Distributed user-centric scheduling for visible light communication networks

LINGJIAO CHEN,[1] JIAHENG WANG,[1,*] JIANTAO ZHOU,[2]
DERRICK WING KWAN NG,[3] ROBERT SCHOBER,[4,5] AND
CHUNMING ZHAO[1]

[1]*National Mobile Communications Research Laboratory, Southeast University, Nanjing, China*
[2]*Department of Computer and Information Science, University of Macau, Macau*
[3]*School of Electrical Engineering and Telecommunications, University of New South Wales, Australia*
[4]*Institute for Digital Communications, Friedrich-Alexander-UniversitÂÍat Erlangen-NÂÍurnberg, Germany*
[5]*Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, BC, Canada*
[*]jhwang@seu.edu.cn

Abstract: Visible light communication (VLC) networks, consisting of multiple light-emitting diodes (LEDs) acting as optical access points (APs), can provide low-cost high-rate data transmission to multiple users simultaneously in indoor environments. However, the performance of VLC networks is severely limited by the interference between different users. In this paper, we establish a distributed user-centric scheduling framework based on stable marriage theory, and propose a novel decentralized scheduling method to manage interference by forming flexible amorphous cells for all users. The proposed scheduling method has provable low computational complexity and requires only the exchange of a few 1-bit messages between the APs and the users but not the feedback of the channel state information of the entire network. We further show that the proposed method can achieve both user-wise and system-wise optimality as well as a certain level of fairness. Simulation results indicate that our decentralized user-centric scheduling method outperforms existing centralized approaches in terms of throughput, fairness, and computational complexity.

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1. Introduction

Visible light communication (VLC), as a promising solution to indoor short-range wireless communication, has received increasing attention in recent years. Employing light-emitting diodes (LEDs), VLC accomplishes the two-fold goal of illumination and communication and enjoys the advantages of low cost, free spectrum, natural confidentiality, and low energy consumption [1]. Hence, it is widely believed that VLC will play an important role in next generation wireless communication systems [2]. As a result, extensive studies have been dedicated to point-to-point VLC technologies [3, 4]. For example, recently VLC data rates of Gbps have been reported in [5, 6], and a hybrid VLC and RF system has been developed in [7].

The success of point-to-point VLC technologies paves the way for multiuser VLC, where multiple optical access points (APs) transmit data to multiple user receivers simultaneously. In indoor environments, there are usually many LEDs available which can act as APs and form a VLC network that can support high-rate communication for a number of users [8, 9]. On the one hand, this system architecture further increases the potential of VLC, but, on the other hand, it also introduces additional challenges for system design. Firstly, the density of APs and users in VLC networks is much higher than than in conventional cellular communication systems, implying a complicated network topology. Secondly, due to the associated high density, inter-user interference is severe and may cause a serious performance degradation if not properly handled. Thirdly, user fairness becomes a critical issue in multiuser VLC networks due to inter-user inference and limited system resources. Thus, interference management becomes a vital issue in VLC networks.

Existing RF cellular systems (such as the 4G LTE (Long-Term Evolution) [10] and the 3G

CDMA (Code Division Multiple Access) [11] systems) generally adopt cell-centric designs, in which each user is assigned to a base station and interference among cells is suppressed by some frequency reuse methods, i.e., adjacent cells use different frequencies. However, such a design philosophy may cause a low spectrum utilization and limit system performance. Hence, user-centric network designs have been proposed for next generation wireless communication systems [12]. In this case, rather than being a passive endpoint, every user is allowed to participate actively as a network component. Thereby, by utilizing the diversity of the users' locations and service requirements, virtual amorphous cells are formed to provide better service to each user. Compared with traditional cell-centric designs, user-centric designs offer higher spectrum efficiency, better service coverage, and higher flexibility for service provisioning [13].

User-centric scheduling provides a natural solution for managing the complicated interference scenario in VLC networks with high AP and user densities and high throughput demands of heterogeneous users [14]. In addition, the vicinity of APs and users in indoor environments makes user-centric scheduling in VLC networks more practical than in RF cellular networks. However, user-centric scheduling relies on the joint coordination of all APs and all users, which, if designed in a centralized manner (such as in [9] and [15]), will require the gathering of the channel state information of the whole network at a central node and inevitably induce a heavy computational burden at this node. Besides, such a high signaling overhead and computational complexity conflict with the desired simplicity of VLC networks, which, instead, calls for distributed and low-complexity user-centric scheduling methods.

In this paper, we develop a distributed user-centric scheduling method for VLC networks with mutiple APs and mutiple users. Our design goals include: 1) optimizing the network performance; 2) providing fairness among users; 3) decentralized scheduling with limited signaling overhead; 4) low-complexity implementation. To achieve these goals, we establish a VLC scheduling framework based on the elegant concept of stable marriage [16] with general utilities. Based on this framework, a novel user-centric scheduling method is proposed and implemented in a decentralized manner. The proposed method only requires the exchange of a few 1-bit messages between the APs and the users. Neither centralized computation nor the feedback of the channel state information of the entire network are required. It is further shown that the proposed scheduling method can achieve both user-wise and system-wise optimality and has provable low computational complexity. Our simulation results show that, compared to existing centralized approaches, the proposed scheduling method provides better throughput performance and guarantees fairness while requiring lower signaling overheads and lower complexity.

The remainder of the paper is organized as follows. The VLC system model is described in Section 2, and the scheduling problem is stated in Section 3. Section 4 introduces the proposed stable marriage approach. We present and analyse the proposed decentralized user-centric scheduling method in Section 5. Simulation results are presented in Section 6, and conclusions are drawn in Section 7.

2. System model

Consider a downlink VLC network, consisting of a set of VLC APs, each of which employs an LED lamp installed, e.g., on the ceiling of a hall or a room. The layout of the system is illustrated in Fig. 1, where the LEDs are installed in a regular pattern (which is usually the case) on the ceiling of a hall, with the distance between adjacent APs denoted by d. Note that our proposed method is also applicable to irregular configurations of LEDs.

Suppose there are K VLC APs serving L users. Denote the set of all APs by $A = \{a_1, a_2, \cdots, a_K\}$ and the set of all users by $U = \{u_1, u_2, \cdots, u_L\}$. VLC APs generally adopt intensity modulation (IM) techniques, such as on-off keying (OOK) or direct-current-biased optical orthogonal frequency division multiplexing (DCO-OFDM), to embed information into
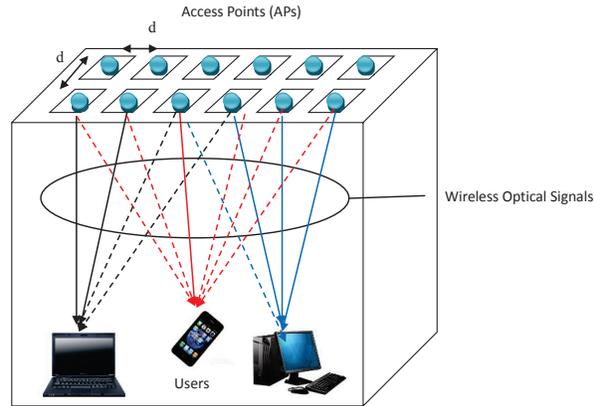
Fig. 1. Layout of the considered indoor VLC system.

optical signals. VLC receivers convert the received optical signals to electrical signals via photodetectors and perform direct detection (DD). In such a system, from a user-centric perspective, a user can be simultaneously served by multiple APs through proper scheduling so that all available resources are fully and intelligently utilized.

