**Title:** Frequency Estimation for 3 Phase Power Systems **Topic number:** DZ29

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## A. Problem statement

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| Due to the safety issues, the stability of a 3-phase power system is very important which could be determined by estimation the frequency of the system. For the unbalanced 3-phase system, there are still some limitations to estimate the frequency under different cases in real life. The challenge is to improve the estimation algorithm for the unbalanced system under some conditions, such as, amplitude and phase unbalanced, harmonic distortion and so on. The improvement of the algorithm will benefit the estimation result to have a more accurate frequency estimation result under more complex conditions in the real world. |

## B. Objective

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| a) Implementing the existing algorithm for both balanced and amplitude unbalanced 3-phase power systems with noise distortion and comparing the estimation results. |
| b) Based on the existing algorithm, finding the better solution for phase and amplitude unbalanced systems and the unbalanced systems with harmonic distortion. |
| c) Achieving the better performance for frequency estimation under more complex unbalanced conditions. |

## C. My solution

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| a) Converting the 3-phase signal into a complex signal by using the Clarke's transform. |
| b) Using 4 different algorithms (both balanced and unbalanced design of Augmented Complex Least Mean Square, Widely Linear Complex Least Mean Square and Coupled Orthogonal Constant Modulus Algorithm) that based on the least square technique to estimate the frequency of the system. |
| c) Deriving and improving the algorithm for phase unbalanced systems and unbalanced systems with harmonic distortion based on the existing algorithms. |
| d) Implementing the improved algorithm based on the previous algorithms. |

## D. Contributions (at most one per line, most important first)

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|---|
| a) Deriving the algorithm expressions for both phase and amplitude unbalanced systems with noise distortion. |
| b) Deriving the algorithm expressions for amplitude unbalanced systems with noise and harmonic distortion. |
| c) Deriving the Clarke's transform expressions for both phase and amplitude unbalanced systems with noise distortion. |
| d) Deriving the Clarke's transform expressions for amplitude unbalanced systems with noise and harmonic distortion. |
| e) Demonstrating the comparison between the previous 4 algorithms (section C "My solution" part b) under different frequency estimating/tracking cases. |

## E. Suggestions for future work

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| a) To reduce the noise for the Coupled Orthogonal Constant Modulus Algorithm. |
| b) To improve the tracking ability for the linear frequency tracking. |

While I may have benefited from discussion with other people, I certify that this report is entirely my own work, except where appropriately documented acknowledgements are included.

Signature: ______________________ Date: 18/10/2016

**School of Electrical Engineering and Telecommunications Engineering**

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# Frequency Estimation For 3 Phase Power Systems

by

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Thesis submitted as a requirement for the degree of

Bachelor of Engineering in Electrical Engineering

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# Abstract

In the present-day society, the 3-phase power system is one of the most common used and important power systems for society to keep a home, a industry or even a country running and operating at its normal conditions. As the stability of the 3-phase power system will be one of the most important elements or conditions that will make sure the system operates normally. Therefore, many power companies will be interested in the determination of the stability for a 3-phase power system. In other words, frequency estimation will be extremely important, as a stable power system requires to have its frequency maintained at a certain value.

Therefore, in this thesis report, we will discuss about the latest methods and algorithms that could achieve the above purpose which is to estimate the frequency for a 3-phase power system. The latest and most efficiency algorithms for the frequency estimation are based on the least square technique and Kalman filter method. For the balanced system case, the least square algorithm has a higher accuracy than the Kalman filter, so it's decided to work on the improvement of the algorithm based on the least square technique for both amplitude and phase unbalanced case even with harmonic distortion.

