Optimal Power Allocation for a Hybrid Energy Harvesting Transmitter

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***Abstract*— In this work, we consider a point–to–point link where the transmitter has a hybrid supply of energy, i.e., the energy is supplied by a constant energy source and an energy harvester, which harvests energy from its surrounding environment. Our goal is to jointly minimize the power consumed by the constant energy source and any possible waste of the harvested energy to ensure their optimum utilization for transmission of a given amount of data in a given number of time intervals. Two scenarios are considered for packet arrival. In the first scenario, we assume that all data packets have arrived before the transmission begins, whereas in the second scenario, we assume that data packets are arriving during the course of data transmission. For both scenarios, we propose optimal offline transmit power allocation schemes which provide insight on how to efficiently consume the energy supplied by the constant energy source and the energy harvester.**

I. Introduction

Green communication has attracted significant attention in academia and industry as the energy consumption of the equipment in wireless communication systems has increased rapidly in recent years which has raised environmental concerns [1], [2]. In the literature, a number of research contributions, which aim to provide a balance between energy consumption and performance have been reported for different wireless communication systems [3]–[6]. Most of these works assume that the energies are supplied by a constant energy source and/or a rechargeable battery. However, the deployment of energy harvesting (EH) nodes can be a potential solution to supplement the traditional energy supply for communication systems. EH nodes harvest energy from their surroundings using solar, thermoelectric, or motion effects or by exploiting some other physical phenomenon. Therefore, the harvested energy is practically free of cost and can ensure a perpetual supply of energy.

Recently, transmission strategies and power allocation policies for EH nodes in wireless communication systems have been studied in [7]–[9]. In [7], a single non–cooperative link with an EH transmitter was considered and an optimal offline along with an optimal and several sub–optimal online transmission policies were provided for maximizing the system capacity. In [8], a similar system model was considered and dynamic programming (DP) was employed to allocate the transmit power for the case when causal channel state information (CSI) was available. On the other hand, transmission time minimization and transmission packet scheduling in EH systems were considered in [9]. In [10], a source–relay–destination link with an EH source and an EH relay was considered and an offline and online power allocation schemes were proposed to maximize the end–to–end system throughput. The concept of energy transfer in EH relay systems was considered in [11], where an offline power allocation scheme was proposed.

All the above works on systems with energy harvesting capability assume that energy harvesters are the only source of energy for the transmitter. From a practical point of view, to achieve both reliable and green communication, it is desirable to have a hybrid source of energy, cf. [12]. Motivated by this fact, in this paper, we consider a single link where the transmitter has a hybrid energy source. The hybrid source of energy includes a constant energy source and an energy harvester which harvests energy from the surroundings. The constant energy source is assumed to be operated by a costly and/or non–environmentally friendly generator, e.g., a diesel fuel power generator in a remote location or a nuclear power plant. In contrast, the harvested energy is green and stems from a sustainable source of energy. Thus, our aim is to jointly minimize the amount of energy drawn from the constant source and any possible waste of the harvested energy so that the harvested energy is utilized as efficiently as possible for transmitting a given number of data packets over a finite number of transmission intervals. Note that our problem formulation is different from that in [7]–[11], as [7]–[11] considered throughput maximization and/or minimization of transmission time for communication systems employing energy harvesting sources but no constant energy source. The solution of the optimization problem considered in this paper provides us with insight regarding the optimal power allocation policy and thereby will facilitate the design of reliable green communication systems.

We consider two scenarios for the arrival process of the data packets into the data queue at the transmitter. In the first scenario, the data packets that have to be transmitted arrive before the transmission begins and no packets arrive during the transmission. In the second scenario, the data packets may arrive during the course of transmission. For both scenarios, we derive offline power allocation schemes that minimize the total amount of energy drawn from the constant energy source and the amount of wasted harvested energy. Offline

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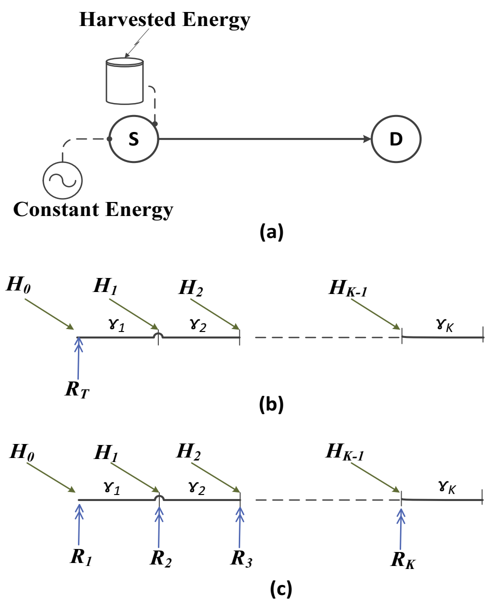

