THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF ELECTRICAL ENGINEERING AND TELECOMMUNICATION

Frequency Estimation for Three-Phase Power System

by

Zuguang Chen

Thesis submitted as a requirement for the degree

Bachelor of Engineering (Electrical Engineering)

Submitted: October 2015 Student ID: z3384687
Supervisor: Dr. Elias Aboutanios Topic ID: EAB22

Title: Frequency Estimation for Three-Phase Power System   Topic number: EAB22

Student Name: Zuguang Chen   Student ID: z3384687

A. Problem statement

Frequency estimation of three-phase power system is tools provide protection for the power system machine against loss of synchronism. The frequency estimation error determined the condition of power system health. Thus, the stability of the power system can be guaranteed with high performance of frequency estimator. Also the unbalance condition of three-phase power system causes power loss. The main challenge is estimating frequency of three-phase signals simultaneously and improve the frequency estimation algorithms design for balanced system can be applying to unbalanced three-phase system. Due to limited of resource and computational load, the estimator design must reach requirement of having high efficiency. Therefore, simplify the process of three-phase signal measurements benefit the project design.

B. Objective

Implementation of different frequency estimators for both signal amplitude balanced and unbalanced system cases under noise situation
Demonstrate estimators that designed for balanced system have good performance in amplitude unbalanced system.
Extended the research on frequency estimation of phase unbalanced system;
Find the proper solution to improve the estimation results;

C. My solution

Using Clarke’s Transform simplified the three-phase system in voltage amplitude unbalanced condition. Derive the Clarke’s Transform expressions for phase unbalanced three-phase system.
Implement Balancing Voltage Transform and follow by DFT-based iterative interpolation estimation method to improve estimation performance in unbalanced condition.
Implement Matrix Pencil and A&M estimator and compare them with “BVT” approach in phase unbalanced condition.

D. Contributions (at most one per line, most important first)

Derive the Clarke’s Transform expressions for phase unbalanced three-phase system, provide new direction for phase unbalanced frequency estimation research.
Demonstrate the advantage and disadvantage of BVT-DFT, Matrix Pencil and A&M approaches.
Implement the new approaches in phase unbalanced condition in high efficiency.

E. Suggestions for future work

BVT with DFT iterative approach without comparison of coefficients.
Hybrid unbalanced system research.
Tracking ability of frequency estimation in unbalanced condition.

While I may have benefited from discussion with other people, I certify that this report is entirely my own work, except where appropriately documented acknowledgements are included.

Signature: ______________________________   Date: 28/ 10/ 2015

Pointers

List relevant page numbers in the column on the left. Be precise and selective: Don't list all pages of your report!

3-4	Problem Statement
4	Objective

Theory (up to 5 most relevant ideas)

2	Motivation of Three-Phase Power Frequency Estimation research
15	Clarke's Transform
16	Balancing Voltage Transform
9	DFT-iterative interpolation frequency estimation
20	Matrix Pencil and A&M

Method of solution (up to 5 most relevant points)

16,19	Utilization of Clarke's Transform in unbalanced system
18	DFT-based iterative interpolation frequency estimation
16	Balancing voltage transform to improve performance in unbalanced condition
20	Comparison of BVT, Matrix pencil and A&M

Contributions (most important first)

19	Derive the Clarke's Transform expressions for phase unbalanced three-phase system, provide new direction for phase unbalanced frequency estimation research.
20	Demonstrate the advantage and disadvantage of BVT-DFT, Matrix Pencil and A&M approaches.
19-20	Implement the new approaches in phase unbalanced condition in high efficiency.

My work

14-23	System block diagrams/algorithms/equations solved
27-33	Description of assessment criteria used
25-26	Description of procedure (e.g. for experiments)

Results

27-33	Succinct presentation of results
27-33	Analysis
32-33	Significance of results

Conclusion

34-35	Statement of whether the outcomes met the objectives
35	Suggestions for future research

Literature: (up to 5 most important references)

15	[10] E.Clarke. 1997
10	[5] E. Aboutanios and B. Mulgrew. 2005
16	[11] Yili Xia; Kai Wang; Danilo P. Mandic; Wenjiang Pei. 2014
22	[13] Shanglin Ye and Elias Aboutanios 2015
10	[4] E. Aboutanios. 2010

Acknowledgements

I would like to express my very great appreciation to my supervisor Dr. Elias Aboutanios offer many helps and suggestions in thesis research when I was in times of difficulty. I would also like to extend my thanks to Shanglin Ye and Jiadong Sun who put a large amount of effort in helping me with theory work.

- Zuguang Chen

Abstract

Various frequency estimation methods for power systems have been extensively developed in recent years. However, few of those algorithms are specifically designing for three-phase unbalanced system, and this may cause the low efficiency and high difficulty of frequency estimation. In this report, many literature frequency estimation techniques based on the Fast Fourier Transform are reviewed. Illustrate the Clarke's transformation which extremely reduce the difficulty of frequency estimation for three-phase system. The proposed Clarke's Transform expression in phase unbalanced system and Balancing Voltage Transform is introduced for the purposed of improving estimation accuracy in both voltage and phase unbalanced system. The comparison of various frequency estimation approaches also involved in the report.

Contents

List of Figures

Abbreviations

AC	**A**lternating **C**urrent
AWGN	**A**dditive **W**hite **G**aussia **N**oise
BVT	**B**alancing **V**oltage **T**ransform
CRLB	**C**ramer **R**ao **L**ower **B**ound
DFT	**D**iscrete **F**ourier **T**ransform
MLE	**M**aximum **L**ikelihood**E**stimate
MBS	**M**aximum **B**in **S**earch
RMSE	**R**oot **M**ean **S**quare **E**rror
SNR	**S**ignal-to-**N**oise **R**atio
WLS	**W**eighted **L**east **S**quare
ZC	**Z**ero-**C**rossing
ZP	**Z**ero **P**adding

Chapter 1

Introduction

The modern power gird, or the smart grid combines varies of technology such as control system technology, electrical power system and information technology. The smart grid also create new jobs for its maintenance and operation and boom the economic development. Compare with the traditional power system, the smart grid can integrate energy sources which are renewable, the system continuously monitoring, receive the feedback elements from the central network and keep self-learning. In addition, smart gird lower the energy costs, save customers money and have higher customer satisfaction compare with traditional power system. The power supply demands keep rising with industry highly development. At the same time, the advanced technology brings disadvantages to the power system. The complexity of smart grid is one of them. How to keep power system working properly as long as possible becomes a research direction. The smart grid requires power generation and loads having continuous balanced matches have a stable operation. And the problem of protecting electrical power system has been a major concerned today.

The measurement of power quality and power system heath rely on frequency estimation. Therefore, it is essential for power system to have accurate and efficient frequency estimation technology monitored in real time.

1.1 Research Motivation

1.1.1 The Importance of Power Frequency

AC power frequency is important and sensitive parameter in power system. In many countries, the standard of AC power frequency is 50Hz. Frequency of power system varies when power supply generation and load changes. The reliability of power system closely related to the stabilization of power frequency. In other words, the frequency of power system is maintaining within a tiny range of the ratted power frequency. So variation of frequency need to be monitored.

1.1.2 Negative Effects from Power System Faults

- **Frequency fluctuation** :

  Frequency fluctuation can be treating as random variation in power frequency, due to an imbalance of load and generation. It happens when loads draw currents change suddenly[2].

- **Loss of synchronization**

  Loss of synchronization of frequency and phase in power system also may cause the frequency fluctuation if the phase different is large enough. Even more damage the system device, because significant rise in current.

- **Harmonics**

  In addition, electrical power frequency harmonics can cause serious problem in power system, it mainly effect the power quality and also result in current has a significant increase, the temperature of system devices and conductors rise up, finally damage those equipment.

1.1.3 Frequency Estimation for Power Systems

In smart grid or power system, undesirable frequency changes can result in power system unbalance between the generation and the load, bring abnormal consequence. It's very important for power system to maintain the system frequency, and have stable working condition. Frequency estimation provide a reflection consequence of dynamic unbalance between the generation and consumption. So have an accurate monitor on power frequency in dynamics is essential. Doing research on frequency estimation contribute to extent the efficiency and reduce the computation cost for frequency estimation methods.

In recent research on frequency estimation, single phase frequency estimation and three-phase frequency estimation have been advanced in many applications. But most of the estimation methods suffer from unbalanced three phase power system.

1.2 Requirements of Frequency Estimation in Power System

- accurate estimation in system with noise and harmonics;
- effective and fast processing in real-time;
- specifically design in three-phase power system for both balanced and unbalanced case;
- estimator performance need to approach to the theoretical boundary.

1.3 Problem Statement

Frequency estimation of three-phase power system is tools provide protection for the power system machine against loss of synchronism.The frequency estimation error determined the condition of power system health. Thus, the stability of the power system can be guaranteed with high performance of frequency estimator. Also the unbalance condition of three-phase power system causes power loss. The main challenge is estimating frequency of three-phase signals simultaneously and improve the frequency estimation algorithms design for balanced system can be applying to unbalanced three-phase system. Due to limited of resource and computational load, the estimator design must reach requirement of having high efficiency. Therefore, simplify the process of three-phase signal measurements benefit the project design.