# Abbreviation

ECKF: Extended complex Kalman filter

LMS: Least mean square algorithm

WLS: Weighted least square algorithm

RAFE: Robust adaptive frequency estimation

ANF: Adaptive notch filter

ACLMS: Augmented Complex Least Mean Square

WLCLMS: Widely Linear Complex Least Mean Square

CO-CMA: Coupled Orthogonal Constant Modulus Algorithm

# Contents

# 1 Introduction

With the development of the modern society, the demand for electricity to maintain a whole society running under the normal conditions is growing faster than ever before. Not only the industry, home, government facilities, shops, but also almost everything in today's society, they all require power source like electricity to supply them for the daily operation and all the electricity requires a power system to generate, transfer and distribute electricity so that it could be used by people or the equipments and loads at the distribution terminal end. Therefore, the power system is one of the most important things for human beings in today's society, especially the 3-phase power system. A 3-phase power system is usually used to transfer AC over a large distance and to supply the heavy loads, and because of this, it's much more effective and economical than single phase.

However, if the 3-phase power system works under an unstable conditions for either steady state, transient or dynamic, a series of serious issues will occur. For example, during the transmission, the high order harmonic current will lead to the damage of the transformer and some other equipments like capacitors. Also, if the output of the distribution terminal is not stable, it will cause the damage of the loads and some other safety issues for both equipments and people who are using them. In other words, to determine whether a 3-phase power system is stable or not is extremely important when using the 3-phase power system. Therefore, many power companies will be interested in these kinds of topics.

From the research, one way to figure out whether a 3-phase power system is stable or not is to estimate the frequency of this system as a stable and high quality system is required to have its frequency maintained at a certain value. Also, the parameters of the power system, such as, amplitude, phase and so on are important as well, as they could insure the safety of the interconnection of 2 systems. Therefore, these drive the motivation to research for this thesis topic.

By knowing all the informations mentioned above, some researching works have been done.

It is shown that there are some latest existing algorithms that could be used to estimate the frequency for both the balanced and the unbalanced 3-phase power system.

**Problem Statement:** For the unbalanced system, there are still some limitations exist which will influence the performance of the results. Therefore, these limitations are needed to be found and solved so that the stability of the power system could be defined by a much more accurate result and to insure the proper operating conditions and some other safety issues for people who are working within the area. The main challenge is to improve the frequency estimation technique of existing algorithm in order to have a better performance for some complex unbalanced system with harmonic distortion. In the end, in order to make sure the result after the improvement is correct, the process is required to be tested by the computer simulation test through some special softwares, like Matlab.

The following chapters will start from the discussion of the latest existing algorithms which will estimate the frequency for the 3-phase power system by using different kinds of algorithms. Then, in the "Methodology" chapter, the problem of this thesis research will be defined and both of the theory consideration and solution method will be discussed. After that, the implementation plan and the results will be mentioned in the following chapters. In the end, the conclusion will summarize all the important ideas and the future work of this thesis. Furthermore, for this thesis, the work will meanly focus on the simulation by using the software in the laptop, instead of experimenting in the lab. Therefore, no risk assessment will be needed.

# 2 Background

As the estimated frequency for a 3 phase power system is used to determine whether the system is stable or not, the estimation of the frequency is extremely important alone with other parameters if necessary, such as, amplitude, phase and so on. In this chapter, we'll first introduce the basic model of the 3-phase power system and the structure of a standard algorithm.

## 2.1 Basic introduction for 3-phase power system

For a balanced 3-phase power system, each phase will have the same voltage peak amplitude of 230V(RMS) with 50Hz in Australia. However, in reality, due to the faults or loads, more unbalanced 3-phase power system will be dealt with which means that for each phase, they won't have to have the same amplitude and more noise error or phase difference error will occur. Therefore, in general, the voltage of a 3-phase power system can be represented as:

$$\begin{cases} v_{a,i} = V_a cos(2\pi f \Delta T + \phi_{a,i}) + \xi_{a,i} \\ v_{b,i} = V_b cos(2\pi f \Delta T + \phi_{b,i} + \frac{2\pi}{3}) + \xi_{b,i} \\ v_{c,i} = V_c cos(2\pi f \Delta T + \phi_{c,i} - \frac{2\pi}{3}) + \xi_{c,i} \end{cases} \tag{1}$$

where $v_{a,i}, v_{b,i}, v_{c,i}$ are the 3-phase signals; $V_a, V_b, V_c$ are the amplitudes of signals; $\phi_{a,i}, \phi_{b,i}, \phi_{c,i}$ are the phases of signals; $\xi_{a,i}, \xi_{b,i}, \xi_{c,i}$ are the noises; f is the system frequency; $\Delta T$ is the sampling time interval.