Fig. 1. (a) System model for a communication link where the transmitter has a hybrid energy supply. (b) Illustration of Scenario 1 with K transmission intervals, channel SNRs γ_k, and harvested energies H_k, where $k \in \{1, 2, \cdots, K\}$. Here, R_T bits arrive before transmission begins. (c) Illustration of Scenario 2. Here, R_k bits arrive just before time interval, k.

schemes are of interest when the amount of harvested energy, the channel signal-to-noise ratio (SNR), and the amount of incoming data for all transmission intervals are known a priori. Considering offline schemes is important since they provide performance upper bounds for more practical online schemes. Moreover, offline schemes also provide valuable insight for the design of efficient online algorithms, which is an interesting topic for future research.

II. System Model

We consider a single communication link, where a transmitter (source), S, communicates with a receiver (destination), D, as shown in Fig. 1(a). We assume that S has a data queue with infinite capacity which can store data packets temporarily before their transmission. The energy required by S for signal transmission and processing is supplied by a hybrid source of energy. The hybrid source includes a constant energy source, possibly connected through a cable to the power grid, and an EH module which harvests energy from the surroundings. The harvested energy is stored in a battery which can store at most B_{max} Joules of energy. We consider a deadline of K transmission intervals and assume that our data transmission is packet based. The duration of each transmission interval is T and without loss of generality, we assume $T = 1s$.

We consider two scenarios for packet arrivals. In the first scenario, we assume that R_T bits have arrived at S before the transmission starts and have to be transmitted in K transmission intervals, cf. Fig. 1(b). We assume that additional bits do not arrive during the transmission. On the other hand, in the second scenario, we assume that R_k bits arrive immediately before transmission interval k, where $k \in \{1, 2, \cdots, K\}$, and all bits have to be transmitted by the end of the last transmission interval K, cf. Fig. 1(c).

We assume that the transmitted packets contain Gaussian–distributed symbols and the transmission is impaired by additive white Gaussian noise (AWGN). Let γ_k denote the channel SNR of the S–D link, which is assumed to be independent and identically distributed (i.i.d.) over the time intervals. For future reference, we introduce the average SNR of the S–D link as $\bar{\gamma}$. The transmit power for each transmission interval, $k \in \{1, 2, \cdots, K\}$, is the summation of the transmit powers $P_{E,k}$ and $P_{H,k}$ drawn from the constant energy source and the energy harvesting source, respectively. We denote the extra amount of harvested energy, which cannot be stored in the battery in transmission interval k due to its limited storage capacity as $\delta_{H,k}$. We assume that the power required for signal processing, which is constant in each time interval, is supplied by the constant energy source. Since the power amplifier used for transmission is not ideal, the total powers drawn from the constant energy source and the energy harvesting source are given by $\rho P_{E,k}$ and $\rho P_{H,k}$, respectively. Here, $\rho \geq 1$ is a constant that accounts for the power amplifier inefficiency. For instance, if $\rho = 2$, 100 Watts of power are consumed in the power amplifier for every 50 Watts of power radiated in the radio frequency, and the efficiency of the power amplifier in this case is $\frac{1}{\rho} = 50\%$.

We assume that E_k is the maximum transmit energy that can be drawn from the constant energy source in each interval, excluding the required constant signal processing power. On the other hand, the energy harvester at S collects $H_k \leq B_{max}$ Joules of energy during the kth interval. Let $H_R \triangleq \mathcal{E}\{H_k\}$ denote the average EH rate, where $\mathcal{E}\{\cdot\}$ denotes statistical expectation. Similar to [8], we assume that the harvested energy stored in the battery increases and decreases linearly provided the maximum storage capacity B_{max} is not exceeded, i.e.,

$$B_{k+1} = \min\{(B_k - P_{H,k} + H_k), B_{max}\}, \ \forall k. \tag{1}$$

Furthermore, $B_1 = H_0 \geq 0$ denotes the available energy before the transmission starts.