1.4 Thesis Objectives

Basic Objective

- To understand the basic background knowledge of frequency estimation for the three-phase AC power system;
- Implementation of different frequency estimators for both signal amplitude balanced and unbalanced system cases under noise situation;

Advanced Objective

- Demonstrate estimators that designed for balanced system have good performance in amplitude unbalanced system.
- Extended the research on frequency estimation of phase unbalanced system;
- Find the proper solution to improve the estimation results;

1.5 Report Format

- Chapter 2 (Background): the review of background knowledge and literature methods of frequency estimation.

- Chapter 3 (Thesis Research Methodology): possible methods and solution for thesis problem.
- Chapter 4 (Implementation Plan): provide evidence and implementation for methods which mentioned in the report, the research plan for next session.
- Chapter 5 (Simulation Results): provide simulation results according to the methodology.
- Chapter 6 (Conclusion): overall review of this paper.

Chapter 2

Background

It has been greatly long period since frequency estimation of power systems was obtained considerable development. The research on frequency estimation methods have been devised deeply in many regions including transformation and modelling techniques, famous estimators and its performance. This report focus on frequency estimation techniques as a main interest, those previous contributions for frequency estimation will be reviewed in chapter.

2.1 Model of Three-Phase Power System

In the three-phase power system, the voltage signal amplitude should be the same with RMS voltage ratted as 240V with freuqncy 50Hz in Australia if the system assumed to be under the balance condition. The measured three-phase signal model used here is as follows:

$$\begin{aligned} v_a(n) &= V_a cos(\omega t_n + \varphi_a) + \zeta_a \\ v_b(n) &= V_b cos(\omega t_n - \frac{2\pi}{3} + \varphi_b) + \zeta_b \\ v_c(n) &= V_c cos(\omega t_n + \frac{2\pi}{3} + \varphi_c) + \zeta_c \end{aligned} \tag{2.1}$$

Where v_a,v_b,v_c are the sampled signal values in phases a, b and c, V_a,V_b,V_c are the signal amplitudes in each phase, φ_a,φ_a,φ_a are the phases of signals in each phase, ζ_a,ζ_a,ζ_a are the noise terms in each phase, which generally represented as Additive White Gaussian Noise, the variance of noise is $\sigma^2/2$. ω is system frequency, t_n is discrete time instant.

2.2 Frequency Estimation Methods and Techniques

2.2.1 Fourier Transform

Continuous Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt \tag{2.2}$$

The Fourier Transform converts the time domain signal, x(t), into its frequency domain representation, it is the fundamental of frequency analysis.

Discrete Fourier Transform

$$X(n) = \sum_{k=0}^{N-1} x[k]\, e^{-j2\pi k \frac{n}{N}}, k = 0....N-1 \tag{2.3}$$

The DFT denoted by X[k], allows to evaluate the Fourier Transform X(θ). This complex valued sequence $X[k]$ is obtained by sampling the Fourier Transform $X(\theta)$ at a finite number of frequency points. it sampling the points equally spaced over $-\pi \leq \theta \leq \pi$.

DFT allows us to determine the frequency content of a signal, that is, to perform spectral analysis. The DFT is a powerful tool in implementation of amount of digital signal processing algorithms.

2.2.2 Maximum Likelihood Estimate

Maximum likelihood estimation basically achieves the highest performance of measurements that give , it obtains the sample numbers which are exactly the same as the sample numbers are given at the beginning. Which means the frequency of MLE is given by the maximiser of the periodogram [3].

$$\hat{f}_{ML} = arg_{\lambda} max X\left(\lambda\right) \tag{2.4}$$

$$X\left(\lambda\right) = \sum_{k=0}^{N-1} x\left[k\right] e^{-j2\pi k\lambda} \tag{2.5}$$

2.2.3 The Cramer Rao Lower Bound

The Cramer Rao Lower Bound is a method to determine a lower bound of variance and provides a benchmark to assess the performance of an estimator so that the frequency can be estimated given by,

$$\sigma^2 = \frac{6f_s^2}{\left(2\pi\right)^2 \rho N \left(N^2 - 1\right)} \tag{2.6}$$

2.2.4 Zero-Padding Method

The zero padding method is basically implemented by padding the data with zeroes to some length L $\geq$ N. Zero padding estimator can improved te resolution by 1/L without adding the information signal, which the frequency can be measured as,

$$f = \frac{m + \delta}{L} \tag{2.7}$$

the variance of zero padding estimation is,

$$\hat{\sigma}_f^2 = \frac{1}{12L^2} \tag{2.8}$$

The zero-padding estimation has poor performance with short length of L padding in. To improve the performance of estimator, large quantity of zeroes need to pad in, and the computational cost is significant.

2.2.5 Method based on using Recursive-Least-Squares Approach

Frequency is a key parameter in power system, and the frequency estimation of power system need to be accurate. The Recursive-Least-Squares (RLS) algorithm is applied to frequency estimation of power system. Compare with the Least Mean Square (LMS) algorithm which is also an adaptive algorithm for frequency estimation, RLS has better convergence rate and good robustness, that make it becomes a good tools of frequency estimation.The basic model of voltage signal given by[1]:

$$v(n) = Acos\left(wt_n + \varphi_0\right) = Acos\left(wnT_s + \varphi_0\right) \tag{2.9}$$

The frequency can be obtained by adding $v\left(n-1\right) + v\left(n+1\right)$,

$$v(n-1) + v(n+1) = 2Acos\left(wnT_s + \varphi_0\right)cos(wT_s) = 2v(n)cos(wT_s) = \eta v(n) \tag{2.10}$$

where η = 2cos(wT_s). And the frequency f is given as,

$$\hat{f} = \frac{arccos(\frac{\eta}{2})}{2\pi T_s} \tag{2.11}$$

2.2.6 Method based on Iterative Frequency Estimation by Interpolation

The computational cost of maximum likelihood estimator or zero-padding estimator is the real problem for both of methods, and it may have issues with convergence and resolution problem. The method based on iterative frequency estimation

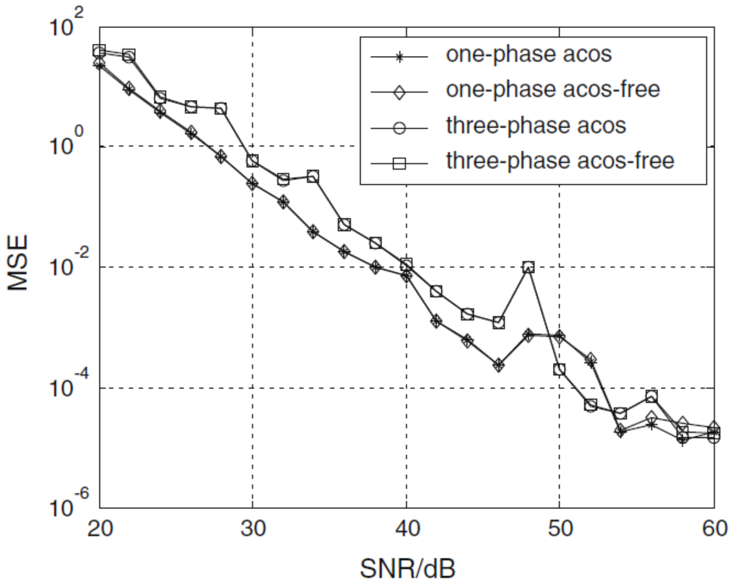

Figure 2.1: Plot of Mean Square Error verses different SNRs [1]

by interpolation has been detailed explain in [4], [5]. It has two approaches of using the interpolation. The first algorithm uses two complex DFT coefficients to calculate midway between standard DFT coefficients while the second algorithm is more focus on magnitude of DFT coefficients, and it resolves the problem that occur in the ML and ZP estimator.

The iterative frequency estimator works as using the coarse search to find the $\hat{m}$ of bin which is the maximum firstly. Introduce two DFT coefficents X_p where p = ±0.5, on the edges of the maximum bin. For $|\delta|$ is the residual in [-0.5,0.5], the estimation of residual frequency δ, $\hat{\delta}$ can be represented as,

$$\hat{\delta}_i = \hat{\delta}_{i-1} + h\left(\hat{\delta}_i\right) \tag{2.12}$$

where,

$$h\left(\hat{\delta}_{i-1}\right) = \frac{1}{2} Re\left\{\frac{X_{0.5} + X_{-0.5}}{X_{0.5} - X_{-0.5}}\right\} \tag{2.13}$$

$$h\left(\hat{\delta}_{i-1}\right) = \frac{1}{2}\frac{|X_{0.5}| + |X_{-0.5}|}{|X_{0.5}| - |X_{-0.5}|} \tag{2.14}$$

and the final frequency can be estimated as,

$$\hat{f} = \frac{\hat{m} + \hat{\delta}_Q}{N} f_s \tag{2.15}$$

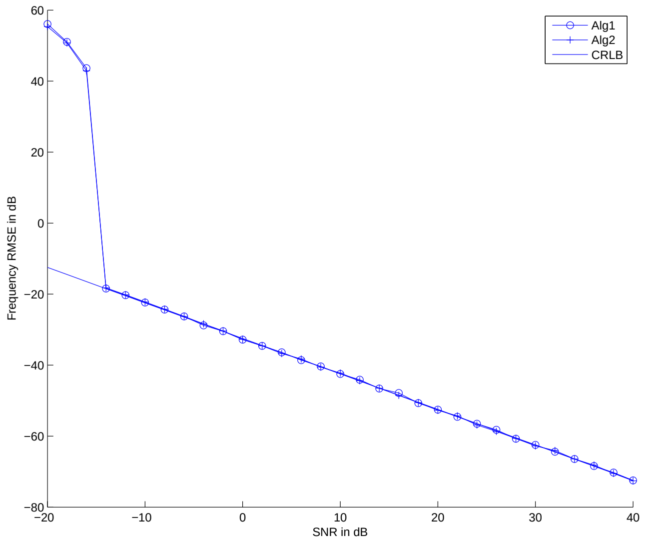

Figure 2.2: Plot of the exponential signal frequency estimation error for Alg1 and Alg2 verses SNR with CRLB with N = 1024

2.2.7 Method based on Modified Dichotomous Search

For purpose of avoiding divisions and functions with long Taylor series expansions, the dichotomous search in-depth extensively explained in [6], [7]. It uniformly accomplish the CRLB and reduce the computational complexity with various interpolation techniques combined [8]. And it reduces the number of iterations for convergence which is compulsory to accomplish the CRLB before.