**Balanced system:** The amplitudes and phases for the balanced 3-phase power system will be the same, in other words, $V_a = V_b = V_c$ and $\phi_{a,i} = \phi_{b,i} = \phi_{c,i}$.

**Unbalanced system:** For an unbalanced 3-phase power system, we could have amplitude unbalanced, phase unbalanced or amplitude and phase unbalanced system. In other words, amplitudes and phases will not necessarily equal to each other.

**Distorted system:** A distorted system will contain noise distortion or harmonic distortion and the distorted situation could happen to both balanced and unbalanced 3-phase power system.

## 2.2 Basic introduction for algorithm structure

From the research, generally, there are 2 ways to estimate the frequency for both balanced and unbalanced 3-phase power system. First method is to estimate the frequency for each phase separately and then combine them in the end. Second method is to firstly combine all 3 phases into a single signal and then estimate its frequency.

It's decided to work on the second method, because it will give a higher accuracy than the first method as it contain more information, although it's more complicated. For a completed algorithm using the second method, it should contain 3 main parts: Transformation, Mosel and Estimator.

**Transformation:** The transformation part is used to convert the 3-phase signals into a combined signal. For example, by using the Clarke's transform, the 3-phase signal could be transfer into one complex signal $V_k$ as showing below:

$$\begin{aligned} v_{\alpha k} &= \sqrt{\frac{2}{3}}(V_a - 0.5V_b - 0.5V_c) \\ &= \frac{\sqrt{6}V_m}{2} cos(2\pi f k \Delta T + \phi) \end{aligned} \tag{2}$$

$$\begin{aligned} v_{\beta k} &= \sqrt{\frac{2}{3}}(\frac{\sqrt{3}}{2}V_b - \frac{\sqrt{3}}{2}V_c) \\ &= \frac{\sqrt{6}V_m}{2} sin(2\pi f k \Delta T + \phi) \end{aligned} \tag{3}$$

Therefore,

$$\begin{aligned} V_k &= v_\alpha + j v_\beta \\ &= \frac{\sqrt{6}V_m}{2} e^{j(2\pi f k \Delta T + \phi)} + \xi(k) \end{aligned} \tag{4}$$

**Model:** After transformation, the combined signal will then need to be converted into a certain model or form (linear model) so that the frequency could be estimated by the estimator.

**Estimator:** This last part is the place where the frequency will be estimated from the signal.

# 3 Literature review

After understanding the basic knowledge of the 3-phase power system and algorithm structure, the research on different techniques must be done in order to estimate the frequency. From research, the Kalman filter and the least square technique are the 2 most popular methods for frequency estimation of the 3-phase power system. The following sections will summarize and discuss the existing techniques.

## 3.1 Kalman filter

### 3.1.1 Extended complex Kalman filter

First, the algorithm based on Kalman filter is introduced. By using the Clarke's transform, the $\alpha$-$\beta$ components can be found from the 3-phase voltage:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{bmatrix} \tag{5}$$

From this, we can have a complex voltage V(k):

$$\begin{aligned} V(k) &= v_\alpha(k) + jv_\beta(k) \\ &= Ae^{j(2\pi f k\Delta T+\phi)} + \xi(k) \end{aligned} \tag{6}$$

where A is a constant and $\eta(k)$ is the noise.

