposed scheme performs well with the following zero-forcing beamforming scheme [9].

Step 2 (Zero-Forcing Beamformimg): In fact, the multi-cell network with full BS cooperation can be considered as a MIMO broadcast channel. It can be shown that dirty paper coding (DPC) is optimal in achieving the multiuser broadcast capacity region. However, DPC requires a very high complexity which is considered impractical. On the contrary, although ZFBF is a suboptimal precoding scheme, it has been considered as a practical precoding solution, due to its linear complexity and promising performance. Besides, it can be shown that the near-orthogonal user selection algorithm together with ZFBF can achieve the same asymptotic sum capacity performance as DPC [9]. Therefore, we focus on ZFBF in the rest of the paper.

Since ZFBF is used for transmission, the capacity equation in (4) can be rewritten as

$$C^k(i) = \frac{\mathcal{B}}{n_F}\log_2\Big(1+\Gamma^k(i)\Big) \text{ and} \tag{17}$$

$$\Gamma^k(i) = \frac{\Big|\sum_{c=1}^{M}\sqrt{l_{B_c}^k}H_{B_c}^k(i)w_{B_c,B_m}^k(i)\Big|^2 P_B^k(i)}{\sigma_z^2}, \tag{18}$$

where $P_B^k(i) = P_{B_1}^k(i) = P_{B_2}^k(i) = \dots = P_{B_M}^k(i)$ due to ZFBF transmission. On the other hand, without loss of generality, we assume that user 1 to user k are selected for using subcarrier i by searching the orthogonal set $\mathcal{S}_\perp(i)$. Define a selected user set as $\widetilde{\mathcal{S}}_\perp(i) \subset \mathcal{S}_\perp(i)$ where $|\widetilde{\mathcal{S}}_\perp(i)| \le M$. Then, we define a super channel matrix $\mathbf{H}_B(i) \in \mathbb{C}^{|\widetilde{\mathcal{S}}_\perp(i)|\times M}$ such that

$$\mathbf{H}_B^T(i) = \Big[\big(H_B^1(i)\big)^T\big(H_B^2(i)\big)^T\dots\big(H_B^k(i)\big)^T\Big]. \tag{19}$$

Here, $\mathbb{C}^{|\widetilde{\mathcal{S}}_\perp(i)|\times M}$ and $(\cdot)^T$ are the space of all $|\widetilde{\mathcal{S}}_\perp(i)|\times M$ matrices with complex entries and the matrix transpose operation, respectively. Then, the corresponding ZFBF super matrix $\mathbf{B}(i) \in \mathbb{C}^{M\times|\widetilde{\mathcal{S}}_\perp(i)|}$ can be calculated in the centralized unit and is given by

$$\mathbf{B}(i) = \mathbf{H}_B^\dagger(i)\Big(\mathbf{H}_B(i)\mathbf{H}_B^\dagger(i)\Big)^{-1}\mathbf{D}(i), \tag{20}$$

where $\mathbf{D}(i) \in \mathbb{C}^{|\widetilde{\mathcal{S}}_\perp(i)|\times|\widetilde{\mathcal{S}}_\perp(i)|}$ is a diagonal matrix with diagonal elements $\gamma^k(i) = 1/\sqrt{\Big[\Big(\mathbf{H}_B(i)\mathbf{H}_B^\dagger(i)\Big)^{-1}\Big]_{k,k}} = \Big|\sum_{c=1}^{M}\sqrt{l_{B_c}^k}H_{B_c}^k(i)w_{B_c,B_m}^k(i)\Big|$. Here, operator $\big[\cdot\big]_{a,b}$ refers to the element in row a and column b of a matrix. Note that $\gamma^k(i)$ represents the equivalent channel gain between all BSs and user k on subcarrier i. Hence, the ZFBF coefficient $w_{B_c,B_m}^k(i)$ is given by

$$w_{B_c,B_m}^k(i) = \Big[\mathbf{B}(i)\Big]_{c,k}. \tag{21}$$

The centralized unit delivers the relevant ZFBF coefficients to each BS via another backhaul connection, cf. Figure 1.