According to [3], the signal attenuation of the optical channel from AP a_i to the receiver of user u_j is given by

$$h_{ij} = \begin{cases} \frac{(l+1)S_j}{2\pi r_{ij}^2} \cos^l(\psi)\cos(\varphi)T_s(\varphi)G(\varphi), & \varphi \le \varphi_F \\ 0, & \varphi > \varphi_F. \end{cases} \tag{1}$$

Here, l is the Lambert index and is given by $l = -(\log_2(\cos \psi_{1/2}))^{-1}$, with $\psi_{1/2}$ being the semi-angle at half-illuminance of the LED source. S_j is the physical area of the photodiode of user u_j's detector, and r_{ij} is the distance between user u_j and AP a_i. ψ is the angle of irradiance, φ is the angle of incidence, and φ_F is half of u_j's Field of View (FOV). $T_s(\varphi)$ is the gain of the optical filter, and $G(\varphi)$ represents the gain of the optical concentrator and is given by [17]

$$G(\varphi) = \begin{cases} \frac{RI^2}{\sin^2 \varphi_F}, & \varphi \le \varphi_F \\ 0, & \varphi > \varphi_F, \end{cases} \tag{2}$$

where RI is the refractive index of the lens at the photodiode. From the above channel model, a VLC AP can serve a user only if the AP is within the FOV of the user's receiver.

In VLC, the performance of a user is determined by its signal-to-interference-plus-noise ratio (SINR). As a user could be served by multiple APs, the signal power of user u_j is the aggregate electrical power received from the APs in set $A_j \subseteq A$, where A_j denotes the set of APs serving u_j. The interference power impairing u_j is the sum of the electrical powers received from the APs in set A_j^c, which is the complementary set of A_j, i.e., $A_j^c = A \setminus A_j$. Since VLC systems employ intensity modulation, the transmitted optical signals must be real and nonnegative. Consequently, the optical signals from different APs are added in phase at the photodetector, and the received signal power is the sum power of all received optical signals. More in detail, let p_i be the transmitted optical power of AP a_i, which is typically non-adaptive due to the illumination requirement [14]. Then, the optical powers impinging on the photodetector of u_j

from the APs in A_j and A_j^c are given respectively by

$$P(u_j, A_j) = \sum_{i \in A_j} p_i h_{ij} \tag{3}$$

and

$$P^c(u_j, A_j) = \sum_{i \in A_j^c} p_i h_{ij}. \tag{4}$$

$P(u_j, A_j)$ and $P^c(u_j, A_j)$ are thereby the optical signal power and the optical interference power of user u_j, respectively.

According to [4], the corresponding electrical currents generated by the photodetector are $\gamma \cdot P(u_j, A_j)$ and $\gamma \cdot P^c(u_j, A_j)$, respectively, where γ [A/W] represents the photodetector's responsivity. Since the electrical power is proportional to the square of the amplitude of the current, we can express u_j's SINR as [3]

$$\xi_j = \frac{\gamma^2 \cdot P(u_j, A_j)^2}{\sigma_N^2(u_j, A_j) + \gamma^2 \cdot P^c(u_j, A_j)^2}, \tag{5}$$

where $\sigma_N^2(u_j, A_j) = \sigma_s^2(u_j, A_j) + \sigma_t^2(u_j)$ is noise at the receiver and consists of two main parts [1], namely, the shot noise

$$\sigma_s^2(u_j, A_j) = 2q\gamma P(u_j, A_j) + 2I_{bg}I_2 B \tag{6}$$

and the thermal noise

$$\sigma_t^2(u_j) = \frac{8\pi k_B T_k \eta_c I_2 B^2 S_j}{OVG} + \frac{16\pi^2 k_B T_k \Gamma \eta_c^2 B^3 I_3 S_j^2}{g_m}, \tag{7}$$

where $q = 1.6 \times 10^{-19} C$ is the electronic charge constant, I_{bg} is the background current, I_2 is the noise bandwidth factor, B is the noise bandwidth, k_B is Boltzmann's constant, T_k is the absolute temperature, η_c is the fixed capacitance of the photo detector per unit area, OVG is the open-loop voltage gain, Γ is the field-effect transistor (FET) channel noise factor, $I_3 = 0.0868$, and g_m is the FET transconductance.

3. Problem statement

To arrive at a general problem formulation, we introduce an utility function $f_j(\xi_j)$ to measure the performance of user u_j, where each $f_j(\cdot)$ is only required to be nonnegative and nondecreasing with respect to the input variables. A commonly used utility is the spectral efficiency $f_j(\xi_j) = \log_2(1 + \xi_j)$, which represents an upper bound on the throughput in a unit bandwidth that user j can achieve for a given SINR ξ_j and is often used to evaluate VLC performance [8,9,15]. Note that our framework is general enough to include many other utilities (see, e.g., [18]).

One of our design goals is to properly match each LED or AP to a user so that the users' utilities are maximized. For this purpose, we characterize the system by a graph. Specifically, we introduce a graph $G = (V, E)$, where $V = U \cup A$ is the node set (including all users and APs) and E is the edge set, where $e = (u, a) \in E$ indicates that AP a is within the FOV of user u. Recall that an AP can serve a user only if it is within the user's FOV. Therefore, E actually contains all possible links between the users and the APs. The scheduling problem can be formulated as finding a subset of E, say M, which defines the matching between the users and the APs.

In parallel to utility optimization, fairness is also an important issue in scheduling [19, 20]. Firstly, different users should be provided with similar average utilities over time. Secondly,

serving user u_j should not significantly decrease other users' performance. Based on these two principles, we introduce the *fairness index function* for AP a_i serving user u_j as

$$FI_i(u_j) = \frac{1}{(1+\overline{f}_j)(1+d_i(j))}, \tag{8}$$

where $\overline{f}_j$ denotes the average utility of u_j up to the current time slot and $d_i(j)$ is the number of users that may suffer from inter-user interference due to the service that a_i offers u_j. More specifically, $\overline{f}_j$ is obtained and updated as follows

$$\overline{f}_j = (1 - \frac{1}{T})\overline{f}_{j,T} + \frac{1}{T}f_j, \tag{9}$$

where $\overline{f}_{j,T}$ is the average utility of u_j over the past T time slots and f_j is u_j's current utility. $d_i(j)$ indicates how much interference is generated if a_i serves u_j. Denote the set of the neighbouring users that are affected by user u_j by

$$N(u_j) = \{u \neq u_j \in U \mid \exists a \in A, \{(u,a),(u_j,a)\} \subseteq E\}. \tag{10}$$

Then, $d_i(j)$ is equal to the cardinality of $N(u_j)$, i.e.,

$$d_i(j) = |N(u_j)|. \tag{11}$$

$FI_i(u_j)$ measures the level of fairness when AP a_i serves user u_j. From (8), it is easily seen that $FI_i(\cdot) \in [0,1]$. If $FI_i(u_j) = 0$, then either the utility function of u_j is infinite or serving u_j may affect an infinite number of users and cause severe performance degradation to other users. If $FI_i(u_j) = 1$, then $\overline{f}_{j,T} = 0$ and $d_i(j) = 0$, meaning that u_j has never been served in the past and serving u_j will not generate any interference to other users, which indicates that a_i shall serve u_j.