The complex signal V(k) can be rewritten as following:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & x_1(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{7}$$

$$y(k) = V(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \xi(k) \tag{8}$$

where

$$\begin{cases} x_1(k) = e^{j2\pi f\Delta T} = cos(2\pi f k\Delta T) + jsin(2\pi f k\Delta T) \\ x_2(k) = Ae^{j(2\pi f k\Delta T+\phi)} \end{cases} \tag{9}$$

By using the Kalman filter to the system in (7), the Kalman gain can be obtain as following:

$$K(k) = \hat{P}(k/k-1)H^T(H\hat{P}(k/k-1)H^T + Q)^{-1} \tag{10}$$

where H is the observation vector, Q is the noise covariance of the signal and P is the covariance matrix which is shown below:

$$\hat{P}(k/k) = \hat{P}(k/k-1) - K(k)H\hat{P}(k/k-1) \tag{11}$$

Therefore the new estimated state is given by the following relationship:

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)(V(k) - H\hat{x}(k/k-1)) \tag{12}$$

Finaly, the frequency can be calculated from the $x_1(k)$ of the equation (6)

$$\hat{f} = \frac{1}{2\pi\Delta T}(sin^{-1}Im(\hat{x}_1)) \tag{13}$$

## 3.2 Least square technique

The previous method is using the Kalman filter to obtain the frequency estimation and the following 5 methods are all based on the least square technique. Furthermore, most of them are using the process that first to combine the 3-phase power system into a complex signal and estimate the frequency, except the 3rd method (3.2.3). The process of third method is to estimate the frequency for each of the 3-phase firstly, and then combine the 3 frequencies to get the final frequency. The advantage of doing this is that it has much less complexity than to combine the 3-phase power signal, especially when the signal carries the harmonic distortion. However, the process that combining the signal will increase the signal-noise ratio (SNR) which means that it's more accurate.

### 3.2.1 Least mean square algorithm

After using the same transformation method, the Clarke's transform, as the Kalman filter technique, the complex voltage $V_k$ can be presented as the following equation:

$$\begin{aligned} V_k &= v_{\alpha k} + jv_{\beta k} \\ &= Ae^{j(2\pi fk\Delta T)} + \xi_k \end{aligned} \tag{14}$$

$$\hat{V}_k = V_{k-1} e^{j2\pi f k \Delta T} \tag{15}$$

The error signal will be presented as:

$$e_k = V_k - \hat{V}_k \tag{16}$$

$$\hat{V}_k = W_k \hat{V}_{k-1} \tag{17}$$

where $W_k = e^{j\hat{w}_k \Delta T}$ is the weight of the voltage signal, $\hat{w}$ is the estimated angular frequency. By using the equation below which aim to alter $W_k$, the error can be minimized:

$$W_k = W_{k-1} + \mu_k e_k \hat{V}_k^* \tag{18}$$

where the $\mu$ is the convergence factor which could control the stability and $\mu_k$ can be varied to reach the better convergence:

$$\mu_{k+1} = \lambda \mu_k + \gamma R_k R_k^* \tag{19}$$

where $R_k$ is the autocorrelation of $e_k$ and $e_{k-1}$ as following:

$$R_k = \rho R_{k-1} + (1-\rho) e_k e_{k-1} \tag{20}$$

where $\rho$ is a weighting parameter and $0 < \rho < 1$, and $0 < \lambda < 1$ and $\gamma > 0$ which control the time of convergence. The frequency is calculated as:

$$f_k = \frac{1}{2\pi \Delta T} sin^{-1}[Im(W_k)] \tag{21}$$

This frequency estimation model is using the least mean square technique which is one of the most basic methods that is able to estimate the frequency from the 3-phase power system.