III. Optimal Power Allocation

In this section, we develop offline power allocation strategies for the considered scenarios of packet arrivals. Our goal is to jointly minimize the amount of energy drawn from the constant energy source and the wasted harvested energy in order to exploit both as efficiently as possible for transmission of a required amount of information in K transmission intervals, cf. Figs. 1(b) and 1(c). Due to the finite storage capacity of the battery, it is advantageous to draw the energy as quickly as possible from the battery for packet transmission so that more harvested energy can be stored in the future, and thus the amount of wasted harvested energy is minimized.

A. Scenario 1: Data Packets Arrive Before Transmission Starts

In this scenario, we are required to transmit R_T bits by the deadline of K time intervals. We assume that the R_T bits have arrived before the transmission starts. We formulate the corresponding offline optimization problem as

$$\min_{P_{E,k}\geq 0,\ P_{H,k}\geq 0,\ \delta_{H,k}\geq 0} \quad \sum_{k=1}^{K}(\rho P_{E,k}+\delta_{H,k}) \tag{2}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\log_2(1+\gamma_k(P_{E,k}+P_{H,k}))\geq R_T \tag{3}$$

$$\sum_{k=1}^{l}\rho P_{H,k}\leq\sum_{k=0}^{l-1}(H_k-\delta_{H,k}),\ \forall l \tag{4}$$

$$\sum_{k=0}^{q}(H_k-\delta_{H,k})-\sum_{k=1}^{q}\rho P_{H,k}\leq B_{max},\ \forall q \tag{5}$$

$$\rho P_{E,k}\leq E_k, \qquad \forall k, \tag{6}$$

where $\delta_{H,0}=0$, $l\in\{1,2,\cdots,K\}$, $q\in\{1,2,\cdots,K-1\}$, and $k\in\{1,2,\cdots,K\}$. Constraint (3) ensures that R_T packets are transmitted over K intervals. Constraint (4) stems from the causality constraint on the harvested energy and constraint (5) ensures that the harvested energy does not exceed the limited storage capacity of the battery. The limitedness of the energy drawn from the constant supply is reflected in constraint (6). Note that for a given time interval, for the constant energy supply, any extra amount of energy which is not used for transmission cannot be transferred to the next interval. It is worth mentioning that problem (2)–(6) is not always feasible. Assuming R_T, the channel SNRs, γ_k, and the harvested energies, H_k, are given for all time intervals, a sufficient (but not necessary) condition for feasibility of problem (2)–(6) is

$$\sum_{k=1}^{K}\log_2(1+\frac{\gamma_k}{\rho}(E_k+H_k))\geq R_T. \tag{7}$$

However, if the problem is not feasible, we can optimize K by following similar steps as in [7]. The optimized K, denoted by $K^*\geq K$, will provide an indication for how many time intervals are required for transmission of R_T bits. Note that problem (2)–(6) is always feasible if $E_k\rightarrow\infty$, $\forall k$, i.e., when the constant energy supply is (practically) unlimited. In the following, we assume that problem (2)–(6) is feasible.

Problem (2)–(6) is a convex optimization problem and hence can be solved optimally and efficiently [13]. Moreover, as problem (2)–(6) satisfies Slater's constraint qualification and is jointly convex in $P_{E,k}$, $P_{H,k}$, and $\delta_{H,k}$, the duality gap between the optimum values of the original problem and its dual is zero [13]. Therefore, we solve our problem by solving its dual. For this purpose, we first provide the Lagrangian of problem (2)–(6) which can be written as

$$\begin{aligned}\mathcal{L}_1=&\sum_{k=1}^{K}(\rho P_{E,k}+\delta_{H,k})+\sum_{k=1}^{K}\beta_k\left(\rho P_{E,k}-E_k\right)\\&+\sum_{q=1}^{K-1}\xi_q\left(\sum_{k=0}^{q}(H_k-\delta_{H,k})-\sum_{k=1}^{q}\rho P_{H,k}-B_{max}\right)\\&+\sum_{l=1}^{K}\alpha_l\left(\sum_{k=1}^{l}\rho P_{H,k}-\sum_{k=0}^{l-1}(H_k-\delta_{H,k})\right)\\&-\lambda\left(\sum_{k=1}^{K}\log_2(1+\gamma_k(P_{E,k}+P_{H,k}))-R_T\right),\end{aligned} \tag{8}$$