Power Allocation Solution: It can be observed from (9) that for a joint power constraint of RF transmission power and backhaul transmission power in each BS, the capacity is always limited by a bottleneck link. Therefore, the maximum capacity between BSs and user k in subcarrier i occurs when

$$C_{B_m}^k(i)=C_{B_m,B_d}^k(i),\ \forall d=\{1,\dots,M\}\backslash m,$$
$$\Rightarrow P_{B_m,B_c}^k(i)G_{B_m,B_c}=P_{B_m,B_d}^k(i)G_{B_m,B_d},\forall c,d. \quad (22)$$

In other words, the transmit powers in the backhaul links in subcarrier i from BS m to the other $M-1$ BSs are identical. In order to derive a closed-form solution for power allocation, we define an auxiliary variable

$$\begin{aligned} P_{T_m}^k(i) &= P_{B_m}^k(i)|w_{B_m,B_m}^k(i)|^2+\sum_{c=1,c\neq m}^{M}P_{B_m,B_c}^k(i) \\ &= P_B^k(i)|w_{B_m,B_m}^k(i)|^2+\sum_{c=1,c\neq m}^{M}P_{B_m,B_c}^k(i) \quad (23)\end{aligned}$$

which represents the power consumption of BS m for user k in both RF transmission and backhaul transmission in subcarrier i. Note that since user k is associated with BS m, so BS m will not receive any data symbol of user k from other BSs, i.e, $P_{B_c,B_m}^k(i)=0,\forall c,k\in\mathcal{A}_m$. The problem in (15) with ZFBF and near orthogonal user selection is concave with respect to the power optimization variables, so by standard optimization techniques and Karush-Kuhn-Tucker (KKT) conditions, the closed-form power allocation for BSs to serve user k for a given parameter q is obtained as

$$P_{T_m}^k(i)=\Bigg[\frac{(\mathcal{B}/n_F)(\alpha_k+\eta)/\ln(2)}{q\varepsilon\big(1+\sum_{\substack{c=1\\c\neq m}}^{M}A_{B_c,B_m}^k(i)\big)+\lambda_m+\Omega_m^k(i)} \quad (24)$$

$$-\frac{|w_{B_m,B_m}^k(i)|^2\sigma_z^2+\Xi_{B_m}|\gamma^k(i)|^2}{|\gamma^k(i)|^2}\Bigg]^+, \quad (25)$$

$$\Xi_{B_m}=\sum_{c=1,c\neq m}^{M}\frac{\sigma_B^2}{G_{B_m,B_c}},\quad \Omega_m^k(i)=\sum_{\substack{c=1\\c\neq m}}^{M}\lambda_c A_{B_c,B_m}^k(i), \quad (26)$$

$$A_{B_c,B_m}^k(i)=\frac{\sigma_z^2|w_{B_c,B_m}^k(i)|^2}{|w_{B_m,B_m}^k(i)|^2\sigma_z^2+\Xi_{B_m}|\gamma^k(i)|^2}, \quad (27)$$

$$P_{B_m,B_c}^k(i)=\frac{|\gamma^k(i)|^2P_{T_m}^k(i)\sigma_B^2}{\big(\frac{|w_{B_m,B_m}^k(i)|^2\sigma_z^2}{\Xi_{B_m}}+|\gamma^k(i)|^2\big)(G_{B_m,B_c}\Xi_{B_m})}, \quad (28)$$

$$P_B^k(i)=P_{B_m}^k(i)=\frac{P_{T_m}^k(i)\sigma_z^2}{|w_{B_m,B_m}^k(i)|^2\sigma_z^2+\Xi_{B_m}|\gamma^k(i)|^2}, \quad (29)$$

where $[x]^+=\max\{0,x\}$. λ_m and η are the Lagrange multipliers chosen to satisfy the individual BS power constraint C1 and data rate requirement C2 in (12), respectively. $\Omega_m^k(i)$ represents the influence of the other BSs created by their power allocations on subcarrier i. The optimal values of λ_m and η can be easily found by using numerical methods such as the gradient method or the bisection method, due to the concavity of the transformed problem with respect to the power allocation variables.