### 3.2.2 Extended least square algorithm

To extend the previous model, this extended least square technique introduce a new parameter, $\phi(k)$. In this case, the signal with noise will have the following form:

$$z(k) = \begin{bmatrix} sin\omega_0 k & cos\omega_0 k \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \xi(k) \tag{22}$$

where z(k) is the noisy measurement; $\xi(k)$ is the noise term; $\phi(k)$ is the system structure matrix; $\alpha = A_1 cos\phi_1$ and $\beta = A_1 sin\phi_1$ The frequency can be calculated from $z(k)$:

$$f_0 = \frac{1}{2\pi k}[sin^{-1}(\frac{z(k) - \xi(k)}{A_1} - \phi_1)] \tag{23}$$

#### 3.2.3 Weighted-Least-Square Algorithm and Adaptive FIR Filter

In addition to the previous 2 techniques, the weighted factor and adaptive FIR filter are considered in this method. For this method, the 3-phase signals are firstly required to pass through the FIR filter. Then, the following equation contains three consecutive signals of the output of the FIR filter:

$$\frac{v_{a,i} + v_{a,i-2}}{2} = v_{a,i-1} cos(\omega_1 \Delta t) \tag{24}$$

or

$$y_{a,i} = z_{a,i} x \tag{25}$$

where

$$y_{a,i} = \frac{v_{a,i} + v_{a,i-2}}{2}$$

$$za, i = v_{a,i-1}$$

$$x = cos(\omega_1 \Delta t)$$

For phase b and c, they are similar to this:

$$y_{b,i} = z_{a,i} x \tag{26}$$

$$y_{c,i} = z_{c,i} x \tag{27}$$

Therefore,

$$y_{k,i} = z_{k,i} x \tag{28}$$

where

$$y_k = \begin{bmatrix} W^{k-1}y_{a,1} \\ W^{k-1}y_{b,1} \\ W^{k-1}y_{c,1} \\ W^{k-2}y_{a,2} \\ W^{k-2}y_{b,2} \\ W^{k-2}y_{c,2} \\ . \\ . \\ . \\ y_{a,k} \\ y_{b,k} \\ y_{c,k} \end{bmatrix} \quad z_k = \begin{bmatrix} W^{k-1}z_{a,1} \\ W^{k-1}z_{b,1} \\ W^{k-1}z_{c,1} \\ W^{k-2}z_{a,2} \\ W^{k-2}z_{b,2} \\ W^{k-2}z_{c,2} \\ . \\ . \\ . \\ z_{a,k} \\ z_{b,k} \\ z_{c,k} \end{bmatrix}$$

Therefore, $x_k$ can be calculated using the following formula:

$$x_k = \frac{\sum_{i=0}^{k} W^{2(k-i)}[z_{a,i}y_{a,i} + z_{b,i}y_{b,i} + z_{c,i}y_{c,i}]}{\sum_{i=0}^{k} W^{2(k-i)}[z_{a,i}^2 + z_{b,i}^2 + z_{c,i}^2]} \tag{29}$$

However, if we are looking at a balanced system, then $W = 1$. To implement the above formula, accumulators $AccZY_k$ and $AccZZ_k$ are introduced as:

$$\begin{cases} AccZY_k = W^2 AccZY_{k-1} + z_{a,k}y_{a,k} + z_{b,k}y_{b,k} + z_{c,k}y_{c,k} \\ AccZZ_k = W^2 AccZZ_{k-1} + z_{a,k}^2 + z_{b,k}^2 + z_{c,k}^2 \end{cases} \tag{30}$$

$$x_k = \frac{AccZY_k}{AccZZ_k} \tag{31}$$

Once $x_k$ is obtained, the frequency is then calculated from:

$$f_k = \frac{arccos(x_k)}{2\pi\Delta t} \tag{32}$$

The detail processing diagram is shown in [4] which describe the whole estimation algorithm. Before the frequency calculation (equation 30 - 32), the 3-phase signals will first need to pass through the FIR filters. The transfer function below is used to eliminates the dc component:

$$H_{01}(z) = \frac{1 - z^{-2}}{|1 - z_1^{-2}|} \tag{33}$$

where

$$z = e^{j\omega\Delta t}$$

$$z_1 = e^{j\omega_1\Delta t}$$

$$|1 - z_1^{-2}| = 2sin(\omega_1\Delta t)$$

The equation below is used to reject harmonic $\omega_1$:

$$H_{1i}(z) = \frac{1 - 2cos(\omega_i\Delta t)z^-1 + z^{-2}}{|1 - 2cos(\omega_i\Delta t)z^-1 + z_1^{-2}|}, i = 2, 3, ..., M \tag{34}$$

where

$$|1 - 2cos(\omega_1\Delta t)z^-1 + z^{-2}| = 2|cos(\omega_1\Delta t) - cos(i\omega_1\Delta)|$$