where $\lambda\geq 0$, $\alpha_l\geq 0$, $\xi_q\geq 0$, and $\beta_k\geq 0$ are the Lagrange multipliers associated with constraints (3), (4), (5), and (6), respectively. Note that the boundary conditions $P_{E,k}\geq 0$, $P_{H,k}\geq 0$, and $\delta_{H,k}\geq 0$ are absorbed into the Karush–Kuhn–Tucker (KKT) conditions for deriving the optimal $P_{E,k}$ and $P_{H,k}$. The dual of problem (2)–(6) can be stated as

$$\max_{\lambda\geq 0,\ \alpha_l\geq 0,\ \xi_q\geq 0,\ \beta_k\geq 0} \quad \min_{P_{E,k}\geq 0,\ P_{H,k}\geq 0,\ \delta_{H,k}\geq 0}\mathcal{L}_1. \tag{9}$$

Using standard optimization techniques and the KKT optimality conditions, the optimal $P_{E,k}$, $P_{H,k}$, and $\delta_{H,k}$ can be obtained as

$$P^*_{E,k}=\left[\Lambda_{E,k}-\frac{1}{\gamma_k}-P_{H,k}\right]^+, \tag{10}$$

$$P^*_{H,k}=\left[\Lambda_{H,k}-\frac{1}{\gamma_k}-P_{E,k}\right]^+,\text{and} \tag{11}$$

$$\delta^*_{H,k}=\left[\sum_{i=0}^{k-1}(H_i-\delta_{H,i})-\sum_{i=1}^{k}\rho P_{H,i}+H_k-B_{max}\right]^+, \tag{12}$$

respectively, where $[x]^+=\max\{x,0\}$. The power allocation solutions in (10) and (11) can be interpreted as a form of water–filling, where $\Lambda_{E,k}=\frac{\lambda}{\rho\ln(2)(1+\beta_k)}$ and $\Lambda_{H,k}=\frac{\lambda}{\rho\ln(2)\left(\sum_{j=k}^{K}\alpha_j-\sum_{j=k}^{K-1}\xi_j\right)}$ are the water levels associated with $P^*_{E,k}$ and $P^*_{H,k}$, respectively. Note that $P^*_{E,k}$ and $P^*_{H,k}$ depend on each other due to constraint (3). We observe from (10) that whenever constraint (6) is not satisfied with equality, i.e., $\beta_k=0$, $\forall k$, the optimum water level $\Lambda_{E,k}$ is constant. From (11), we observe that when $B_{max}=\infty$, we have $\xi_q=0$, $\forall q$, and in this case, the optimum water level, $\Lambda_{H,k}$, is monotonically non–decreasing. However, when B_{max} is finite and if constraint (5) is satisfied with equality, i.e., at least one $\xi_q\neq 0$, $\forall q$, then the monotonicity of the optimum water level no longer holds. However, when constraint (5) is not satisfied with equality for finite B_{max}, the optimum water level is monotonically non–decreasing similar to the case when $B_{max}=\infty$.

We adopt the Lagrange dual decomposition method and obtain the optimal $P_{E,k}$, $P_{H,k}$, and $\delta_{H,k}$ via an iterative procedure [13]. We define t as the iteration index. For a given set of Lagrange multipliers $(\lambda(t),\alpha_l(t),\xi_q(t),\beta_k(t))$ and a given value of $P^*_{H,k}(t-1)$, we obtain $P^*_{E,k}(t)$ using (10) and then calculate $P^*_{H,k}(t)$ based on (11) by using $P^*_{E,k}(t)$ as $P_{E,k}$. We also calculate $\delta^*_{H,k}(t)$ based on (12) by using $P^*_{H,k}(t)$ as $P_{H,k}$. The initial set of Lagrange multipliers $(\lambda(1),\alpha_l(1),\xi_q(1),\beta_k(1))$ are chosen from the feasible set, i.e., $\lambda(1)\geq 0,\ \alpha_l(1)\geq 0,\ \xi_q(1)\geq 0,\ \beta_k(1)\geq 0$. However, to calculate $P_{E,k}(1)$ for $t = 1$, $P_{H,k}(0) \geq 0$ is chosen such that (4) and (5) are satisfied. We update the Lagrange multipliers as follows:

$$\lambda(t+1) = \left[\lambda(t) - \Upsilon(t)\left(\sum_{k=1}^{K}\log_2\left(1+\gamma_k\left(P_{E,k}^*(t) + P_{H,k}^*(t)\right)\right) - R_T\right)\right]^+, \quad (13)$$