V. Results and Discussions

In this section, we evaluate the system performance with the proposed resource allocation and scheduling algorithm using simulations. A multi-cell system with 3 cells is considered. Each cell has a radius of 1 km. The number of subcarriers is $n_F=64$ with carrier center frequency 2.5 GHz, system bandwidth $\mathcal{B}=5$ MHz, and $\alpha_k=1,\forall k$. Each subcarrier for both RF transmission and backhaul connection has a bandwidth of 78 kHz and the noise variance is $\sigma_B^2=\sigma_z^2=-125$ dBm. The 3GPP path loss model is used [10]. The small scale fading coefficients of the BS-to-users links are generated as independent and identically distributed (i.i.d.) Rayleigh random variables with zero means and unit variances. We assume that all BSs have the same maximum transmit power P_T. Each backhaul connection is assumed to be implemented by DSL with a 24 American Wire Gauge copper cable. The signal attenuation G_{B_c,B_m} for the corresponding backhaul connection is in the order of -20 dB/km. The average system energy efficiency is obtained by counting the number of packets which are successfully decoded by the users over the total power consumption averaged over both macroscopic and microscopic fading. Unless specified otherwise, we assume a static circuit power consumption of $P_C=50$ dBm [11], a data rate requirement of $R=2$ bit/s/Hz/cell, and an orthogonality parameter of $\delta=0.1$. On the other hand, we assume a power efficiency of 20% for the power amplifiers used in both the backhaul and RF, i.e., $\varepsilon=\frac{1}{0.2}=5$.

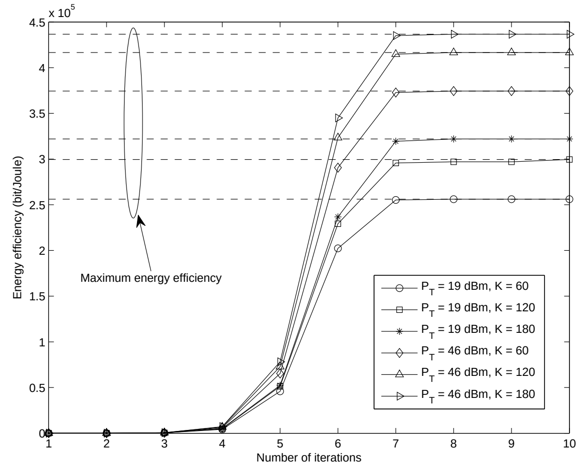

Fig. 2. Energy efficiency (bit-per-Joule) versus number of iterations with different maximum transmit powers per BS, P_T, and different numbers of users K. The dashed lines represent the maximum achievable energy efficiencies for different cases.

A. Convergence of Iterative Algorithm 1

Figure 2 illustrates the evolution of the proposed iterative algorithm for different numbers of users and different maximum transmit powers at each BS. The results in Figure 2 were averaged over 100000 independent adaptation processes where each adaptation process involves different realizations for the path loss and the multipath fading. It can be observed that the iterative algorithm converges to the optimal value[5] within 10 iterations for all considered numbers of transmit antennas. In other words, the maximum system energy efficiency can be achieved within a few iterations on average with a superlinear convergence rate [12].

B. Energy Efficiency versus Transmit Power

Figure 3 illustrates the average energy efficiency versus the total transmit power in each cell, P_T, for different numbers

[5]Here, the optimality is with respect to the optimization of power allocation given a selected user set and ZFBF transmission.