Consequently, the scheduling problem for the VLC network comprises several objectives. In particular, the utilities of all users should be maximized, which essentially amounts to multi-objective optimization. At the same time, fairness among users should also be achieved. Consequently, it is difficult to formulate the scheduling design for VLC networks as a single optimization problem as is done in the traditional scheduling approaches that can only deal with a single objective (e.g., the sum rate or fairness objectives). Traditional scheduling problem formulations lead to nonlinear discrete optimization problems, which are often NP-hard and entail prohibitive computational complexity to obtain the globally optimal solution. Furthermore, solving these problems requires centralized processing and collecting the channel state information of all APs and users. These requirements conflict with the prospect of VLC as a low-cost communication technique. Therefore, low-complexity and decentralized scheduling methods that only need very limited signaling exchange are preferable in practice. In the following, by borrowing elegant concepts from stable marriage theory, we will propose a novel decentralized scheduling method for VLC networks with provable user-wise and system-wise performance.

4. Stable marriage approach

Stable marriage, a nobel prize winning framework, was first proposed by Shapley and Gale [16], and facilitates an in-depth analysis of matching men and women from two distinct sets. The model includes a number of men and women who would like to be matched based on their preference list for each other. Such a matching problem is often not formulated as a single utility optimization problem, but seeks an efficient solution via individual decision making by the men and women according to their preferences. Thus, a very interesting question, which

may lead to an objective, is that whether there is a *stable* way to match each gentleman with a lady. A matching is *unstable* if there is a man and a woman who prefer each other over their current partners. Such a pair is called a *rogue couple*. Thus, a matching of N men with N women is stable if and only if there is no rogue couple.
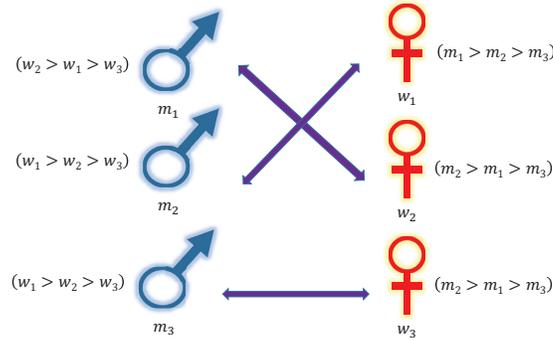

Fig. 2. An example of a stable marriage.

An example of the marriage market is shown in Fig. 2. Three men, namely, m_1, m_2, m_3, and three women, namely, w_1, w_2, w_3, are involved in this instance. Each person's preference list is also illustrated in the Fig., e.g., $(w_2 > w_1 > w_3)$ on the top-left means that man m_1 loves w_2 most, and prefers w_1 to w_3. The matching shown in this instance, $((m_1, w_2), (m_2, w_1), (m_3, w_3))$, is stable. Note that (m_1, w_1) is not a rogue couple. It is true that lady w_1 would rather be with gentleman m_1 than with her current partner. Unfortunately for her, man m_1 would rather be with his current partner than with her. For the same reason, (m_2, w_2) is also not a rogue couple. Note that both m_3 and w_3 are paired with their least favorite choices in this matching. Nonetheless, the given matching is stable, since none of their preferred choices would rather be with them. One extension of stable marriage is college admission, where each college is able to enroll multiple students while each student can only enter one college. Further details regarding the college admission problem can be found in [16, 22].

From the above example, one can see that stable marriage is an elegant framework to handle discrete multi-agent competition with conflicting objectives, which is particularly suitable to address our scheduling problem in the VLC network. Recall that our design goal includes both optimizing the users' utilities and maintaining fairness among the users. Therefore, we can view the users and APs in the VLC network as the men and women in the marriage market (or, more exactly, colleges and students in college admission). The users want to maximize their utilities by choosing their preferred APs, while the APs are responsible for achieving fairness by choosing their preferred users. Consequently, the user-centric scheduling problem is transformed into finding a good matching of the APs to the users according to the stable marriage concept. To this end, we shall first answer the questions outlined in the following three subsections.

4.1. How to formally describe a matching?

Our first mission is to describe the matching of users and APs mathematically. Recall that sets U and A represent all users and APs in the VLC network, respectively. Based on this notation, the matching problem is specified by the tuple

$$(U, A, \{>_u\}_{u \in U}, \{>_a\}_{a \in A}, \{q_u\}_{u \in U}) \tag{12}$$

consisting of the user set U, the AP set A, the set of preference relations of the APs $\{>_u\}_{u \in U}$, the set of preference relations of the users $\{>_a\}_{a \in A}$, and the quota q_u, where q_u indicates how many APs a user u can have at most. A matching is then described as follows.

Definition 1 *A matching M is a function from the set $U \cup A$ into the set of unordered families of elements of $U \cup A$ such that:*
(1). $|M(a)| = 1$ for each AP $a \in A$ and $M(a) = a$ if and only if $M(a) \notin U$;
(2). $1 \le |M(u)| \le q_u$ for each user $u \in U$ and $M(u) = \{u\}$ if and only if $M(u) \nsubseteq A$;
(3). $M(a) = \{u\}$ if and only if $a \in M(u)$.

Note that the user-centric VLC network scheduling is a many-to-one matching problem. Thus, $M(a)$ represents the matched user of AP a and if AP a is not matched with any user, a self-matching $M(a) = a$ is conducted; $M(u)$ represents the *set* of APs matched to user u and if user u is not matched with any AP, a self-matching $M(u) = \{u\}$ is conducted. Note that if $|M(u)| < q_u$, then u is called an *under-subscribed* user.

4.2. How to properly define preferences?

Since the APs are responsible for maintaining fairness among the users, their preferences shall be based on fairness, which can be described by the fairness index function introduced in Section 3. Thus, it is natural to assign the AP preferences according to the fairness index function. For an AP a_i, the higher the value of its fairness index function when serving a user u, the higher the preference given to user u. More specifically, given any $u, u' \in U$, $u >_{a_i} u'$, if and only if $FI_i(u) > FI_i(u')$. *Self-matching* indicates that an AP does not match any user. Each AP a is assumed to prefer serving to self-matching, so for an AP a in the FOV of user u, we have $u >_a a$. Also, the users will not be served by an AP outside their FOVs, so for an AP outside the FOV of user u, we have $a >_a u$.

The preferences of the users shall reflect their utilities. Existing works on resource assignment often adopt a matching independent utility function for each user, i.e., a user's utility does not depend on a specific matching of users and APs. For example, in [23] each AP is assumed to provide a fixed gain to a user, which does not depends on other APs. However, such an assumption is not justified for VLC networks. Recall from Section 3 that the utility function of user u_j, $f_j(\xi_j)$, where the SINR $\xi_j = \xi_j(A_j)$, depends on A_j, i.e., the set of APs that serve user u_j. Moreover, $f_j(\xi_j)$ is a general nonnegative and nondecreasing function without specific form. Hence, $f_j(\xi_j)$ cannot be directly used to define the preferences of the users.