$$\alpha_l(t+1) = \left[\alpha_l(t)+\Psi(t)\left(\sum_{k=1}^{l}\rho P_{H,k}^*(t) - \sum_{k=0}^{l-1}(H_k - \delta_{H,k}^*(t))\right)\right]^+, \quad (14)$$

$$\xi_q(t+1) = \left[\xi_q(t)+\Omega(t)\left(\sum_{k=0}^{q}(H_k - \delta_{H,k}^*(t)) - \sum_{k=1}^{q}\rho P_{H,k}^*(t) - B_{max}\right)\right]^+, \quad (15)$$

$$\beta_k(t+1) = \left[\beta_k(t) + \Phi(t)\left(\rho P_{E,k}^*(t) - E_k\right)\right]^+, \quad (16)$$

where $l \in \{1, 2, \cdots, K\}$, $q \in \{1, 2, \cdots, K-1\}$, and $k \in \{1, 2, \cdots, K\}$. Here, $\Upsilon(t)$, $\Psi(t)$, $\Omega(t)$, and $\Phi(t)$ are positive step sizes. With the updated Lagrange multipliers, we solve $P_{E,k}^*(t+1)$ and $P_{H,k}^*(t+1)$ again and the same procedure continues until convergence. Note that, because of the convexity of problem (2)–(6), the convergence to the optimal solution is always guaranteed as long as the step sizes satisfy the infinite travel condition [13].

B. Scenario 2: Data Packets Arrive During the Transmission

In this scenario, data packets arrive during the course of transmission and hence the transmit powers have to be adjusted accordingly during the transmission intervals, cf. Fig. 1(c). Similar to Scenario 1, we also formulate an offline optimization problem for Scenario 2

$$\min_{P_{E,k}\geq 0,\ P_{H,k}\geq 0,\ \delta_{H,k}\geq 0} \quad \sum_{k=1}^{K}(\rho P_{E,k} + \delta_{H,k}) \quad (17)$$

$$\text{s.t.} \quad \sum_{k=1}^{q}\log_2(1+\gamma_k(P_{E,k}+P_{H,k})) \leq \sum_{k=1}^{q} R_k, \ \forall q \quad (18)$$

$$\sum_{k=1}^{K}\log_2(1+\gamma_k(P_{E,k}+P_{H,k})) = \sum_{k=1}^{K} R_k \quad (19)$$

$$\text{Constraints (4)} - (6), \quad (20)$$

where $q \in \{1, 2, \cdots, K-1\}$. Constraint (18) provides the flexibility to transmit the incoming data packets in future time intervals. Constraint (19) ensures that all the arrived data packets are transmitted by the deadline of K transmission intervals. Note that problem (17)–(20) is not a convex optimization problem because of the non–convexity of constraint (18) and the non–affinity of constraint (19). We combine (18) and (19) and transform problem (17)–(20) into the following problem:

$$\min_{P_{E,k}\geq 0,\ P_{H,k}\geq 0,\ \delta_{H,k}\geq 0} \quad \sum_{k=1}^{K}(\rho P_{E,k} + \delta_{H,k}) \quad (21)$$

$$\text{s.t.} \quad \sum_{k=l}^{K}\log_2(1+\gamma_k(P_{E,k}+P_{H,k})) \geq \sum_{k=l}^{K} R_k, \ \forall l \quad (22)$$

$$\text{Constraints (4)} - (6), \quad (23)$$

where $l \in \{1, 2, \cdots, K\}$. Constraint (22) is an equivalent representation of constraints (18) and (19), and hence problem (21)–(23) is equivalent to problem (17)–(20), i.e., both problems have the same optimal solution. Similar to the optimization problem for Scenario 1, problem (17)–(20) is not always feasible. Assuming R_k, channel SNRs, γ_k, and harvested energies, H_k, are given for all time intervals, a sufficient condition for feasibility of problem (17)–(20) is

$$\sum_{k=1}^{K}\log_2(1+\frac{\gamma_k}{\rho}(E_k+H_k)) \geq \sum_{k=1}^{K} R_k. \quad (24)$$