Dichotomous Search of the Periodogram Peak Algorithm

The dichotomous search frequency estimator [6] consists of a coarse-frequency search followed by a fine-estimation stage. The MBS is used to obtain an initial coarse-frequency estimate.

The binary search method use in [8], locate the true DFT peak, using Y(m - 1) and Y(m + 1), which are FFT coefficients either side of the maximum, get larger coefficient. Then compare those two coefficients in order to half the frequency resolution.

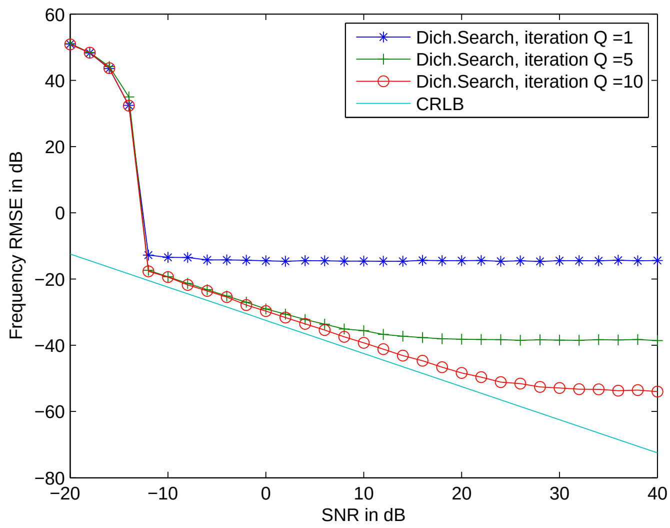

FIGURE 2.3: Plot of sinusoidal signal frequency estimation for the dichotomous search algorithm verses SNR with CRLB with N = 1024

For the sake of approaching the CRLB, Zakharov and Tozer found that, at least the length L = 1.5N must be reached to pad into data, and The frequency error variance asymptotes represented by

$$\sigma_f^2 = (\frac{f_s}{\sqrt{12}L2^{Q-1}})^2 \tag{2.16}$$

Modified Dichotomous Search of the Periodogram Peak Algorithm

To modified the dichotomous search, and remove the data that required to pad with zeros in the previous algorithm, the modified dichotomous search was introduced by Aboutanios [7].

The performance of the previous dichotomous search is going to be poor after the first iteration because δ close to zero, which cause by two DFT coefficients either side of the maximum are dominated by the noise. So the estimator variance is given by,

$$\sigma_f^2 = 0.25\frac{f_s^2}{N^2} \tag{2.17}$$

If an incorrect decision is made on the first iteration, the error is not recoverable unless the true frequency lies on the common point.

The parameter Δ determines the degree of overlap of the two intervals. For instance, setting the initial value of Δ to 0.75 results in an overlap region of half a bin width

The modified algorithm achieves the CRB without the need to pad the data with zeros. This results in a reduction of the computational load. Although only applicable if L is a power of 2, we will assume here, for the purpose of comparison, and the FFT requires $(L/2)log_2(L)$ complex operations.

Chapter 3

Thesis Research Methodology

Using the Clarke's transform, frequency from multiphase can be easily measured by applying the frequency estimation algorithms in balanced condition. However, the performance of frequency estimation algorithms from the unbalanced system condition. The unbalanced system includes with amplitude unbalanced system and phase unbalanced system. Which means, the three-phase are not equal and phase difference between each phase voltage signal is not 120 degree. Therefore, the output of Clarke's transform cannot form a single complex exponential signal. The transform algorithm breaks in this stage and form one positive and negative sequence, many of the frequency estimation methods are designed for only positive sequence condition, so the performance of those estimators have bias results. In this chapter, the proposed solution is introduced.

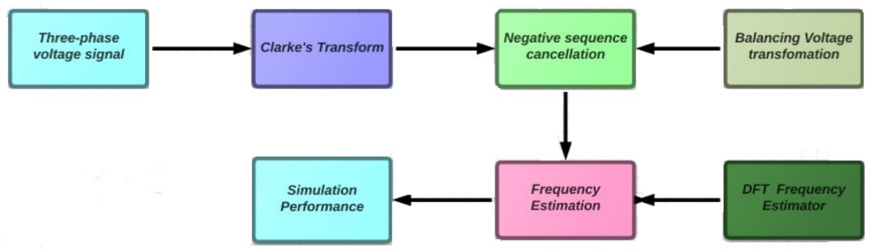

FIGURE 3.1: Block Diagram of the proposed solution

3.1 Frequency Estimation for Voltage Amplitude Unbalanced Three-Phase Power System

3.1.1 Transformation of Three-Phase System Clarke's Transform

Doing frequency analysis to three-phase signals separately is difficult to implement with limited resource. Thus, Clarke's transform is a powerful tool changing the representation of three-phase signal into only one complex exponential signal, it was developed by Edith Clarke [10]. Many literature researches in frequency estimation of three-phase system established above the Clarke's transform. The output of Clarke's transform has three components, which are direct, quadrature, and DC- component.

$$V_{0\alpha\beta} = \begin{pmatrix} v_0(k) \\ v_\alpha(k) \\ v_\beta(k) \end{pmatrix} = \frac{2}{3} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & cos(\frac{2\pi}{3}) & cos(\frac{-2\pi}{3}) \\ 0 & sin(\frac{2\pi}{3}) & sin(\frac{-2\pi}{3}) \end{pmatrix} \begin{pmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{pmatrix} = CV_{0\alpha\beta} \tag{3.1}$$

Where the C is Clarke's Transform matrix.

When it comes to the balanced three-phase system, three sinusoidal signals have the same amplitude which given by,

$$V_a = V_b = V_c \tag{3.2}$$

Then v_α and v_β components can be represented as,

$$v_\alpha(k) = \frac{2}{3}(v_a(k) - \frac{1}{2}v_b(k) - \frac{1}{2}v_c(k)) \tag{3.3}$$

$$v_\beta(k) = \frac{2}{\sqrt{3}}v_b(k) - \frac{2}{\sqrt{3}}v_b(k) \tag{3.4}$$

v_α and v_β components in an orthogonal reference frame and v_0 the homopolar component of the system.The homopolar component is less important in many applications, so $v_0 = 0$. Under sinusoidal and balanced conditions, three signals have the same amplitude, and the system could be represent as a complex signal,

$$v_s(k) = v_\alpha(k) + jv_\beta(k) \tag{3.5}$$

3.1.2 Balancing Voltage Transformation Method

Most frequency estimation algorithms work well under balanced three phase power system conditions after using the Clarke's transform. However, most of these methods suffer from performance with unbalanced three phase voltage conditions. When deviations happens for example the amplitude unbalanced, and then negative sequence and phase angle estimation error occurred, which represented by oscillations at double the system frequency[11], [9].

$$v(k) = v_\alpha + jv_\beta = Ae^{j(\omega k+\phi)} \tag{3.6}$$

Under unbalanced conditions, the Clarke's transformed voltage is second order noncircular (improper). When three phase power system operates in amplitude unbalanced condition, then the complex Clarke's transformation will be,

$$v(k) = Ae^{j\omega k} + Be^{-j\omega k} \tag{3.7}$$

Where,

$$A = \frac{\sqrt{6}(V_a + V_b + V_c)}{6} e^{j\phi} B = \frac{\sqrt{6}(2V_a + V_b + V_c)}{6} e^{j\phi} \tag{3.8}$$

This expression is theoretically accurate for both the balanced and unbalanced system conditions. For balanced system, $B = 0$, on the unbalanced system, $B \neq 0$. In this case, we need to cancel out the effect of negative sequence $(Be^{j\omega k})$ in order to avoid the bias frequency estimation.

So the balancing voltage transform (BVT) method is introduced below,

$$\begin{aligned}\tilde{v} &= v(k) - av^*(k)\\ &= v(k) - \frac{B}{A^*}(A^*e^{-j\omega k} + B^*e^{j\omega k})\\ &= (A - \frac{|B|^2}{A^*}e^{j\omega k})\\ &= (1-|a|^2)Ae^{j\omega k}\end{aligned} \tag{3.9}$$

where the ratio $a = \frac{B}{A^*}$ are the degree of system imbalance. And $\tilde{v}$ get only the positive sequence.