Therefore, the overall filter equation is given below:

$$H_1(z) = H_{01}(z)\prod_{i=2}^{M} H_{1i}(z) \tag{35}$$

### 3.2.4 Robust Frequency Estimation using correntropy-based adaptive filter

According to the published paper, previous 3 techniques only consider the balanced 3-phase voltage input signal. However, this method takes unbalanced cases into account. In other words, this paper introduce the frequency estimation designs for both balanced and unbalanced cases.

For the balanced system, the three phase input will first pass through the Clarke's transform(2-3) to become a complex combined signal (4). Then, by using the weight vector ,$w_{CLMS}(k)$, to the prediction of the next sample ,$y_{CLMS}(k)$, as following:

$$y_{CLMS}(k + 1) = w_{CLMS}(k)v(k) \tag{36}$$

By taking the error between the actual signal sample and the prediction, the weight vector can be updated as the following equation:

$$e_{CLMS}(k) = v(k + 1) - y_{CLMS}(k + 1) \tag{37}$$

$$w_{CLMS}(k + 1) = w_{CLMS}(k) + \mu e_{CLMS}(k)v^*(k) \tag{38}$$

Finally, the frequency can be estimated by the equation below:

$$f_(k) = \frac{1}{2\pi T} sin^{-1}(Im(w_{CLMS}(k))) \tag{39}$$

For the unbalanced system, in order to deal with the amplitude and phase unbalance, 3 more elements are required, h(k), g(k) and s1(k). The detail information will be shown in [5]. The following equations show how to update those 3 elements.

$$h(k+1) = h(k) + \mu_h v^*(k)e(k)exp(-\frac{|e(k)|^2}{2\sigma^2}) \tag{40}$$

$$g(k+1) = g(k) + \mu_g v^*(k)e(k)exp(-\frac{|e(k)|^2}{2\sigma^2}) \tag{41}$$

$$s1(k) = \frac{-jIm(h(k)) + j\sqrt{Im^2(h(k)) - |g(k)|^2}}{g(k)} \tag{42}$$

Finally, the frequency can be estimated by the equation below:

$$f_(k) = \frac{1}{2\pi T} sin^{-1}(Im(h(k) + s1(k)g(k))) \tag{43}$$

### 3.2.5 Coupled Orthogonal Constant Modulus Algorithm

This CO-CMA method is developed based on the previous method (3.2.4). The improvement is that in this paper, after the 3-phase signal has passed through the Clarke's transform, Instead of using equation (4), the below equation is used:

$$r(k) = W_1^* v_{\alpha(k)} + jW_2^* v_{\beta(k)} = W^H V(k) \tag{44}$$

where $W = [W_1, W_2]$ is a one by two matrix contains the complex coefficients $W_1$ and $W_2$. Also, the matrix W will be undated by the following equations:

$$W = W + \mu_r e(k) r^*(k) R(k)^{-1} V(k) \tag{45}$$

$$R(k) = \lambda R(k-1) + V(k)V(k)^H r(k) r^*(k) \tag{46}$$

where the error signal e(k) and adaptive coefficient h(k) could be achieved as below:

$$e(k) = h(k) - r(k)r^*(k-1) \tag{47}$$

$$h(k+1) = h(k) - \mu e(k) \tag{48}$$

In the end, the frequency can be estimated by:

$$f_(k) = \frac{1}{2\pi T} sin^{-1}(Im(h(k))) \tag{49}$$

# 4 Methodology

After doing the research to enrich the knowledge about frequency estimation, this chapter will introduce the method that is used for this thesis.