To overcome this difficulty, we define the preferences of the users not based on their utilities, but based on the power received from an AP. Specifically, a user prefers one AP to another one if and only if the optical power received from the former is great than that from the latter. Recall that in Section 2, p_i denotes the transmit power of AP a_i, and h_{ij} represents the channel gain from AP a_i to user u_j. Hence, the received power of u_j from AP a_i is $p_i h_{ij}$. Thereby, for a fixed user u_j and two given APs a_i and a_k, $a_i >_{u_j} a_k$ if and only if $p_i h_{ij} > p_k h_{kj}$. The intuition behind this definition is that the more power a user can receive from an AP, the higher the SINR that the user can attain. Yet, there arises a question: Does the preference of a user reflect its utility function, or in other words, are users' utilities optimized by using this definition of preference? Surprisingly, the answer to this question is positive. We will consider this aspect more in detail in Section 5.

Similar to the APs, self-matching of a user means that the user does not match any AP. Each user u is assumed to prefer being served to no service. Thus, for any AP a within the FOV of user u, $a >_u u$. User u will not attempt to call for service from APs that are outside its FOV, so for any AP a outside the FOV of user u, $u >_u a$.

4.3. How to formally describe stability?

To formally describe *stability*, we first introduce the following two concepts.

1) A matching M is *individually rational* if there exists no AP a for which $a >_a M(a)$, and there exists no user u such that $u >_u a'$ for some $a' \in M(u)$. Individual rationality avoids invalid service, which occurs when a user is matched to an AP outside its FOV. In such a case, no signal can be received by the user from the AP, so the service from the AP to the user is invalid. In other words, individual rationality prevents a matching M from providing invalid service.

2) A matching M is *blocked* by a pair $(u,a) \in U \times A$ if one of the following two conditions holds: (i) $u >_a M(a)$ and $|M(u)| < q_u$; (ii) $u >_a M(a)$ and for some $a' \in M(u)$, $a >_u a'$. If $|M(u)| < q_u$, then user u has $(q_u - |M(u)|)$ additional partners which are all by themselves. Thus, condition (i) can be explained as that AP a prefers user u over its current partner $M(a)$ and user u also prefers to be matched to AP a compared to itself. Condition (ii) indicates that AP a prefers user u over its current partner $M(a)$, and user u is willing to replace one of its partners with AP a. A stable matching shall exclude any blocked pair, i.e., there is no single pair of user and AP which prefer being matched to each other instead of being matched to their current partners.

Therefore, a stable matching is defined as follows.

Definition 2 *A matching M is stable if and only if M is individually rational and not blocked by any pair $(u,a) \in U \times A$.*

Consequently, the user-centric scheduling problem is transformed into finding a stable matching according to the APs' and users' preferences. Note that this is a many-to-one matching problem and finding its stable solution is difficult. In the next section, we provide a decentralized method to achieve this goal.

5. User-centric scheduling

In this section, we present a user-centric scheduling algorithm along with its decentralized implementation to achieve a stable matching for the VLC network. We further analyze the optimality and complexity of the proposed algorithm.

5.1. Distributed scheduling algorithm

For clarity, we first show the principle of the proposed algorithm, which we refer to as the Decentralized Stable Matching Scheduling Algorithm (DSMSA), in Fig. 3.

DSMSA works as follows. Recall from Section 4.1 that a user u is called under-subscribed if $|M(u)| < q_u$. In every iteration, each under-subscribed user attempts to link to the favorite AP in its Potential Partner List (PPL), which includes the APs within its FOV, and then deletes it from the PPL, while each AP decides to accept or reject these requests. The users and APs make their decisions based on their preferences (as introduced in Section 4). The mechanism terminates either when all users are matched with sufficiently many APs (i.e., are not under-subscribed) or each under-subscribed user has been rejected by every AP. Therefore, DSMSA will terminate within a finite number of steps.

Algorithm 1 describes in detail how DSMSA is implemented in a distributed manner. In each iteration, each under-subscribed user requests a new link based on its preferences, which, according to Section 4.2, depend on its received power from each AP and can be measured locally. An AP who receives a link request decides whether or not to accept the request and replace its current partner with the new one. From Section 4.2, an AP's preference is determined by the fairness index function which depends on the users' average utilities and the numbers of their neighbors. Such information can be locally inferred by the APs if they are interconnected or periodically reported by the users at a low rate. Therefore, in each iteration, APs and users
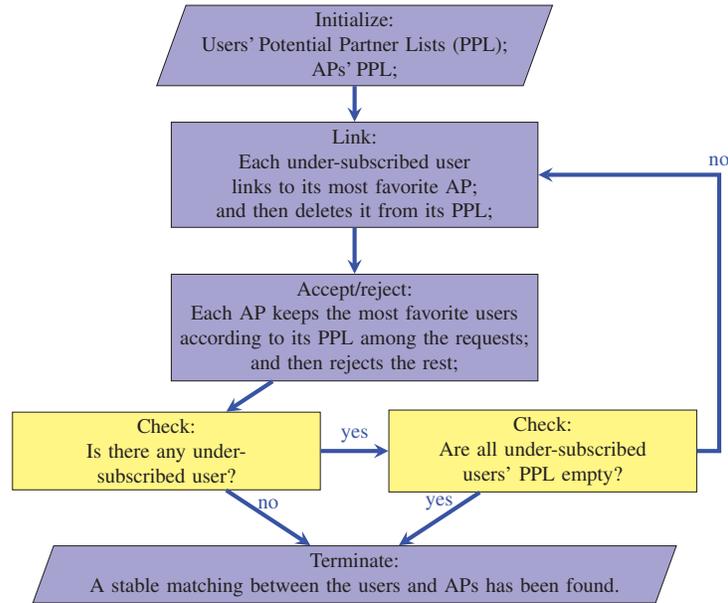
Fig. 3. The flowchart of DSMSA.

Algorithm 1: Decentralized Stable Matching Scheduling Algorithm (DSMSA)

Input: User set U, AP set A, set of the users' preference relations $\{>_u\}_{u \in U}$, set of the APs' preference relations $\{>_a\}_{a \in A}$, and quota $\{q_u\}_{u \in U}$.

Output: A stable matching M.

1 each user u sets its PPL $p(u) = \{a \in A | a >_u u\}$ and $M(u) = \phi$;

2 each AP a sets its PPL $p(a) = \{u \in U | u >_a a\}$ and $M(a) = a$;

3 **while** *some user u is under-subscribed (i.e., $|M(u)| < q_u$) and $p(u)$ is not empty* **do**

4   **for** *any such user u* **do**

5   request a link from its most preferred AP in $p(u)$;

6   add this AP to $M(u)$, and then delete it from $p(u)$;

7   **end**

8   **for** *any AP a^* that receives a link request from some user u^** **do**

9   **if** $u^* >_a M(a)$ **then**

10   send a reject message to $M(a)$;

11             $M(a) = u^*$;

12   **else**

13   send a reject message to u^*;

14   **end**

15   **end**

16   **for** *any user $\hat{u}$ that receives a reject message from some AP $\hat{a}$* **do**

17   delete $\hat{a}$ from $M(u)$

18   **end**

19 **end**

20 return M;

only need to exchange the request and reject messages, each of which requires only 1 bit. Consequently, DSMSA entails a very limited signalling overhead, and does not require the gathering of the channel state information of all APs and users.

Theorem 1 *The matching M generated by DSMSA is stable.*

Proof: Please see Appendix A1.   ■

Theorem 1 indicates that DSMSA always leads to a stable matching. Note that in DSMSA a user will never request an AP to which the user prefers itself. Thus, the resulting matching M must be individually rational. There will also not be any blocked pair in M, which ensures that DSMSA generates a stable matching. The formal proof is provided in Appendix A1.