However, the estimation of system imbalance ratio a will be the next concerned. Basically, we need to resort to the full autocorrelation structure of $v(k)$, by using the lag m, we find the autocorrelation coefficient $r(m)$ and pseudo-autocorrelation coefficient $p(m)$ in (5),(6).

$$\begin{aligned}r(m) =& E[v(k)v^*(k-m)\\ &= E[|A|^2e^{j\omega m}] + E[|B|^2e^{-j\omega m}]\\ &+ E[AB^*e^{j\omega(2k-m)}] + E[A^*Be^{-j\omega(2k-m)}]\\ &\approx |A|^2e^{j\omega m} + |B|^2e^{-j\omega m}\end{aligned} \tag{3.10}$$

$$\begin{aligned}p(m) =& E[v(k)v(k-m)\\ &= E[A^2e^{j\omega(2k-m)}] + E[B^2e^{-j\omega(2k-m)}]\\ &+ E[ABe^{j\omega m}] + E[ABe^{-j\omega m}]\\ &\approx ABe^{j\omega m} + ABe^{-j\omega m}\end{aligned} \tag{3.11}$$

And the final solution of system imbalance ratio a is given below,

$$a = \frac{r(m) + r^*(m) \pm \sqrt{(r(m)+r^*(m))^2 - 4|p(m)|^2}}{2p^*(m)} \tag{3.12}$$

Where, the $r^*(m)$ and $p^*(m)$ are the conjugate of r(m) and p(m).

DFT interpolation

Follow by the DFT coefficient interpolation frequency estimators, the frequency of balancing voltage transform's output can be estimated. These kind of estimators basically using the coarse search to find sample index with largest magnitude inside N bins. The fine search can be applied to get the true frequency in interval of $[p_N - 0.5, p_N + 0.5]$. In this report, we use the DFT interpolation method proposed by Aboutanios and Mulgrew[4], [5].

Alg1 and Alg2 are two algorithms approach which is given below

$$h\left(\hat{\delta}_{i-1}\right) = \frac{1}{2} Re \left\{ \frac{X_{0.5} + X_{-0.5}}{X_{0.5} - X_{-0.5}} \right\} \tag{3.13}$$

$$h\left(\hat{\delta}_{i-1}\right) = \frac{1}{2} \frac{|X_{0.5}| + |X_{-0.5}|}{|X_{0.5}| - |X_{-0.5}|} \tag{3.14}$$

and the final frequency can be estimated as,

$$\hat{f} = \frac{\hat{m} + \hat{\delta}_Q}{N} f_s \tag{3.15}$$

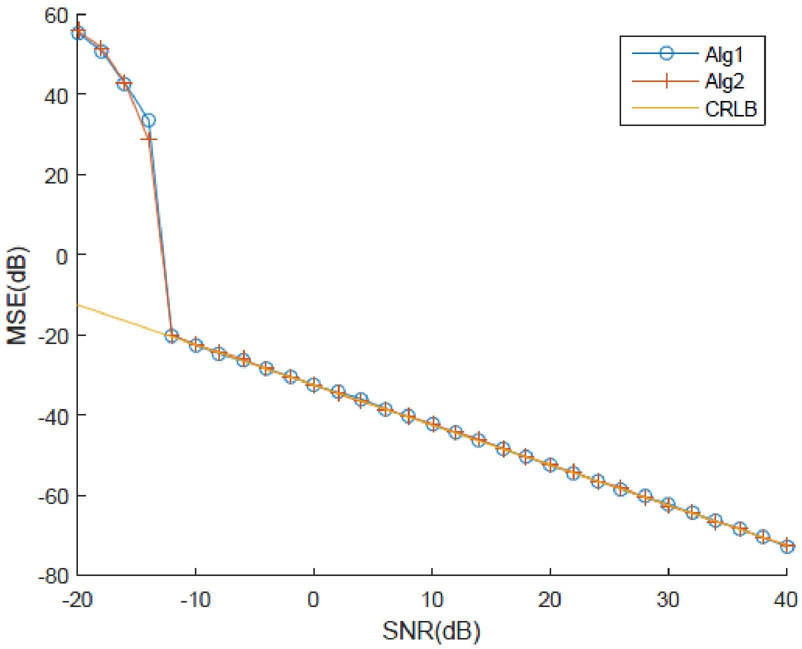

FIGURE 3.2: Plot of the exponential signal frequency estimation error for Alg1 and Alg2 verses SNR with CRLB with N = 1024 (BVT improved)

The above method achieve almost the same performance as DFT interpolation in balanced conditions, and provide the evidence that balancing voltage method can efficiently improve the accuracy of estimation in voltage amplitude unbalanced three-phase system.

3.2 Frequency Estimation for Phase Unbalanced Three-Phase System

3.2.1 Proposed Three-Phase Model for Phase Unbalanced Three-Phase System

For the purpose of extent the research on phase unbalanced three-phase system, we assume that all of phase angle for this condition will be ϕ_1,ϕ_2,ϕ_3, while other parameters remain the same as in balanced condition. The three-phase voltage signals for phase unbalanced condition model given as:

$$\begin{aligned} v_a(n) &= V_a cos(\omega t_n + \phi_1) + \zeta_a \\ v_b(n) &= V_b cos(\omega t_n + \phi_2) + \zeta_b \\ v_c(n) &= V_c cos(\omega t_n + \phi_3) + \zeta_c \end{aligned} \tag{3.16}$$

Apply the Clarke's transform for generating the complex exponential, we find the complex exponential expression is given by,

$$\begin{aligned} v &= v_\alpha + j v_\beta \\ v_\alpha &= \sqrt{\frac{2}{3}}(v_a - \frac{v_b}{2} - \frac{v_c}{2}) \\ v_\beta &= \sqrt{\frac{2}{3}}(\frac{\sqrt{3}v_b}{2} - \frac{\sqrt{3}v_c}{2}) \end{aligned} \tag{3.17}$$

Substitute the three-phase into the expression,

$$
\begin{aligned}
v &= v_\alpha + jv_\beta \\
&= \sqrt{\frac{2}{3}}(V_a cos\phi_1 - \frac{V_b}{2}cos\phi_2 - \frac{V_c}{2}cos\phi_3)\frac{e^{j(\omega t_n)} + e^{-j(\omega t_n)}}{2} \\
&- \sqrt{\frac{2}{3}}(V_a sin\phi_1 - \frac{V_b}{2}sin\phi_2 - \frac{V_c}{2}sin\phi_3)\frac{e^{j(\omega t_n)} - e^{-j(\omega t_n)}}{2j} \\
&+ j(\sqrt{\frac{2}{2}}V_b(cos\phi_2 - \sqrt{\frac{2}{2}}cos\phi_2)\frac{e^{j(\omega t_n)} + e^{-j(\omega t_n)}}{2} \\
&- j(\sqrt{\frac{2}{2}}V_b(sin\phi_2 - \sqrt{\frac{2}{2}}sin\phi_2)\frac{e^{j(\omega t_n)} - e^{-j(\omega t_n)}}{2j}
\end{aligned}
\tag{3.18}
$$

Finally, the expression can be conclude as:

$$v = (A_1 + jA_2)e^{j(\omega t_n)} + (B_1 + jB_2)e^{-j(\omega t_n)} \tag{3.19}$$

And we find this equation has the similar structure with the original output from the Clarke's transform that we have used in voltage amplitude unbalanced three-phase system. Moreover, in balancing voltage transform. To obtain the system imbalance ratio a, we need to find the absolute value of $A_1 + jA_2$ and $B_1 + jB_2$, besides which one is larger need to be determined. According to the equation (3.18), the value of $A_1 + jA_2$ and $B_1 + jB_2$ can be only determined once we get the phase angle information.

3.2.2 Matrix Pencil and A&M Approach with Proposed Modeling

Matrix Pencil

It is based on the characteristic of the underlying signal, X_0 and X_1 are two noise-free data matrices with dimension (N - L)$\times$ L and which are given by

$$x_t = [x_t, x_{t+1}, ..., X_{N-L+t-1}]^T. \tag{3.20}$$

$$\underset{(N-L)\times L}{X_0} = [x_{L-1}, x_{L-2}, ..., x_0] \tag{3.21}$$

$$\underset{(N-L)\times L}{X_1} = [x_L, x_{L-1}, ..., x_1]. \tag{3.22}$$

Where pencil parameter define by L, and

$$X_0 = \begin{bmatrix} x_{L-1} & x_{L-2} & \cdots & x_0 \\ x_L & x_{L-1} & \cdots & x_1 \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-L-1} \end{bmatrix} \tag{3.23}$$

$$X_1 = \begin{bmatrix} x_L & x_{L-1} & \cdots & x_1 \\ x_{L+1} & x_L & \cdots & x_2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-L} \end{bmatrix} \tag{3.24}$$

where L is pencil parameter.

$$X_0 = Z_L B Z_R \tag{3.25}$$

$$X_1 = Z_L B Z Z_R \tag{3.26}$$

$$Z_L = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \cdots & z_M^{N-L-1} \end{bmatrix} \tag{3.27}$$

$$B = \begin{bmatrix} a_1 & 0 & \dots & 0 \\ 0 & a_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_M \end{bmatrix} \tag{3.28}$$

$$Z_R = \begin{bmatrix} z_1^{L-1} & z_1^{L-2} & \dots & 1 \\ z_2^{L-1} & z_2^{L-2} & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ z_M^{L-1} & z_M^{L-2} & \dots & 1 \end{bmatrix} \tag{3.29}$$

$$Z = \begin{bmatrix} z_1 & 0 & \dots & 0 \\ 0 & z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_M \end{bmatrix} \tag{3.30}$$

Where Z_L and Z_R are Vandermonde matrices, and B and Z are diagonal matrices. By going through the whole method mention in [12].Finally, frequency can be estimated as

$$f_i = arg(z_i)/2\pi \tag{3.31}$$

Matrix pencil method has high efficiency in computation. Thus, it has low sensitive to noise and restrict to the signal.