If the problem is not feasible, we can use a similar approach as for Scenario 1 to obtain $K^* > K$ to avoid infeasibility. Note that if we optimize K and find $K^* > K$, then during time intervals $k' \in \{K+1, K+2, \cdots, K^*\}$, we have to assume that no additional data packets arrive at S to avoid the possibility of facing further infeasibility. In the following, we assume that problem (21)–(23) is feasible. Problem (21)–(23) is a convex optimization problem and can be solved optimally and efficiently [13]. The Lagrangian of problem (21)–(23) can be written as

$$\mathcal{L}_2 = \sum_{k=1}^{K}(\rho P_{E,k} + \delta_{H,k}) + \sum_{l=1}^{K}\alpha_l\left(\sum_{k=1}^{l}\rho P_{H,k} - \sum_{k=0}^{l-1}H_k - \delta_{H,k}\right) + \sum_{q=1}^{K-1}\xi_q\left(\sum_{k=0}^{q}(H_k - \delta_{H,k}) - \sum_{k=1}^{q}\rho P_{H,k} - B_{max}\right) + \sum_{k=1}^{K}\beta_k(\rho P_{E,k} - E_k) - \sum_{l=1}^{K}\lambda_l\left(\sum_{k=l}^{K}\log_2(1+\gamma_k(P_{E,k}+P_{H,k})) - \sum_{k=l}^{K}R_k\right). \quad (25)$$

Here, λ_l denotes the Lagrange multiplier associated with (22). The dual of problem (21)–(23) can be stated as

$$\max_{\lambda_l\geq 0,\ \alpha_l\geq 0,\ \xi_q\geq 0,\ \beta_k\geq 0} \quad \min_{P_{E,k}\geq 0,\ P_{H,k}\geq 0,\ \delta_{H,k}\geq 0} \mathcal{L}_2. \quad (26)$$

Using standard optimization techniques and the KKT optimality conditions, the optimal $P_{E,k}$ and $P_{H,k}$ can be obtained as

$$P_{E,k}^* = \left[\Xi_{E,k} - \frac{1}{\gamma_k} - P_{H,k}\right]^+ \text{ and} \quad (27)$$

$$P_{H,k}^* = \left[\Xi_{H,k} - \frac{1}{\gamma_k} - P_{E,k}\right]^+, \quad (28)$$

respectively, where $\Xi_{E,k} = \frac{\sum_{j=1}^{k}\lambda_j}{\rho\ln(2)(1+\beta_k)}$ and $\Xi_{H,k} = \frac{\sum_{j=1}^{k}\lambda_j}{\rho\ln(2)\left(\sum_{j=k}^{K}\alpha_j - \sum_{j=k}^{K-1}\xi_j\right)}$ are the optimum water levels associated with $P_{E,k}^*$ and $P_{H,k}^*$, respectively. Moreover, $\delta_{H,k}^*$ is

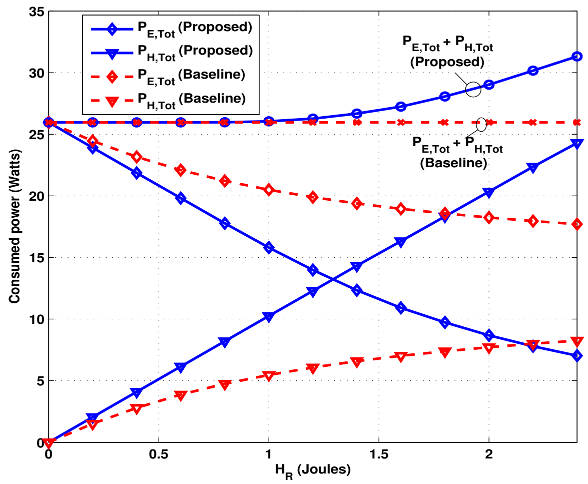

Fig. 2. Consumed power for Scenario 1 vs. H_R for $K = 10$ and $R_T = 75$ bits.