5.2. Optimality analysis

In VLC networks, each user wishes to maximize its own utility function and each AP is dedicated to maintaining fairness among the users. Therefore, the user-AP matching problem is essentially a multi-objective optimization problem, characterized by different levels of optimality. In the following, we shall analyze the optimality of the proposed DSMSA from both the user-wise and system-wise perspectives.

5.2.1. User-wise optimality

The user-wise optimality can be naturally characterized by *Pareto optimality*, one of the most widely-used optimality concepts in multi-objective optimization [24]. Pareto optimality implies that no individual's utility can be improved without making at least one other individual worse off. In the VLC network, a matching M is *Pareto dominated* by another matching M' if and only if (i) each user's utility satisfies $f_j(\xi_j(M')) \geq f_j(\xi_j(M))$ for $j = 1, 2, \cdots, m$ and each fairness index function satisfies $FI_i(M'(a_i)) \geq FI_i(M(a_i))$ for $i = 1, 2, \cdots, k$; (ii) there exists some user $u_{j'}$ such that $f_{j'}(\xi_{j'}(M')) > f_{j'}(\xi_{j'}(M))$ or some AP $a_{i'}$ such that $FI_{i'}(M'(a_{i'})) > FI_{i'}(M(a_{i'}))$. Note that SINR $\xi_j(M)$ is defined in (5) and depends on matching M. A matching is *Pareto optimal* if and only if it is not Pareto dominated by any other matching.

Theorem 2 *The matching M generated by DSMSA is Pareto optimal.*

Proof: Please see Appendix A2.   ■

We would like to note that the Pareto optimality in Theorem 2 is in terms of the users' utilities as well as fairness. Recall that in Section 4.2, we defined the preferences of the users based on their received powers instead of directly based on their utilities, which depend on the users' SINRs, and left the question of whether the users' utilities can be optimized or not. Here, Theorem 2 gives a positive answer from the user-wise perspective. In the following, we will further answer this question from the system-wise perspective.

5.2.2. System-wise optimality

The system-wise optimality often represents the performance of the entire system. It is natural to use the sum of the users' utilities, i.e., $\sum_{j=1}^{k} f_j(\xi_j)$, as the measure of the performance of the entire system. Note that in general multiple stable matchings may exist. Thereby, system-wise optimality is achieved by the matching that maximizes the sum utility $\sum_{j=1}^{k} f_j(\xi_j)$ among all stable matching.

Theorem 3 *The matching generated by DSMSA achieves the maximum sum utility $\sum_{j=1}^{m} f(\xi_j)$ among all stable matchings.*

Proof: Please see Appendix A3.   ■

Theorem 3 confirms that DSMSA also provides system-wise optimality in terms of the sum utility of all users. Although it is possible to find among all possible matchings (i.e., also including the non-stable ones) the one with the maximum sum utility, this requires an exhaustive search that has prohibitive complexity (usually increasing exponentially with the number of users and APs). In addition, an exhaustive search is generally centralized. On the other hand, the proposed DSMSA is a decentralized scheme and guaranteed to attain the highest sum utility among all stable matchings, which are already Pareto optimal. We will further show that DSMSA enjoys low computational complexity in the next subsection.

5.3. Computational complexity

As mentioned above, the proposed DSMSA can not only be implemented in a decentralized manner but also enjoys the advantage of low complexity. Note that in each iteration, at least one AP is deleted from some user's PPL. Thus, the computational complexity of DSMSA is bounded by the sum of the initial cardinalities of all PPLs. Furthermore, in the distributed implementation, each user is actively involved in the scheduling process, so the complexity only depends on the maximum run times of the individual users. Moreover, each user u's run time is proportional to the cardinality of the user's initial PPL, i.e., $|\{a \in A | a >_u u\}|$. Hence, we have the following result.

Theorem 4 *The complexity of DSMSA is $O(\max_{u \in U} |p^*(u)|)$, where $p^*(u) = \{a \in A | a >_u u\}$.*

Proof: Please see Appendix A4. ∎

Theorem 4 indicates that the run time of DSMSA is proportional to the maximum number of APs inside a single user's FOV. For example, there are typically 4 to 6 APs inside a user's FOV observed in the simulation. Thus, DSMSA will terminate within at most 6 iterations. The low computational complexity and decentralized implementation make DSMSA particularly suitable for VLC networks that aim to provide high quality wireless service at low cost.

6. Performance evaluation

In this section, we present comprehensive simulation results to evaluate the proposed scheduling method. We consider both regular and irregular arrangements of APs adopting the same configurations as in [9, 25]. Specifically, we first consider a regular arrangement of 8×8 APs installed equidistantly on the ceiling of a square room as shown in Fig. 4. The simulation parameters are the same as in [9] and are listed in Table 1, where the distance between any two adjacent APs is denoted by d and the height from the ceiling to the horizontal plane on which the receivers of the users are placed is denoted by h. The transmitted optical power of each AP is set to P_{to}, i.e., $p_1 = \cdots = p_K = P_{to}$, and each receiver has an FOV of 50 degrees. The LED deployment has been checked to satisfy illumination requirements. The distribution of the received optical power is shown in Fig. 5, where the entire area can be roughly divided into two parts, namely, the central area and the edge area. The received power in the 12m × 12m central area varies between -26 to -23 dBm, while the edge area has significantly lower received power.

To demonstrate the performance of the proposed distributed user-centric scheduling algorithm, i.e., DSMSA, the simulation results are averaged over 5000 independent tests, in each of which a certain number of users are randomly distributed in the room. Our proposed method is compared with three baseline scheduling methods. In the first baseline method, referred as APRS, each AP is randomly chosen to serve one of the users whose FOVs cover this AP. The second baseline method, referred as FR, is the traditional frequency reuse method from RF cellular systems, which can be realized via DCO-OFDM techniques [21] in VLC networks. The key idea of FR is to eliminate interference by forcing adjacent APs to use different frequencies, where two APs are adjacent if and only if they are within the FOV of the same user. In FR,

Table 1. Parameters used in the simulation.

Parameter	Value	Parameter	Value	Parameter	Value
d	2 [m]	h	2.2 [m]	γ	0.54 [A/W]
S	0.785 [mm^2]	P_{to}	25 [W]	g_m	0.03
$\psi_{1/2}$	50 [deg.]	φ_F	50 [deg.]	B	100 [MHz]
$T_s(\varphi)$	1.0	RI	1.6	I_3	0.0868
T_K	300 [K]	η_c	1.12×10^{-6} [F/m^2]	OVG	10
Γ	1.5	I_{bg}	5.1×10^{-3} [A]	I_2	0.562
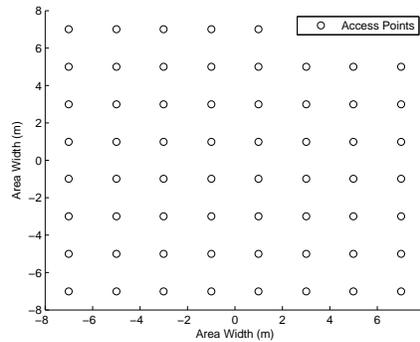
Fig. 4. Regular access point arrangement.
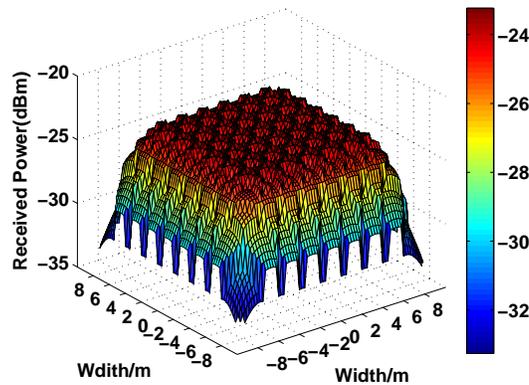
Fig. 5. Received power distribution.

to maximize the sum rate, the frequency reuse factor, which is the rate at which the same frequency can be used in the network, is chosen as large as possible under the constraint that there is no interference between any users. The third baseline method is from [9] and referred as GWMIN, whose principle is to force the inter-user interference to zero and then maximize the sum rate using an approximation algorithm based on graph theory. In each time slot, GWMIN first builds an interference graph, where each vertex denotes a user and there is an edge between two vertices if and only if there is potential interference between the corresponding users, i.e., their FOVs cover the same AP. Then, GWMIN performs centralized optimization over the interference graph by gathering the channel state information of the entire network.