A&M

This frequency estimation method based on A&M estimtor which has been derived in [13]. The algorithm combines the A&M estimator and iterative leakage subtraction for the purpose of error cancellation in interpolated Fourier coefficients.

The method start with A&M from using the MBS to find the periodogram which given by,

$$\hat{m} = arg_{\lambda} max|X(k)|^2 \tag{3.32}$$

Where X(k) = DFT[x(n)]. And the frequency can be estimated by

$$\hat{f} = \frac{\hat{m} + \hat{\delta}_Q}{N} f_s \tag{3.33}$$

The noise-free interpolated coefficients is required to refined the coarse estimation, and it is given by

$$\begin{aligned} X_{\pm 0.5} =& \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(\hat{m}+\hat{\delta}\pm 0.5)n} \\ =& A \frac{1 + e^{j2\pi(\delta - \hat{\delta})}}{1 - e^{j\frac{2\pi}{N}(\delta - \hat{\delta} \mp 0.5)}} \end{aligned} \tag{3.34}$$

Where $\hat{\delta}$ is estimated residual from above iteration.

The advantage of method of multiple superimposed complex exponentials is that voltage amplitude nor phase angle are not required for frequency estimation. It reduce the estimation bias gradually by iteration with fast process, and can achieve the performance closed to the CRLB.

Chapter 4

Implementation Plan

4.1 Implementation in Thesis A

In order to identify feasible and measurable objectives and goals, and keep the thesis A research progress in track, the below steps must be follow:

- 1. Review the literature about frequency estimation in both single-phase and three-phase system,

  Basic Set Up

- 2. Set up the sampling Rate and natural frequency generate range,
- 3. Build up the three-phase system model,(2.1)
- 4. Demostarte Clarke's transform and find out the $v_\alpha(k)$ and $v_\beta(k)$,
- 5. Using complex signal $v_s(k) = v_\alpha(k) + v_\beta(k)$ as input testing the performance of estimator,

  Dichotomous Search Estimator[6]

- 6. Implement Dichotomous Search estimator start with MBS, S = FFT(s,L) and Y(n) = $|S(n)|^2$,then m = argmaxY[n]

- 7. Set Δ=1, Y_{-1}=Y(m-1) and Y_1=Y(m+1)

- 8. Achieve the iteration of dichotomous search

  $\Delta = \frac{\Delta}{2}$ if, $Y_1 > Y_{-1}$

  then Y_{-1}= Y_0 and m=m+δ

  else Y_1= Y_0 and m=m-δ

- 9. Then $\hat{f} = \frac{m}{L} f_s$

  Iterative Frequency Estimation by Interpolation[5]

- 10. Implement iterative interpolation estimator start with MBS, S = FFT(s,L) and Y(n) = $|S(n)|^2$,then m = argmaxY[n]

- 11. Set $\hat{\delta}_0 = 0$,

- 12. Achieve the iteration interpolation estimation

  $X_p = \sum_{k=0}^{N-1} s\left[k\right] e^{-j2\pi k \frac{\hat{m}+\hat{\delta}_{i-1}+p}{N}}, p = \pm 0.5$

  $\hat{\delta}_i = \hat{\delta}_{i-1} + h\left(\hat{\delta}_i\right)$

  Where,

  for algorithm 1,

  $h\left(\hat{\delta}_{i-1}\right) = \frac{1}{2} Re\left\{\frac{X_{0.5}+X_{-0.5}}{X_{0.5}-X_{-0.5}}\right\}$

  for algorithm 2,

  $h\left(\hat{\delta}_{i-1}\right) = \frac{1}{2}\frac{|X_{0.5}|+|X_{-0.5}|}{|X_{0.5}|-|X_{-0.5}|}$

- 13. Then $\hat{f} = \frac{\hat{m}+\hat{\delta}_Q}{N} f_s$

  The Gantt Chart of implementation in thesis A is available in Appendix B.

4.2 Implementation Plan for Thesis B

- 1. Research and review the literature about unbalanced three-phase system ,such as [14], [15], [16]

- 2. Demonstrate the Three-phase voltage unbalanced system to Clarke's transform and have

$$v(k) = Ae^{j\omega k} + Be^{-j\omega k} \tag{4.1}$$

Where,

$$A = \frac{\sqrt{6}(V_a + V_b + V_c)}{6} e^{j\phi} B = \frac{\sqrt{6}(2V_a + V_b + V_c)}{6} e^{j\phi} \tag{4.2}$$

- 3. Implement balancing voltage transform and DFT estimator, by calculating the degree of system imbalance $a = \frac{B}{A^*}$ which is given by

$$a = \frac{r(m) + r^*(m) \pm \sqrt{(r(m) + r^*(m))^2 - 4|p(m)|^2}}{2p^*(m)} \tag{4.3}$$

- 4. Test the improvements with BVT and random the voltage to ensure the method reliability.

- 5. Extended the research to phase unbalanced condition by setting up the three-phase model with unknow phase angle.

- 6. Derive the Clarke's transform output exponential expression

$$v = (A_1 + jA_2)e^{j(\omega t_n)} + (B_1 + jB_2)e^{-j(\omega t_n)} \tag{4.4}$$

in phase unbalanced condition.

- 7. Demonstrate the compatibility of new modeling with BVT, implement and test various methods(Matrix Pencil and A&M) which use new modeling, thus compare their performance.

Chapter 5

Simulation Results

5.1 Verification of Clarke's Transform

This chapter investigates the project being put under test in the MATLAB environment, with each section also containing the assessment of the stated results.

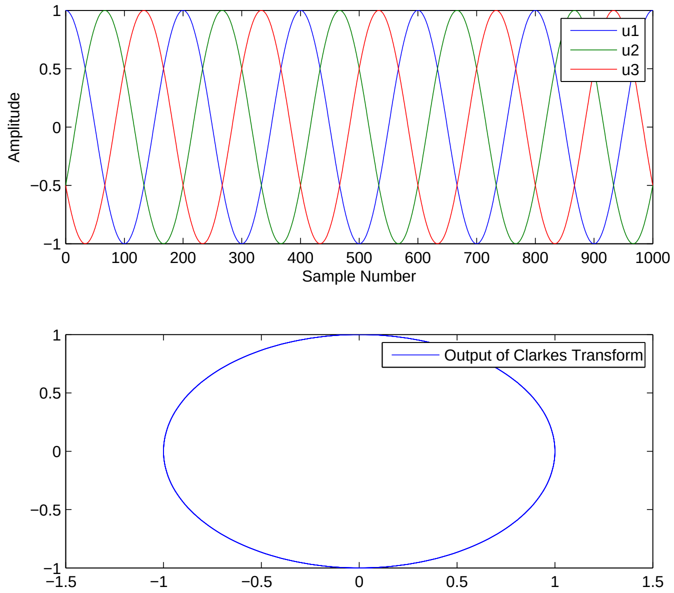

FIGURE 5.1: Plot of the complex exponential s formed using the alpha and beta components of output of the Clarke's Transformation

5.2 Derive of Clarke's Transform for Phase Unbalanced System

Extent the voltage transofrmation method with phase unbalanced, model given below,

$$\begin{aligned} v_a(n) &= V_a cos(\omega t_n + \phi_1) + \zeta_a \\ v_b(n) &= V_b cos(\omega t_n + \phi_2) + \zeta_b \\ v_c(n) &= V_c cos(\omega t_n + \phi_3) + \zeta_c \end{aligned} \tag{5.1}$$