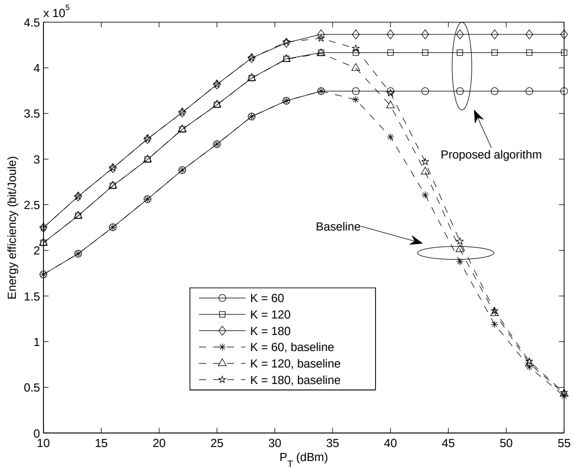

Fig. 3. Energy efficiency (bit-per-Joule) versus maximum transmit power, P_T, for different numbers of users.

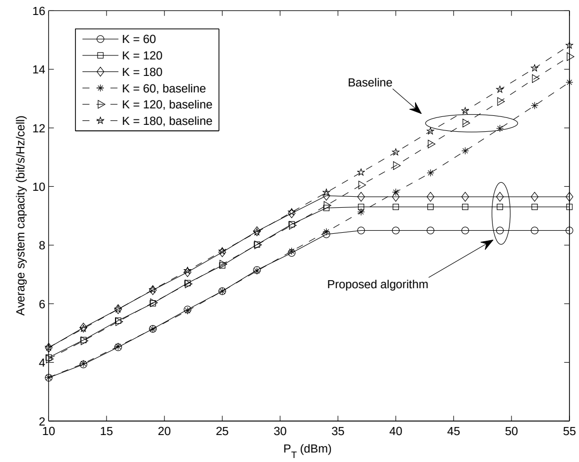

Fig. 4. Average system capacity versus maximum transmit power, P_T, for different numbers of users.

of users. The number of iterations for the proposed iterative resource allocation algorithm is 10. It can be observed that an increasing number of users benefits the system in terms of energy efficiency. This is because the proposed resource allocation and scheduling algorithm is able to exploit multi-user diversity (MUD). Indeed, MUD introduces an extra power gain [13, Chapter 6.6] in the system which provides further energy savings. Yet, the power gain due to MUD is diminishing when K is large. Figure 3 also contains the energy efficiency of a baseline resource allocation scheme. For the baseline scheme, we maximize the average system capacity (bit/s/Hz) with constraints C1-C5 in (12), instead of the energy efficiency. Simulation results in this figure demonstrate that in the low transmit power regime, both algorithms perform nearly the same in terms of energy efficiency. Yet, the proposed algorithm provides a significant performance gain in the high transmit power regime. This is because the baseline scheme uses excess power to increase the average system capacity by sacrificing the system energy efficiency.

C. Average System Capacity versus Transmit Power

Figure 4 shows the average system capacity versus maximum transmit power P_T for different numbers of users. We compare the system performance of the proposed algorithm again with the baseline scheme. The number of iterations in the proposed algorithm is set to 10. It can be observed that the average system capacity of the proposed algorithm approaches a constant in the high transmit power regime. This is because the proposed algorithm clips the power in both backhaul transmission and RF transmission at the BS, in order to maximize the system energy efficiency. We note that, as expected, the baseline scheme achieves a higher average system capacity than the proposed algorithm since the former scheme consumes all the available transmit power in all scenarios. However, the superior average system capacity of the baseline scheme comes at the expense of low energy efficiency. On the other hand, an increasing number of users benefit the average capacity in all cases due to MUD.

VI. Conclusions

In this paper, we formulated the resource allocation and scheduling design for multi-cell OFDMA networks as a non-convex and combinatorial optimization problem, in which the circuit power dissipation, the limited backhaul capacity, and the system data rate requirement were taken into consideration. By exploiting the properties of fractional programming, the considered problem was transformed into an equivalent problem with a tractable solution. An efficient iterative resource allocation algorithm with closed-form power allocation and low complexity user selection was derived for maximization of the energy efficiency. Simulation results did not only show that the proposed algorithm converges to the optimal solution within a small number of iterations, but demonstrated also the achievable maximum energy efficiency in BS cooperation with limited capacity backhaul connections.