For both the proposed DSMSA and the state-of-the-art GWMIN, the scheduling in the current time slot depends on the average rate achieved so far, which is computed based on a time window. Thus, it is necessary to run the algorithms for a number of time slots in each experiment so that the average rate of a certain time slot can be computed based on the rates of the previous slots. Following the setting in [9], each test runs for 50 time slots.

and has to collect the channel state information of all users at a central node.
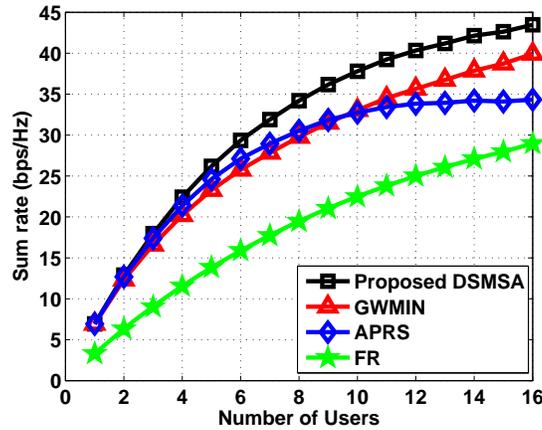
Fig. 6. Sum rate performance.

6.1. Sum rate performance

Figure 6 shows the sum rate of the four considered scheduling schemes. The data rate of each user u_j is obtained by adopting the utility function $f_j(\xi_j) = \log_2(1 + \xi_j)$, which is normalized by the bandwidth and has the unit bps/Hz. It can be observed that the sum rate of FR is relatively low since the frequency reuse factor must be large enough to avoid inter-user interference. Although APRS can achieve a higher sum rate than FR when the number of users is small, the gap decreases when the number of users increases and eventually APRS is outperformed by FR. The reason for this behavior is that as the user density increases, inter-user interference becomes the dominant factor limiting the system performance, which is not taken into account by APRS. GWMIN may obtain a higher sum rate than FR and APRS, since it applies an effective approximation algorithm to mitigate interference. Nevertheless, DSMSA can achieve an even higher sum rate than GWMIN. This is because the proposed method intelligently manages the inter-user interference instead of forcing it to zero or using rigid frequency reuse to avoid it. More importantly, the proposed scheme can be implemented in a distributed manner requiring a small signalling overhead, while GWMIN is a centralized algorithm

6.2. Fairness

Fairness is another important factor in VLC networks. To measure fairness among users, we adopt the *service fairness index (SFI)* [26], one of the most widely used fairness indicators, as performance metric, which is defined as follows

$$SFI = \max_{i,j} \left| \frac{\bar{r}_i}{w_i} - \frac{\bar{r}_j}{w_j} \right| \left(\frac{1}{L} \sum_{l=1}^{L} \frac{\bar{r}_l}{w_l} \right)^{-1} \tag{13}$$
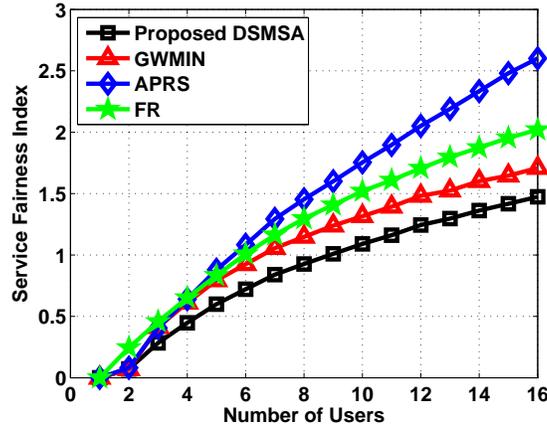
Fig. 7. Service fairness index.

where w_j is the service weight factor, L is the number of users, and $\bar{r}_j$ is the average rate of user u_j obtained by averaging the utility value of the user u_j over the 50 past time slots. For clarity, we assume all users have the same service weights and thus set the service weight factors to $w_1 = \cdots = w_L = 1$. Therefore, (13) can be simplified to

$$SFI = \max_{i,j} \frac{L|\bar{r}_i - \bar{r}_j|}{\left(\sum_{l=1}^{L} \bar{r}_l\right)}. \tag{14}$$

Apparently, a lower SFI indicates higher fairness. If SFI is 0, then all users have the exact same data rate, which means absolute fairness is achieved.

The service fairness indices of the four methods are shown in Fig. 7. APRS has the largest SFI since it does not take into account fairness but serves users randomly. Both FR and GWMIN have a smaller SFI since they aim at proportional fairness. Compared with FR, APRS, and GWMIN, DSMSA achieves a significantly lower SFI and thus maintains a higher degree of fairness.
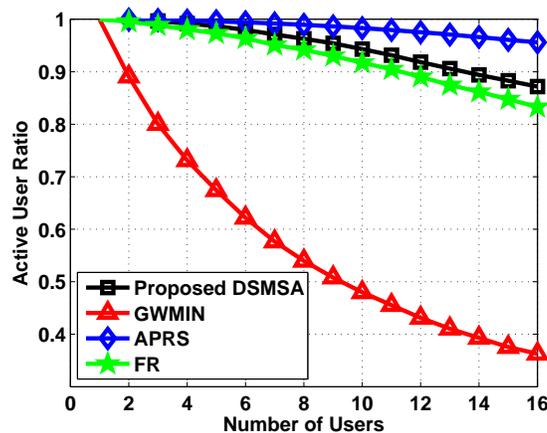

Fig. 8. Active user ratio.

Besides sum rate and fairness, the *active user ratio (AUR)* also plays an important role in

VLC networks. The active user ratio is the ratio of the average number of active users to the total number of users and is defined as

$$AUR = \frac{\sum_{k=1}^{K} N_k}{K \cdot L} \qquad (15)$$

where K is the number of time slots, N_k is the number of active users in each time slot, and L is the total number of users. In the following simulation, $K = 5000 \times 50 = 250000$ since there are 5000 independent tests and 50 time slots in each test. The active user ratio is a measure for short-time fairness as well as communication delay. Obviously, a larger active user ratio indicates that more users can be served in a given time slot and thus a higher fairness is achieved. More importantly, it also means that an individual user is active in more time slots and thus the delay is shorter.