From the standard three phase system,

$$v = v_\alpha + jv_\beta = Ae^{j(\omega k+\phi)} \tag{5.2}$$

The v_α and v_β components can be derived as

$$\begin{aligned} v_\alpha &= \sqrt{\frac{2}{3}}(v_a - \frac{v_b}{2} - \frac{v_c}{2}) \\ &= \sqrt{\frac{2}{3}}(V_a cos(\omega t_n + \phi_1) - \frac{1}{2}V_b cos(\omega t_n + \phi_2) - \frac{1}{2}V_c cos(\omega t_n + \phi_3)) \\ &= \sqrt{\frac{2}{3}}(V_a(cos(\omega t_n)cos\phi_1 - sin(\omega t_n)sin\phi_1) \\ &- \frac{V_b}{2}(cos(\omega t_n)cos\phi_2 - sin(\omega t_n)sin\phi_2) \\ &- \frac{V_c}{2}(cos(\omega t_n)cos\phi_3 - sin(\omega t_n)sin\phi_3)) \\ &= \sqrt{\frac{2}{3}}((V_a cos\phi_1 - \frac{V_b cos\phi_2}{2} - \frac{V_c cos\phi_3}{2})cos(\omega t_n) \\ &- (V_a sin\phi_1 - \frac{V_b sin\phi_2}{2} - \frac{V_c sin\phi_3}{2})sin(\omega t_n)) \end{aligned} \tag{5.3}$$

$$\begin{aligned} v_\beta &= \sqrt{\frac{2}{3}}(\frac{\sqrt{3}v_b}{2} - \frac{\sqrt{3}v_c}{2}) \\ &= \sqrt{\frac{2}{2}}(v_b - v_c) \\ &= \sqrt{\frac{2}{2}}(V_b(cos(\omega t_n)cos\phi_2 - sin(\omega t_n)sin\phi_2) \\ &- V_c(cos(\omega t_n)cos\phi_3 - sin(\omega t_n)sin\phi_3)) \end{aligned} \tag{5.4}$$

We have,

$$\begin{aligned} cos(\omega t_n) &= \frac{e^{j(\omega t_n)} + e^{-j(\omega t_n)}}{2} \\ sin(\omega t_n) &= \frac{e^{j(\omega t_n)} - e^{-j(\omega t_n)}}{2j} \end{aligned} \tag{5.5}$$

Then,

$$\begin{aligned} v &= v_\alpha + jv_\beta \\ &= \sqrt{\frac{2}{3}}(V_a cos\phi_1 - \frac{V_b}{2}cos\phi_2 - \frac{V_c}{2}cos\phi_3)\frac{e^{j(\omega t_n)} + e^{-j(\omega t_n)}}{2} \\ &- \sqrt{\frac{2}{3}}(V_a sin\phi_1 - \frac{V_b}{2}sin\phi_2 - \frac{V_c}{2}sin\phi_3)\frac{e^{j(\omega t_n)} - e^{-j(\omega t_n)}}{2j} \\ &+ j(\sqrt{\frac{2}{2}}V_b(cos\phi_2 - \sqrt{\frac{2}{2}}cos\phi_2)\frac{e^{j(\omega t_n)} + e^{-j(\omega t_n)}}{2} \\ &- j(\sqrt{\frac{2}{2}}V_b(sin\phi_2 - \sqrt{\frac{2}{2}}sin\phi_2)\frac{e^{j(\omega t_n)} - e^{-j(\omega t_n)}}{2j} \end{aligned} \tag{5.6}$$

We found that $v_\alpha + jv_\beta$, could be summarize as, and the model we derived can use balancing voltage transformation method to do frequency estimation in phase unbalanced situation, because both the method need to cancel out the effect of negative sequence ($Be^{j\omega k}$) in order to avoid the bias frequency estimation.

$$v = (A_1 + jA_2)e^{j(\omega t_n)} + (B_1 + jB_2)e^{-j(\omega t_n)} \tag{5.7}$$

5.3 Performance of Estimator as a Function of SNR

BVT Simulation

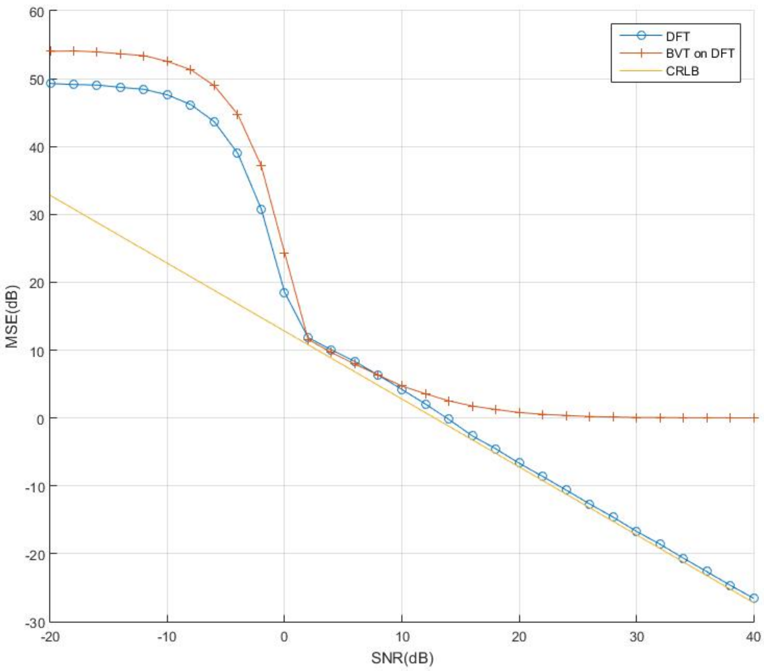

Figure 5.2: Plot of the exponential signal frequency estimation error for Non-BVT and BVT improved, MSE verses SNR with CRLB with N = 20

The graph on the above shows the frequency estimation performance of using balancing voltage transform method. The simulation results with BVT performs better than original frequency estimator that without BVT. The performance can almost achieve the CRLB.

Various Approaches Comparison under Phase Unbalanced Condition

The graph compare performance of three frequency estimation methods using the proposed modeling. It compares the estimation errors against signal to noise ratio. As N increase, all three methods have extremely low errors and close to CRLB. However, BVT with iterative interpolation method has better performance when N is low.

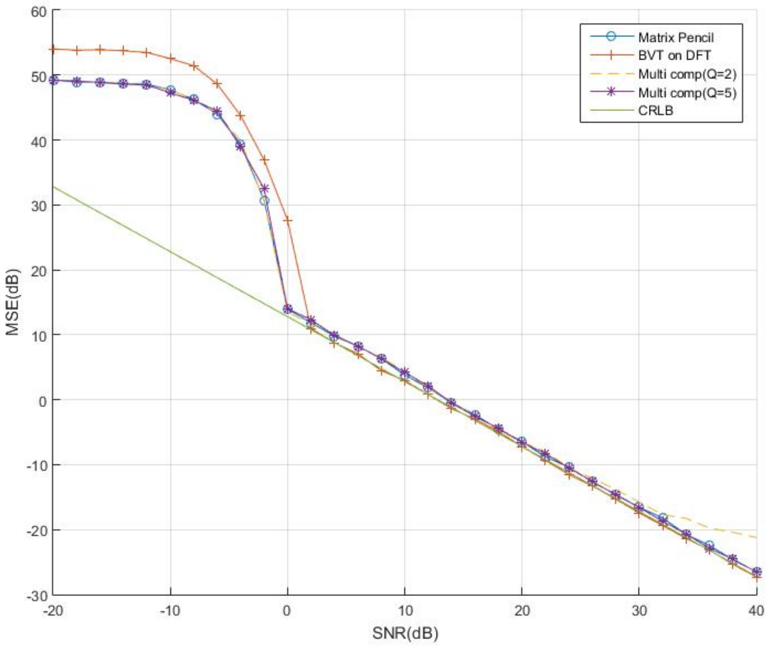

FIGURE 5.3: Plot of the exponential signal frequency estimation error for DFT(Q=1) that with BVT , Matrix pencil, A&M verses SNR with CRLB with N = 20 (correct sequence is canceled)

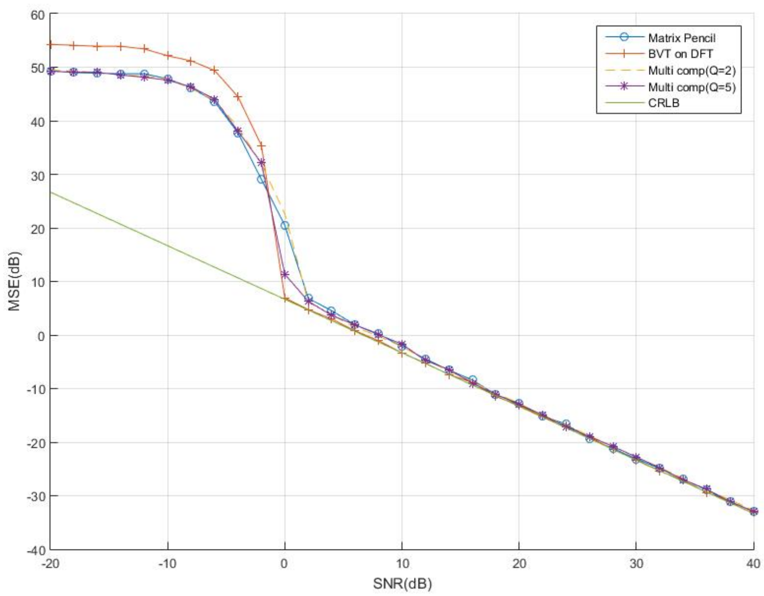

FIGURE 5.4: Plot of the exponential signal frequency estimation error for DFT(Q=1) that with BVT , Matrix pencil, A&M verses SNR with CRLB with N = 32 (correct sequence is canceled)