Appendix

A. Proof of Theorem 1

We now prove the forward implication of Theorem 1 by following a similar approach as in [8]. Without loss of generality, we define q^* and $\{\mathcal{P}^*, \mathcal{W}^*, \mathcal{S}^*\} \in \mathcal{F}$ as the optimal energy efficiency and the optimal resource allocation policy of the *original objective function* in (12), respectively. Then, the optimal energy efficiency can be expressed as

$$\begin{aligned}&q^* = \frac{U(\mathcal{P}^*, \mathcal{W}^*, \mathcal{S}^*)}{U_{TP}(\mathcal{P}^*, \mathcal{W}^*, \mathcal{S}^*)} \geq \frac{U(\mathcal{P}, \mathcal{W}, \mathcal{S})}{U_{TP}(\mathcal{P}, \mathcal{W}, \mathcal{S})}, \forall \{\mathcal{P}, \mathcal{W}, \mathcal{S}\} \in \mathcal{F},\\ &\Longrightarrow U(\mathcal{P}, \mathcal{W}, \mathcal{S}) - q^* U_{TP}(\mathcal{P}, \mathcal{W}, \mathcal{S}) \leq 0 \text{ and}\\ &U(\mathcal{P}^*, \mathcal{W}^*, \mathcal{S}^*) - q^* U_{TP}(\mathcal{P}^*, \mathcal{W}^*, \mathcal{S}^*) = 0. \qquad (30)\end{aligned}$$

Therefore, we conclude that $\max\limits_{\mathcal{P}, \mathcal{W}, \mathcal{S}} U(\mathcal{P}, \mathcal{W}, \mathcal{S}) - q^* U(\mathcal{P}, \mathcal{W}, \mathcal{S}) = 0$, which is achievable by resource allocation policy $\{\mathcal{P}^*, \mathcal{W}^*, \mathcal{S}^*\}$. This completes the forward implication.

Next, we prove the converse implication of Theorem 1. Suppose $\{\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*\}$ is the optimal resource allocation policy of the *equivalent objective function* such that $U(\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*) - q^* U_{TP}(\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*) = 0$. Then, for any feasible resource allocation policy $\{\mathcal{P}, \mathcal{W}, \mathcal{S}\} \in \mathcal{F}$, we can obtain the following inequality

$$\begin{aligned}& U(\mathcal{P}, \mathcal{W}, \mathcal{S}) - q^* U_{TP}(\mathcal{P}, \mathcal{W}, \mathcal{S})\\ \leq\quad & U(\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*) - q^* U_{TP}(\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*) = 0. \qquad (31)\end{aligned}$$

The above inequality implies

$$\begin{aligned}\frac{U(\mathcal{P}, \mathcal{W}, \mathcal{S})}{U_{TP}(\mathcal{P}, \mathcal{W}, \mathcal{S})} &\leq q^* \quad \forall \{\mathcal{P}, \mathcal{W}, \mathcal{S}\} \in \mathcal{F} \quad \text{and}\\ \frac{U(\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*)}{U_{TP}(\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*)} &= q^*. \qquad (32)\end{aligned}$$

In other words, the optimal resource allocation policy $\{\mathcal{P}_e^*, \mathcal{W}_e^*, \mathcal{S}_e^*\}$ for the equivalent objective function is also the optimal resource allocation policy for the original objective function.

This completes the proof of the converse implication of Theorem 1. In summary, the optimization of the original objective function and the optimization of the equivalent objective function result in the same resource allocation policy. $\square$

B. *Proof of Algorithm Convergence*

We follow a similar approach as in [8] for proving the convergence of Algorithm 1. We first introduce the following two propositions. For the sake of notational simplicity, we define the equivalent objective function in (15) as $F(q') = \max_{\mathcal{P},\mathcal{W},\mathcal{S}} \{U(\mathcal{P},\mathcal{W},\mathcal{S}) - q' U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})\}$.