The corresponding simulation results are shown in Fig. 8. Thereby, GWMIN has the smallest active user ratio, meaning that many users in the VLC network have to wait for several time slots to be served, resulting in a larger delay. The active user ratio for APRS is highest, as expected, because of the random service strategy. The proposed DSMSA scheduling method achieves a higher active user ratio than FR and GWMIN. In fact, the active user ratio of DSMSA is no less than 90% when the number of users does not exceed 14 and 87% when there are 16 users.

6.3. FOV impact

Next, we would like to investigate the impact of the FOV on the performance of the proposed DSMSA. As shown in Fig. 9, the sum rate decreases for large FOVs and has a maximum at a FOV of around 35 to 25 degrees. This is because as the FOV increases, the interference between the users also increases. It is interesting to see that the lowest SFI and the highest active user ratio are also achieved at a FOV of 35 degrees. Notice that DSMSA maintains a good active user ratio, which is always greater than 0.75, for all considered FOVs.

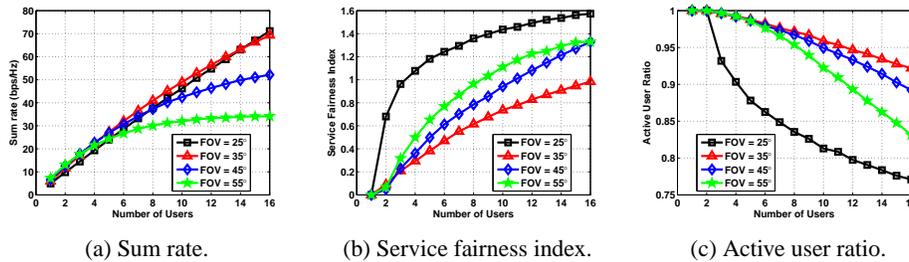(a) Sum rate. (b) Service fairness index. (c) Active user ratio.

Fig. 9. Effects of the FOV.

6.4. Irregular AP arrangement

DSMSA can be applied for any network topology. To demonstrate this, we consider the irregular AP arrangement that was used in [9, 25] to reduce the SNR fluctuation in the room. As shown in Fig. 10, in this arrangement there are 12 circle-APs and 4 corner-APs in a 5m×5m area. The vertical distance from each AP to the receiver of a user is 2.2m. The radius of the AP-circle is 2.0m and the distance between the corner-APs and their nearest walls is 0.1m. The power of each AP is 2W and the FOV is set to be 40 degrees.

As shown in Fig. 11, the proposed DSMSA still achieves a similar performance advantage compared to the three baseline schemes as for the regular AP arrangement. More exactly, DSMSA achieves the highest sum rate, the lowest SFI, and the second highest active
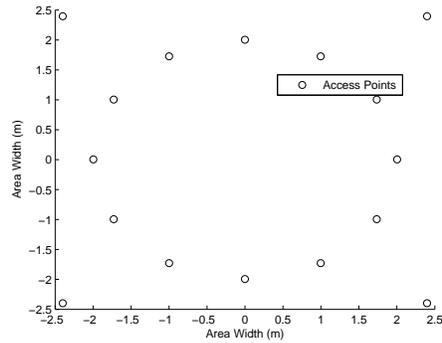
Fig. 10. Irregular Access Points arrangement.

user ratio among the considered state-of-the-art scheduling methods, despite the distributed implementation of DSMSA. We also show the impact of the FOV in this case in Fig. 12. As the FOV decreases, the sum rate increases as the interference between different users decreases. On the other hand, the SFI increases as the FOV decreases since there are fewer APs available to serve a certain user.
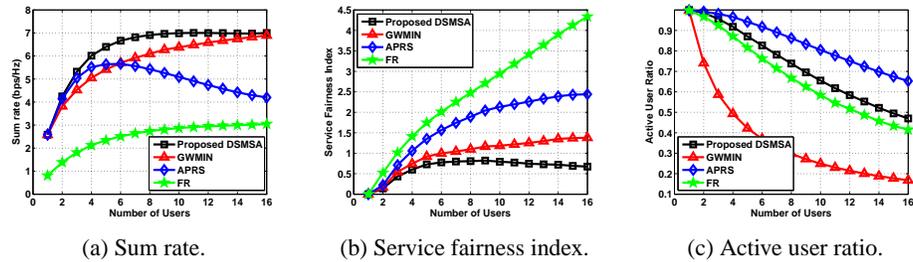

(a) Sum rate.　　　　(b) Service fairness index.　　　　(c) Active user ratio.

Fig. 11. Performance comparison for the irregular AP arrangement.

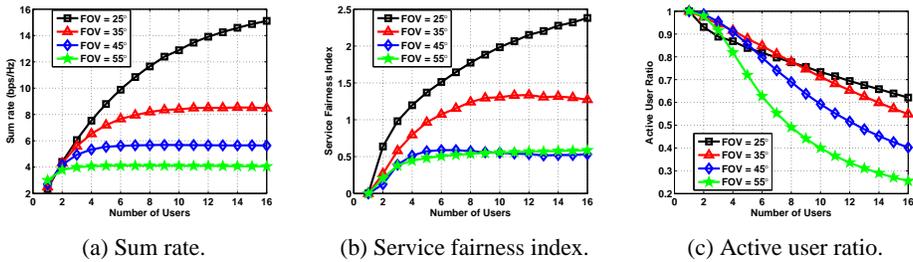(a) Sum rate.　　　　(b) Service fairness index.　　　　(c) Active user ratio.

Fig. 12. Impact of the FOV in the irregular AP arrangement.

6.5. Performance comparison

Our findings regarding the four investigated methods with respect to the four investigated performance metrics are summarized in Table 2. The proposed DSMSA has the highest sum rate and the lowest SFI. Thus, it can simultaneously achieve the largest data throughput and

Table 2. Comparison of the four scheduling methods.

	DSMSA	APRS	FR	GWMIN
Sum Rate	highest	low	lowest	medium
Service Fairness Index (SFI)	lowest	highest	medium	medium
Active User Ratio	high	high	high	low
Decentralized Implementation	Yes	Yes	No	No

the highest degree of long-term fairness among the four considered schemes. Although APRS has the highest active user ratio, its SFI value is the largest, which means that it cannot provide different users with similar data rates and fairness cannot be guaranteed. Yet, the proposed DSMSA also leads to a high active user ratio and thus ensures that user delay is small and short-term fairness is maintained.

7. Conclusion

In this paper, we considered the scheduling problem in VLC networks from a user-centric perspective. We adopted a stable marriage approach and transformed the scheduling problem into a many-to-one matching problem. Then, we proposed a distributed scheduling method to optimize both the users' utilities and fairness while limiting the signaling overhead. The provided simulation results show that, compared to existing centralized methods, the proposed method can achieve a better sum-rate performance while providing both long-term and short-term fairness.

Appendix

A1. Proof of theorem 1

We first prove the following lemmas.