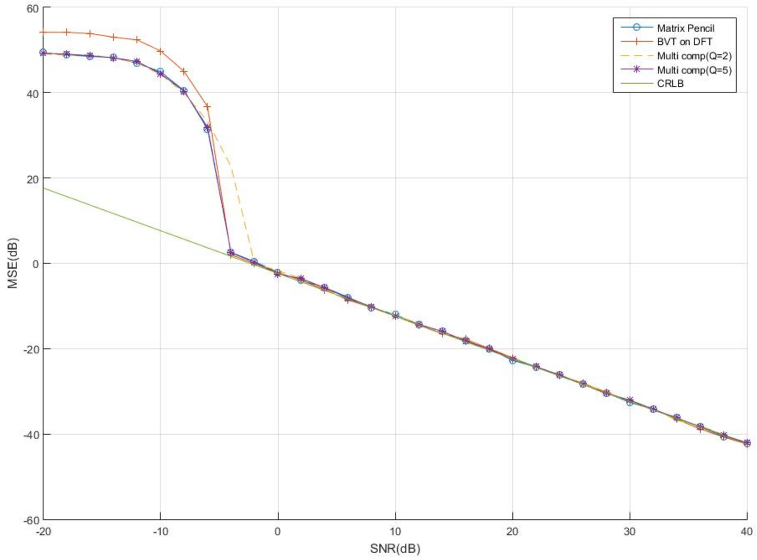

FIGURE 5.5: Plot of the exponential signal frequency estimation error for DFT(Q=1) that with BVT , Matrix pencil, A&M verses SNR with CRLB with N = 64 (correct sequence is canceled)

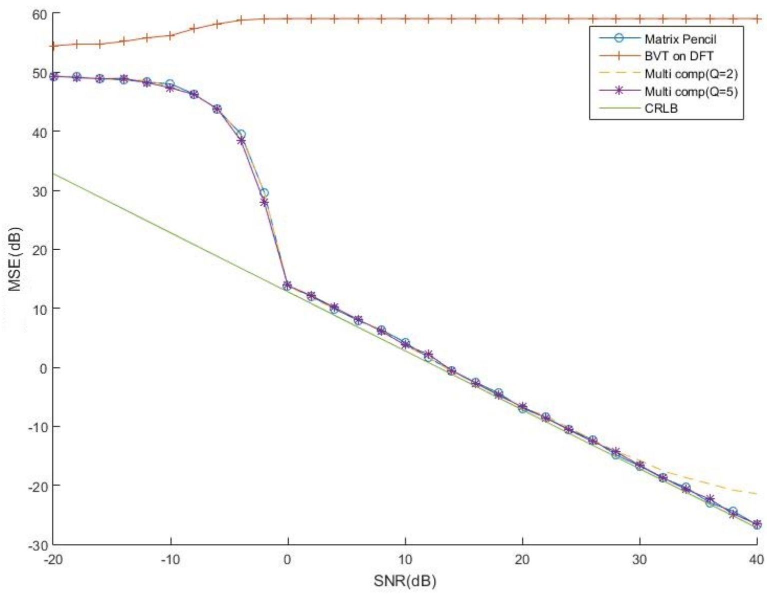

FIGURE 5.6: Plot of the exponential signal frequency estimation error for DFT(Q=1) that with BVT , Matrix pencil, A&M verses SNR with CRLB with N = 20 (incorrect sequence is canceled)

When the wrong sequence exponential is canceled, the frequency estimation cannot approach to the CRLB. Compare with Matrix pencil, A&M estimation, it required the parameters information for the purposed of comparing exponential coefficients A and B. Thus give a proper solution of using correct system imbalanced ratio to cancel the sequence that effect estimation accuracy. In contrast, Matrix pencil, A&M does not require the detail parameter information. However, it needs the numbers of components in exponential signal as a input.

Chapter 6

Conclusion

In conclusion, the motivation of researching the frequency estimation for three-phase power system has been detailed explained. To investigate and research frequency estimation methods for power system in both balanced and unbalanced system which reach the requirements accurately and efficiently is the purpose of doing this project.

The research on various frequency estimation methods for both single-phase and three-phase power systems has been extensively developed in literature, and many of methods and techniques has been reviewed in this report in order to fully understand the basic background knowledge of frequency estimation and take advantage of those methods in future research. In addition, frequency estimation methods that reviewed in this report design to have lower computational load and more accuracy results. However, most of those methods are based on the frequency estimation under three-phase balanced condition, even though the Clarke's transform have already significantly reduce the difficulty of analysing frequency of three-phase system, the reliability of using those algorithms that mentioned previously in unbalanced three-phase system still need to improve.

As a result, the proposed modeling with the benefit from balancing voltage transform for phase three-phase unbalanced system is introduced in the report. It shows that two category unbalanced three-phase system conditions are similar when it comes to the Clarke's transform output expression. To a very great extent provide a direction of phase unbalanced three-phase system research and successfully improve the frequency estimation accuracy under various estimators. Without increasing much of computational load, the design meets the project aims and requirements. It provides unbiased and efficient performance of frequency estimators in the voltage and phase unbalanced three-phase system for power system.

6.1 Future Work

- First of all, the proposed modelling and balancing voltage transform for phase unbalanced three-phase system is not perfect. The frequency estimation and sequence cancelation can be achieve only when phase angle is provided. But the output of Clarke's transform modelling under phase unbalanced gives a direction for frequency estimation of unbalanced three-phase research.

- In the report, all the research and simulation are based on only voltage amplitude unbalanced or phase unbalanced. Practically, two kind of unbalanced conditions could happen at the same time. Hence, the proposed modeling and balancing voltage transform cannot be guarantee to have robustness in the special circumstances.

- In the power systems or smart grids, the tracking ability of frequency estimation is one of the important metrics. Although, most of estimators that mentioned in this report have been prove to be highly efficient in the balanced condition for frequency estimation. However, we still need to test those estimators in the real environment for unbalanced condition to ensure the robustness.

Appendix A

Clarke's Transformation in Balanced and Unbalanced System

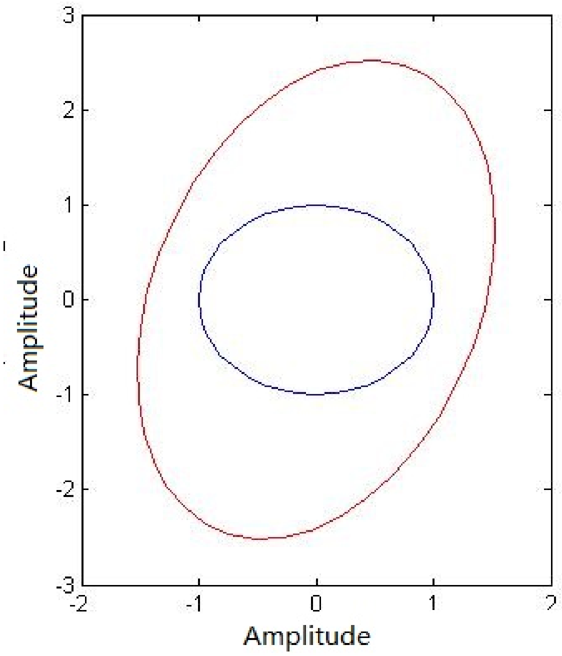

FIGURE A.1: Plot of the output of the Clarke's Transform in balanced and unbalanced condition