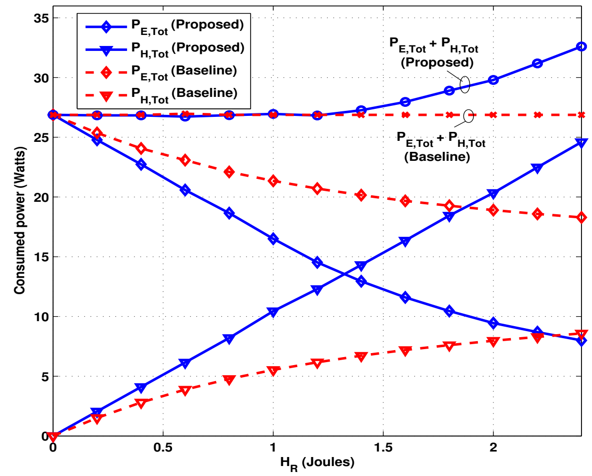

Fig. 3. Consumed power for Scenario 2 vs. H_R for $K = 10$ and $\sum_{k=1}^{K} R_k = 75$ bits.

still given by (12). We note that (27) and (28) have a water–filling structure similar to (10) and (11). We observe from (27) that whenever constraint (6) is not satisfied with equality, i.e., $\beta_k = 0$, the optimum water level, $\Xi_{E,k}$, is monotonically non–decreasing. The observations made about (11) in Scenario 1 hold also true for (28). The optimal solutions for $P_{E,k}$ and $P_{H,k}$ for Scenario 2 can be obtained with Lagrange dual decomposition using a similar method as for Scenario 1.

IV. Simulation Results

In this section, we evaluate the performance of the proposed power allocation schemes for Scenarios 1 and 2. We assume that in each time interval, $H_{k'}$, $k' \in \{0, 1, \cdots, K-1\}$, independently takes a value from the set $\{0, H_R, 2H_R\}$, where all elements of the set are equiprobable. For all presented simulation results, we assume $K = 10$, $E_1 = E_2 = \cdots = 40$ Joules, $B_{max} = 50$ Joules, and the channel SNR follows an exponential distribution with mean $\bar{\gamma} = 25$ dB. For Scenario 2, R_k follows a uniform distribution with mean R_{avg}. We assume $\rho = 2.5$, which corresponds to a power amplifier efficiency of 40%.

For the purpose of comparison, we consider a baseline scheme where we minimize the total consumed power, i.e., the sum of the powers drawn from the constant energy supply and the energy harvester. The optimization problems for the baseline scheme for Scenarios 1 and 2 are obtained by using $\sum_{k=1}^{K} \rho(P_{H,k} + P_{E,k}) + \delta_{H,k}$ as objective function in (2) and (17), respectively. The objective of the baseline scheme is to minimize the total consumed energy rather than to fully exploit the harvested energy. For all simulation results, 10^4 randomly generated realizations of the channel SNRs, harvested energies, and incoming data packets (for Scenario 2) are considered to obtain the average consumed powers.

A. Consumed Power vs. H_R

Figs. 2 and 3 show $P_{E,Tot} = \sum_{k=1}^{K} \rho P_{E,k}$ and $P_{H,Tot} = \sum_{k=1}^{K} \rho P_{H,k}$ vs. the harvesting rate H_R for Scenarios 1 and 2, respectively. Results for both the proposed scheme and the baseline scheme are presented. In Fig. 2, we assume $R_T = 75$ bits, and in Fig. 3, we assume $\sum_{k=1}^{K} R_k = 75$ bits. We observe that in both scenarios, $P_{E,Tot}$ decreases with increasing H_R for the proposed and the baseline schemes. As expected, for $H_R = 0$ Joule, i.e., when there is no harvested energy, the proposed and the baseline schemes yield the same $P_{E,Tot}$ and $P_{H,Tot} = 0$. Since we minimize the power drawn from the constant power supply and the amount of data to be transmitted is constant, as the harvesting rate increases, more harvested energy is available for use which results in increased consumption of harvested energy and decreased consumption from the constant power supply. We observe that the $P_{E,Tot}$ ($P_{H,Tot}$) curve of the proposed scheme always remains below (above) the $P_{E,Tot}$ ($P_{H,Tot}$) curve of the baseline scheme, and that the gap between the two schemes increases as the harvesting rate H_R increases. In particular, for Scenario 1 and $H_R = 1.5$ Joules, we observe that adopting the proposed scheme allows to save 7 Watts of consumed power from the constant energy supply compared to the baseline scheme. In fact, our scheme minimizes only the power drawn from the constant supply and makes maximum use of the harvested energy whereas the baseline scheme minimizes the powers drawn from the constant energy source and the harvested energy simultaneously. Moreover, a comparison of the powers consumed in Scenario 1 and Scenario 2 reveals that Scenario 1 requires a lower $P_{E,Tot}$ ($P_{H,Tot}$) than Scenario 2. This can be explained by the fact that knowing the amount of data to be transmitted before the transmission starts provides more flexibility to allocate the transmit powers over the transmission intervals than when the data packets arrive during the course of transmission.