Proposition 1: $F(q')$ is a strictly monotonic decreasing function in q', i.e., $F(q'') > F(q')$ if $q' > q''$.

Proof: Let $\{\mathcal{P}',\mathcal{W}',\mathcal{S}'\} \in \mathcal{F}$ and $\{\mathcal{P}'',\mathcal{W}'',\mathcal{S}''\} \in \mathcal{F}$ be the two distinct optimal resource allocation polices for $F(q')$ and $F(q'')$, respectively.

$$\begin{aligned} F(q'') &= \max_{\mathcal{P},\mathcal{W},\mathcal{S}} \{U(\mathcal{P},\mathcal{W},\mathcal{S}) - q'' U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})\} \\ &> U(\mathcal{P}',\mathcal{W}',\mathcal{S}') - q'' U_{TP}(\mathcal{P}',\mathcal{W}',\mathcal{S}') \\ &\geq U(\mathcal{P}',\mathcal{W}',\mathcal{S}') - q' U_{TP}(\mathcal{P}',\mathcal{W}',\mathcal{S}') \\ &= F(q'). \end{aligned} \tag{33}$$

$\square$

Proposition 2: Let $\{\mathcal{P}',\mathcal{W}',\mathcal{S}'\} \in \mathcal{F}$ be an arbitrary feasible solution and $q' = \frac{U(\mathcal{P}',\mathcal{W}',\mathcal{S}')}{U_{TP}(\mathcal{P}',\mathcal{W}',\mathcal{S}')}$, then $F(q') \geq 0$.

Proof:
$$\begin{aligned} F(q') &= \max_{\mathcal{P},\mathcal{W},\mathcal{S}} \{U(\mathcal{P},\mathcal{W},\mathcal{S}) - q' U_{TP}(\mathcal{P},\mathcal{W},\mathcal{S})\} \\ &\geq U(\mathcal{P}',\mathcal{W}',\mathcal{S}') - q' U_{TP}(\mathcal{P}',\mathcal{W}',\mathcal{S}') = 0. \end{aligned}$$

$\square$

We are now ready to prove the convergence of Algorithm 1.

Proof of Convergence: We first prove that the energy efficiency q increases in each iteration. Then, we prove that if the number of iterations is large enough, the energy efficiency q converges to the optimal q^* such that it satisfies the optimality condition in Theorem 1, i.e., $F(q^*) = 0$.

Let $\{\mathcal{P}_n,\mathcal{W}_n,\mathcal{S}_n\}$ be the optimal resource allocation policy in the n-th iteration. Suppose $q_n \neq q^*$ and $q_{n+1} \neq q^*$ represent the energy efficiency of the considered system in iterations n and $n+1$, respectively. By Theorem 1 and Proposition 2, $F(q_n) > 0$ and $F(q_{n+1}) > 0$ must be true. On the other hand, in the proposed algorithm, we calculate q_{n+1} as $q_{n+1} = \frac{U(\mathcal{P}_n,\mathcal{W}_n,\mathcal{S}_n)}{U_{TP}(\mathcal{P}_n,\mathcal{W}_n,\mathcal{S}_n)}$. Thus, we can express $F(q_n)$ as

$$\begin{aligned} F(q_n) &= U(\mathcal{P}_n,\mathcal{W}_n,\mathcal{S}_n) - q_n U_{TP}(\mathcal{P}_n,\mathcal{W}_n,\mathcal{S}_n) \\ &= U_{TP}(\mathcal{P}_n,\mathcal{W}_n,\mathcal{S}_n)(q_{n+1} - q_n) > 0 \qquad (34) \\ \Longrightarrow q_{n+1} &> q_n, \qquad \because U_{TP}(\mathcal{P}_n,\mathcal{W}_n,\mathcal{S}_n) > 0. \qquad (35) \end{aligned}$$

By combining $q_{n+1} > q_n$, Proposition 1, and Proposition 2, we can show that as long as the number of iterations is large enough, $F(q_n)$ will eventually approach zero and satisfy the optimality condition as stated in Theorem 1. $\square$

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