Appendix B

Gantt Chart

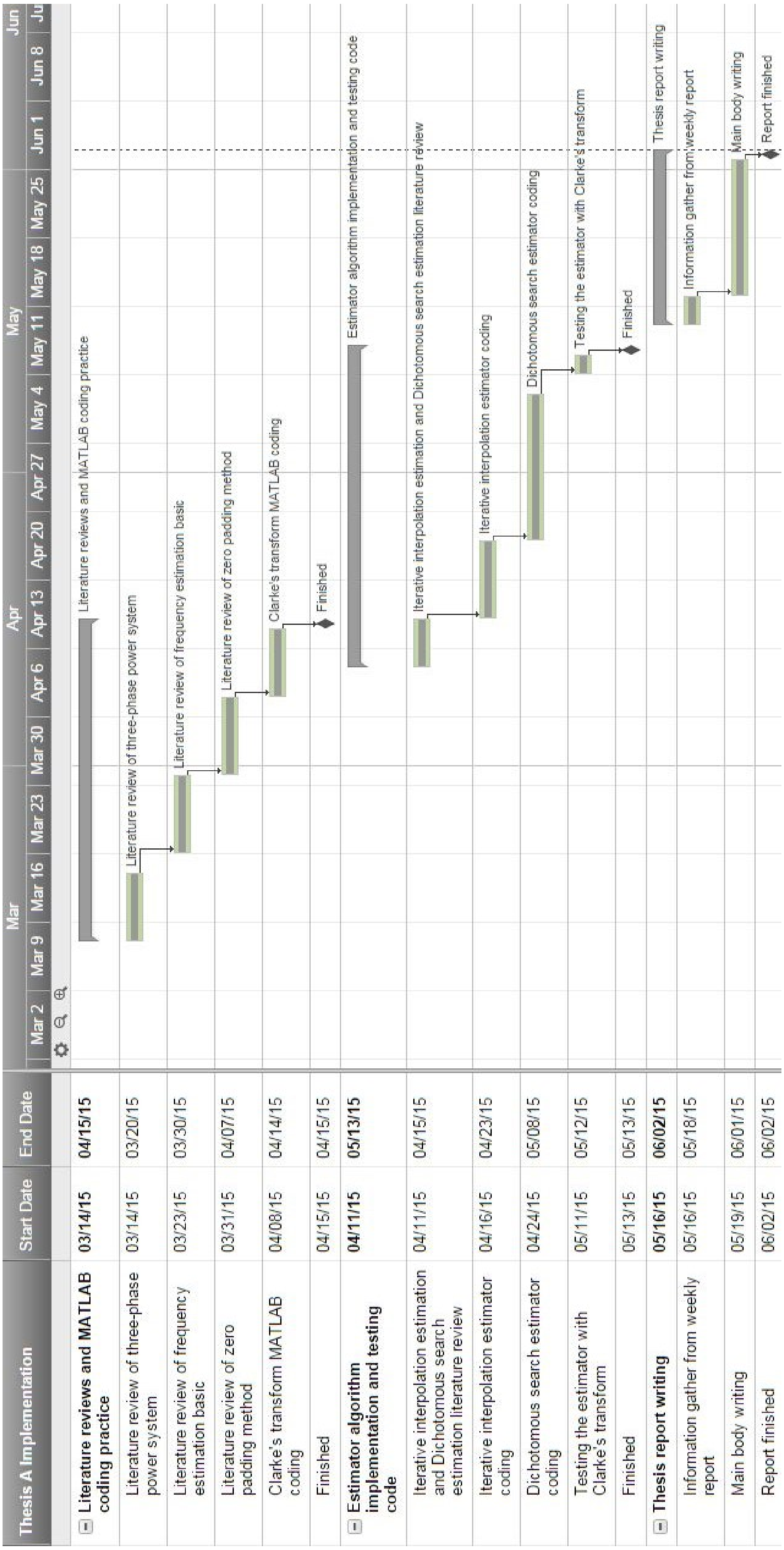

FIGURE B.1: Gantt Chart for Thesis A

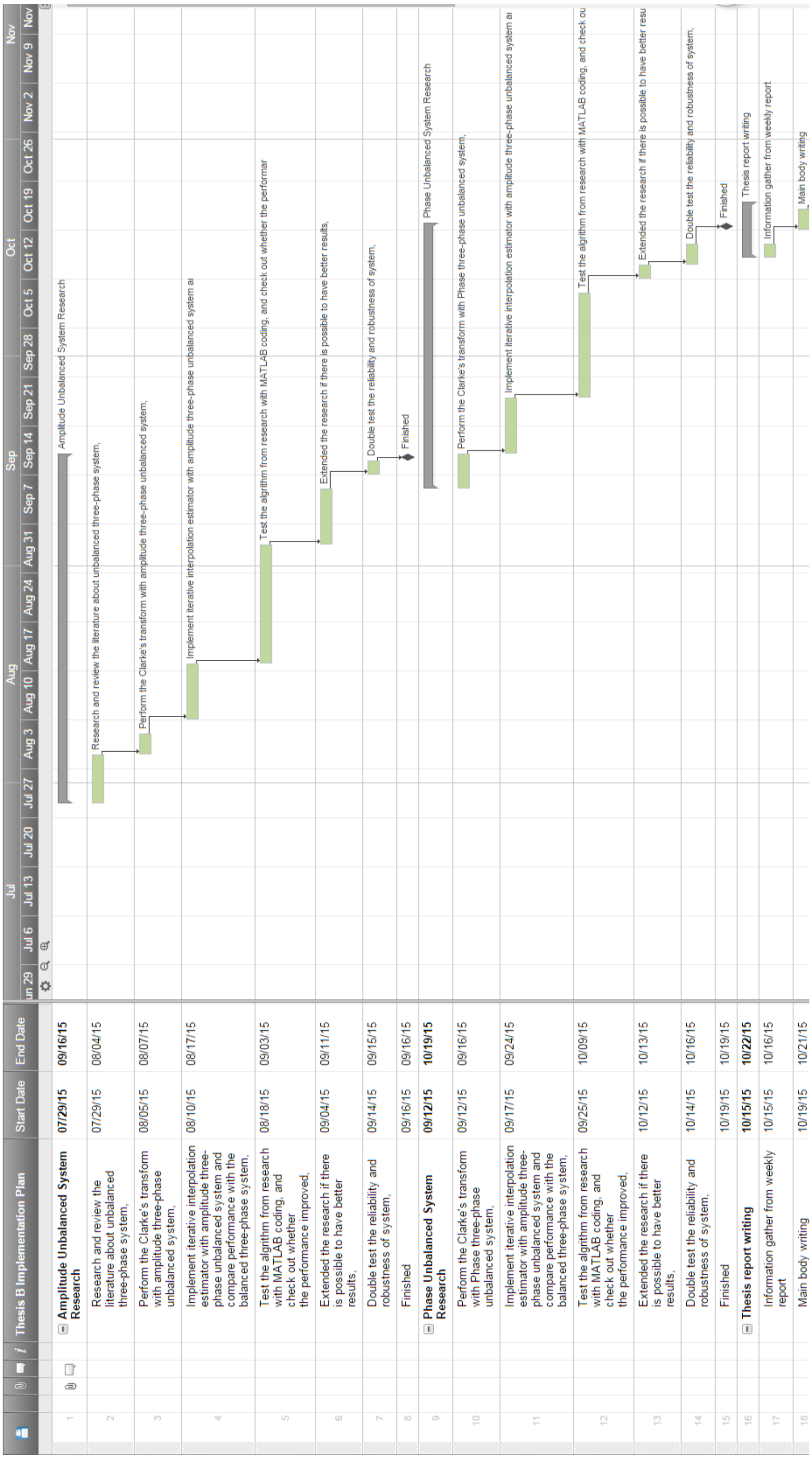

Figure B.2: Gantt Chart for Thesis B

Bibliography

[1] L. Li; W. Xia; D. Shi; J. Li. Frequency estimation on power system using recursive-least-squares approach. *Proceedings of the 2012 International Conference on Information Technology and Software Engineering Lecture Notes in Electrical Engineering*, 211(1):11–18, 2013.

[2] Jin-Lung Guan; Ming-Ta Yang; Jhy-Cherng Gu; Hsin-Hung Chang; and Chin-Lung Huang. The effect of harmonic power fluctuation for estimating flicker. *International Journal of Electrical, Computer, Electronics and Communication Engineering*, 1(4):632–637, April 2007.

[3] D.Rife and R.Boorstyn. Single tone parameter estimation from discrete-time observations. *IEEE Trans.Inf.Theo.*, 20:591–598, September 1974.

[4] E. Aboutanios. Estimating the parameters of sinusoids and decaying sinusoids in noise. *IEEE Trans. Sig. Process*, 58:501–509, 2010.

[5] E. Aboutanios and B. Mulgrew. Iterative frequency estimation by interpolation on fourier coefficients. *IEEE Transactions on Signal Processing*, 53: 1237–1242, April 2005.

[6] T. C. Tozer Y. V. Zakharov. Frequency estimator with dichotomous search of periodogram peak. *Electron. Lett.*, 35:1608–1609, 1999.

[7] E. Aboutanios. Modified dichotomous search frequency estimator. *IEEE SIGNAL PROCESSING LETTERS*, 11(2):186–188, February 2004.

[8] Y. V. Zakharov; V. M. Baronkin; T. C. Tozer. Dft-based frequency estimators with narrow acquisition range. *Proc. Inst. Elect. Eng.—Commun*, 148:1–7, 2001.

[9] Mohsen Mojiri; Davood Yazdani; Alireza Bakhshai. Robust adaptive frequency estimation of three-phase power systems. *IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT*, 59(7), July 2010.

[10] E.Clarke. Frequency estimation by demodulation of two complex signals. *IEEE Trans.Power Delivery*, 12:157–163, 1997.

[11] Yili Xia; Kai Wang; Danilo P. Mandic; Wenjiang Pei. A balancing voltage transformation for robust frequency estimation in unbalanced power systems. *Asia-Pacific Signal and Information Processing Association, 2014 Annual Summit and Conference (APSIPA)*, 1-6, 2014.

[12] Yung-Ya Lin; Paul Hodgkinson; Matthias Ernst; and Alexander Pines. A novel detection–estimation scheme for noisy nmr signals: Applications to delayed acquisition data. *JOURNAL OF MAGNETIC RESONANCE*, 128, June 1997.

[13] Shanglin Ye and Elias Aboutanios. An algorithm for the parameter estimation of multiple superimposed exponentials in noise. *IEEE*, (8), 2015.

[14] H.-SSong and K.Nam. Instantaneous phase-angle estimation algorithm under unbalanced voltag-sag conditions. *IEEE Proc.-Gener. Transm. Distrib*, 147 (6):186–188, November 2000.

[15] Dahir H. Dini and Danilo P. Mandic. Widely linear modeling for frequency estimation in unbalanced three-phase power systems. *IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT*, 62(2), November 2013.

[16] Reza Arablouei; Stefan Werner; and Kutluyil Dogancay. Adaptive frequency estimation of three-phase power systems with noisy measurements. *IEEE ICASSP*, 2013.