## 4.1 Problem definition

For this thesis topic, "Frequency estimation for 3-phase power system", the main structure of the method is to use the Clarke's transform to estimate the frequency from 3-phase power system. However, when doing the frequency estimation, we need to include noise term, amplitude unbalanced signal, phase unbalanced signal and harmonic distortion for the 3-phase power system, as in the real world, all factors will influence the frequency estimation result.

## 4.2 Theory consideration

For the balanced 3-phase input voltage signal, the output of the Clarke's transform could be easily obtained. However, for the unbalanced case, the amplitude and phase of the signal will not necessarily equal to each other which will increase the complexity of the output of the Clarke's transform. In other words, the output will not only be a single complex exponential signal (equation (4)), it will have both positive and negative components. After that, this output will pass through the certain algorithm to estimate the frequency.

Furthermore, the harmonic distortion will be considered. First of all, there are 2 reasons that the harmonic will exist. The first reason is that it will occur when the signal interference with other equipments or systems. The second one is that it's added on purpose by people in order to decrease the weakening effect of the signal. For both of the above reasons, the frequency estimation results will be affected by the small error caused by the harmonic.

## 4.3 Solution method

### 4.3.1 Amplitude unbalanced systems

For the amplitude unbalanced case, the output of the Clarke's transform will have both positive and negative components as below:

$$\begin{aligned} V_k =& v_\alpha + jv_\beta \\ =& Ae^{jwk\Delta T} + Be^{-jwk\Delta T} \end{aligned} \tag{50}$$

where

$$\begin{aligned} A &= \frac{\sqrt{6}(V_a + V_b + V_c)}{6} \\ B &= \frac{\sqrt{6}(2V_a - V_b - V_c)}{12} - \frac{\sqrt{2}V_b - V_c}{4}j \end{aligned} \tag{51}$$

Then, the ACLMS algorithm is chosen to be used as a standard algorithm because it's the most basic and important algorithm based on the LMS technique. To simplify the problem, only $V_k$ is used to express $V_{k+1}$ as following:

$$V_{k+1} = h_k V_k + g_k V_k^* \tag{52}$$

By using the above 2 equations (50 and 51), the following equations could be derived:

$$e^{jw\Delta T} = h_k + \frac{B^*}{A} g_k \tag{53}$$

$$e^{-jw\Delta T} = h_k + \frac{A^*}{B} g_k \tag{54}$$

From the above equation (54), it's easy to get the below equation:

$$e^{jw\Delta T} = h_k^* + \frac{A}{B^*} g_k^* \tag{55}$$

Therefore, equations (55 and 53) should equal to each other, and if we set $s = \frac{B^*}{A}$, the below equation could be obtained:

$$g_k s_k^2 + (h_k - h_k^*)s_k - g_k^* = 0 \tag{56}$$

$$s = \frac{-jIm(h(k)) + j\sqrt{Im^2(h(k)) - |g(k)|^2}}{g(k)} \tag{57}$$

Finally, the frequency can be estimated by the equation below:

$$f_k = \frac{1}{2\pi T} sin^{-1}(Im(h(k) + s(k)g(k))) \tag{58}$$

The simulation result by using Matlab will be shown in Chapter 5 (Evaluation results and discussion). However, for this method, it does not mention anything about the phase unbalanced case and harmonic distortion. Therefore, I decide to consider these 2 cases for this algorithm.

### 4.3.2 Both phase and amplitude unbalanced systems (my work)

After simulating the above amplitude unbalanced design, the frequency estimation result seems to be relatively accurate. Then, I try to improve the previous design for the case with both phase and amplitude unbalanced.