Lemma 1 *DSMSA ends with a matching M.*

Proof: The key is to prove that the loop in DSMSA terminates in a finite number of iterations. Note that in each iteration, $\sum_{u \in U} |p(u)|$ decreases at least by 1. When $\sum_{u \in U} |p(u)| = 0$, we have $p(u) = \phi$ (i.e., an empty set), for all $u \in U$. Thus, DSMSA ends with a matching M. ∎

Lemma 2 *If AP a is user u's most preferred AP in p(u) in the ith iteration of the while loop in DSMSA, then in the subsequent iterations, $M(a) \geq_a u$.*

Proof: We prove the lemma by contradiction. Suppose that the jth iteration for $j > i$ is the first counterexample after which $M(a) <_a u$, while at the $(j-1)$th iteration, $M(a) \geq_a u$. According to the algorithm, $M(a)$ must be changed at the jth iteration. In other words, at the jth iteration, $\exists u'$ such that $p(u') \neq \phi$, $|p(u')| < q_{u'}$, and a is the most preferred AP in $p(u')$, which implies $u' >_a M(a) \geq_a u$. Hence, after the jth iteration, $M(a) = u'$ and $M(a) \geq_a u$. This contradicts the initial assumption and completes the proof. ∎

By Lemma 1, the algorithm will terminate. Recall that it has been shown that M is individually rational in Section 5. We only need to show that there is no blocked pair in M. Suppose that there is a blocked pair $(u,a) \in M$. Then, there are two possibilities.

1. $u >_a M(a)$ and $|M(u)| < q_u$. Note that $u >_a M(a)$ indicates that a is within u's FOV, so u tried to link to a in some earlier iteration and received a reject message. Hence, by Lemma 2, $M(a) \geq_a u$, which leads to a contradiction.

2. $u >_a M(a)$ and there exists some AP $a' \in M(u)$ such that $a >_u a'$. Since u links to APs according to its preference, u tried to link to a before it tried to link to a'. Since u received a reject message, by Lemma 2, $M(a) \geq_a u$, which is a contradiction.

Thus, the assumption of the existence of a blocked pair is false and matching M is stable.

A2. Proof of theorem 2

We prove the result by contradiction. Suppose that there exists another matching M' by which M is pareto-dominated. Let $U^* = \{u|M(u) \neq M'(u)\}$ denote all users that have different partners in M and M'. The total number of APs does not depend on the matching, so we have $\sum_{u \in U^*} |M(u)| = \sum_{u \in U^*} |M'(u)|$. Thus, there exists some $u_j \in U^*$ such that $|M(u_j)| \geq |M'(u_j)|$. M is pareto-dominated by M', so $f_j(\xi_j(M')) \geq f_j(\xi_j(M))$. Also, $u \in U^*$ indicates $M'(u_j) \neq M(u_j)$. Thus, $f_j(M') > f_j(M)$. Let a_i be the AP with the largest $p_i h_{ij}$ which is in $M'(u_j)$ but not in $M(u_j)$. $|M(u_j)| \geq |M'(u_j)|$ and $f_j(\xi_j(M')) > f_j(\xi_j(M))$ indicates that there exists some AP $a_s \in M$ such that $p_i h_{ij} > p_s h_{is}$, i.e., $a_i >_{u_j} a_s$. Since M is pareto-dominated by M', we have $FI_i(M'(a_i)) \geq FI_i(M(a_i))$, which implies $M'(a_i) \geq_{a_i} M(a_i)$. Thereby, $u_j = M'(a_i) >_{a_i} M(a_i)$ because $M(a_i) \neq u_j$. Hence, (a_i, u_j) is a blocked pair in M, which leads to a contradiction since M is stable.

A3. Proof of theorem 3

We first introduce the following two useful definitions.

Definition 3 *Let the reduced AP list of user u be $ral(u) = \{a \in A | \exists$ some stable matching M such that $(u,a) \in M\}$.*

Note that if a does not belong to $ral(u)$, then user u cannot connect to a. For simplicity, let r_u denote the cardinality of $ral(u)$, i.e., $r_u = |ral(u)|$.

Definition 4 *Let the optimal AP list of user u be*

$$oal(u) = \begin{cases} ral(u), & r_u \leq |q(u)| \\ \text{user } u\text{'s } |q(u)| \text{ most preferred APs in } ral(u), & r_u > |q(u)| \end{cases} \tag{16}$$

which represents the best partners that user u can have in a stable matching.

The main proof consists of two parts. Firstly, we prove that the matching M generated by DSMSA satisfies $M(u) = oal(u)$. Suppose that the first iteration when a user u is rejected by an AP a which is in the optimal AP list $oal(u)$ is iteration k. In this iteration, a rejects u in favor of another user u^* since $u^* >_a u$. By definition of $oal(u)$, there exists a stable matching M^* in which $M^*(a) = u$. We will argue that (u^*, a) is a blocked pair in M^*, thus contradicting stability. Since $M^*(a) = u$, we have $u^* >_a M^*(a)$. If $|M^*(u^*)| < q_{u^*}$, then (u^*, a) is a blocked pair in M^*. If $|M^*(u^*)| = q_{u^*}$, then by definition of $oal(u^*)$, $|oal(u^*)| = q_{u^*}$. Iteration k is the first iteration when some user is rejected by one AP in its optimal AP list, so in the kth iteration, u^* has not been rejected by any AP in $oal(u^*)$. Since in the kth iteration u^* is paired with a, u^* likes a at least as much as some AP in $oal(u^*)$. In other words, there are at most $|q_{u^*}| - 1$ elements in $oal(u^*)$ which u^* strictly prefers to a. Thus, there are at most $|q_{u^*}| - 1$ elements in $M^*(u^*)$ which u^* strictly prefers to a. In other words, there exists some AP $a' \in M^*(u^*)$ such that $a >_{u^*} a'$, which indicates that (u^*, a) is a blocked pair in M^*. Thus, the assumption is not true, and $M(u) = oal(u)$.

Secondly, we prove that the sum of the utility functions is maximized. Consider any user u_j. Without loss of generality, we assume $ral(u_j) = \{a_1, a_2, \cdots, a_{r_u}\}$ with $a_1 >_{u_j} a_2 >_{u_j} \cdots >_{u_j} a_{r_u}$. Since $f_j(\xi_j)$ is monotonically nondecreasing, $f_j(\xi_j)$ is maximized if and only if ξ_j is maximized. Let $C_j = \sum_{i \in A} p_i h_{ij}$. By (3) and (4), $P^c(u_j, A_j) = C_j - P(u_j, A_j)$. According to (5), ξ_j can be expressed as

$$\xi_j = \frac{\gamma^2 \cdot P(u_j, A_j)^2}{\sigma_N^2(u_j, A_j) + \gamma^2 \cdot (C_j - P(u_j, A_j))^2} \tag{17}$$

which is monotonically nondecreasing in $P(u_j, A_j)$. Note that $P(u_j, A_j) = \sum_{i \in A_j} p_i h_{ij}$ and $a_k >_{u_j} a_l$ implies $p_k h_{kj} > p_l h_{lj}$, so $P(u_j, A_j)$ is maximized if and only if $A_j = oal(u_j)$. Hence, $\sum_{j=1}^{m} f_j(\xi_j)$ is maximized if and only if $\forall j$, $A_j = oal(u_j)$. A_j is the set of all APs that serve u_j, i.e., $A_j = M(u_j)$. Hence, the matching M generated by DSMSA maximizes the sum utility among all stable matchings.

A4. Proof of theorem 4

By proof of *Lemma 1*, the computational complexity is bounded above by $O(\sum_{u \in U} |p^*(u)|)$, where $p^*(u)$ is the cardinality of user u's initial PPL. In DSMSA's distributed implementation, each user is actively involved in the scheduling process, so the complexity only depends on the maximum run time of the individual users. Each user u's run time is proportional to the cardinality of user u's initial PPL, i.e., $|\{a \in A | a >_u u\}|$. Hence, the computational complexity is bounded above by $O(\max_{u \in U} |p^*(u)|)$.

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