We have also included the total powers consumed by the proposed and the baseline schemes in Figs. 2 and 3. We observe that for the baseline scheme, the increase of $P_{H,Tot}$ with increasing H_R exactly compensates the decrease of

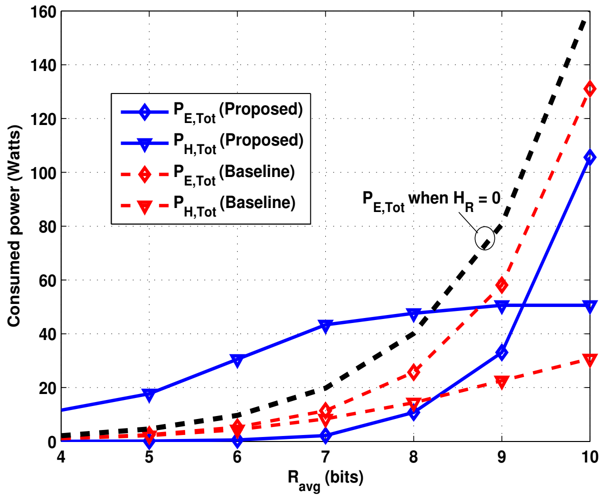

Fig. 4. Consumed power for Scenario 1 vs. R_{avg} for $K = 10$ and $H_R = 5$ Joules.

$P_{E,Tot}$ with increasing H_R, and therefore, the total consumed power does not change with H_R. Intuitively, as the baseline scheme minimizes the total consumed energy, H_R has no impact on the total power consumption. On the other hand, for the proposed scheme, for large H_R, $P_{H,Tot}$ increases faster with H_R than $P_{E,Tot}$ decreases with H_R. As a result, the total consumed power increases with H_R after a certain threshold. The reason for the higher total power consumption is the increase in power drawn from the energy harvester to decrease the consumption from the constant energy source.

B. Consumed Power vs. R_{avg}

Fig. 4 shows $P_{E,Tot}$ and $P_{H,Tot}$ vs. $R_{avg} = (1/K) \sum_{k=1}^{K} R_k$ for Scenario 2. Here, we adopt $H_R = 5$ Joules. We observe that the transmit powers, $P_{E,Tot}$ and $P_{H,Tot}$, increase with increasing R_{avg} because the larger the amount of data that has to be transmitted the higher the required transmit power. However, the rate of increase is not the same for $P_{E,Tot}$ and $P_{H,Tot}$. The rate at which $P_{E,Tot}$ increases is small for low R_{avg}, because for low R_{avg}, the consumed power is mainly drawn from the energy harvester. On the other hand, for large R_{avg}, $P_{E,Tot}$ increases rapidly as the harvested energy alone is not sufficient to supply the required power completely. Moreover, for large R_{avg}, $P_{H,Tot}$ saturates because the maximum energy that the energy harvester can provide is limited by $\sum_{k=1}^{K} H_k$. We observe again that our proposed scheme is more successful than the baseline scheme in reducing $P_{E,Tot}$. For comparison, we also show $P_{E,Tot}$ for the proposed and the baseline schemes for $H_R = 0$ Joule (i.e., no energy harvester) and observe that both schemes yield identical results as expected. Besides, $P_{E,Tot}$ for $H_R = 0$ Joule is always larger than $P_{E,Tot}$ for $H_R = 5$ Joules as without the supplement of the energy harvester, the constant energy source has to supply all the required power.

V. Conclusions

In this paper, we optimized the power allocation for a system with a hybrid energy source. The hybrid energy source includes a constant energy source (non–renewable) and an energy harvester (renewable). We proposed to minimize the amount of power drawn from the constant energy source to make full use of the harvested energy. We presented optimal offline power allocation schemes for two different data arrival scenarios, and derived recursive closed-form solutions. The proposed schemes were compared with baseline schemes, which aimed at minimizing the total consumed power. The comparison revealed that our scheme significantly reduces the power consumption from the constant energy source and utilizes the harvested energy efficiently.

The design of optimal and suboptimal online power allocation schemes for the considered system model is an interesting topic for future research. The optimal online power allocation scheme can be implemented using stochastic DP. However, because of the high complexity of DP, low–complex suboptimal online power allocation schemes are desirable.

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