If the phase unbalanced case is added into consideration, the output equation of the Clarke's transform will need to be set as the equation below:

$$\begin{aligned} V_k =& v_\alpha + jv_\beta \\ =& e^{jwk\Delta T}(A+jB) + e^{-jwk\Delta T}(A^* + jB^*) \end{aligned} \tag{59}$$

where

$$\begin{aligned} A &= \frac{V_a}{\sqrt{6}}e^{j\phi_a} - \frac{\sqrt{6}V_b}{12}e^{j\phi_b} - \frac{\sqrt{6}V_c}{12}e^{j\phi_c} \\ B &= \frac{j\sqrt{2}V_b}{4}e^{j\phi_b} - \frac{j\sqrt{2}V_c}{4}e^{j\phi_c} \end{aligned} \tag{60}$$

By using the same equation (52) to express $V_{k+1}$, the following equations could be derived:

$$e^{jw\Delta T} = h_k + \frac{A-jB}{A+jB}g_k \tag{61}$$

$$e^{-jw\Delta T} = h_k + \frac{A^*-jB^*}{A^*+jB^*}g_k \tag{62}$$

From the above equation (62), it's easy to get the below equation:

$$e^{jw\Delta T} = h_k^* + (\frac{A^*-jB^*}{A^*+jB^*})^* g_k^* \tag{63}$$

where

$$\begin{aligned} (\frac{A^*-jB^*}{A^*+jB^*})^* &= \frac{(A^*-jB^*)^*}{(A^*+jB^*)^*} \\ &= \frac{A+jB}{A-jB} \end{aligned} \tag{64}$$

Therefore, equation (63) will then become

$$e^{jw\Delta T} = h_k^* + \frac{A+jB}{A-jB} g_k^* \tag{65}$$

In this way, equation 61 and 65 should equal to each other, and if we set $s = \frac{A-jB}{A+jB}$, the below equation could be obtained:

$$g_k s_k^2 + (h_k - h_k^*) s_k - g_k^* = 0 \tag{66}$$

$$s = \frac{-jIm(h(k)) + j\sqrt{Im^2(h(k)) - |g(k)|^2}}{g(k)} \tag{67}$$

Surprisingly, noticing that equation (67) is exactly same as equation (57) which is used to estimate the frequency for amplitude unbalanced case. This means that the above amplitude unbalanced design could also be used to estimate the frequency of the phase unbalanced signal. The only difference is that during the process, the parameter s is more complicated in term of A and B, however, it will be canceled out to have the same expression (67) in the end. The frequency estimation results for this case will be shown in Chapter 5.

### 4.3.3 Unbalanced systems with harmonic distortion (my work)

After considering the phase unbalanced systems, I then work on the improvement of the algorithm for the systems with harmonic distortion. For the systems with harmonic distortion, I only consider the first 5th and 7th order of the harmonic, as the effect caused by the other order of the harmonic is very small (less than 3.5 percentage)[8]. Also, the reason why I only consider the order that is non-multiple of 3 is that the harmonic with the order that is multiple of 3 will be canceled out after passing through the Clarke's transform. Therefore, the general 3-phase power system model will then become the following:

$$\begin{cases} v_{a,i} = \sum_{k=1}^{K} V_{a,k} cos(2\pi f \Delta T + \phi_{a,k}) + \xi_{a,i} \\ v_{b,i} = \sum_{k=1}^{K} V_{b,k} cos(2\pi f \Delta T + \phi_{b,k} + \frac{2\pi}{3}) + \xi_{b,i} \\ v_{c,i} = \sum_{k=1}^{K} V_{c,k} cos(2\pi f \Delta T + \phi_{c,k} - \frac{2\pi}{3}) + \xi_{c,i} \end{cases} \tag{68}$$

As mentioned before, if only the first 5th and 7th order of the harmonic are considered, according to [8], the input voltage signal model will become:

$$\begin{cases} v_{a,i} = V_a cos(2\pi f \Delta T + \phi_a) + 0.06 V_a cos(2\pi 5 f \Delta T + \phi_a) + 0.05 V_a cos(2\pi 7 f \Delta T + \phi_a) + \xi_{a,i} \\ v_{b,i} = V_b cos(2\pi f \Delta T + \phi_b + \frac{2\pi}{3}) + 0.06 V_b cos(2\pi 5 f \Delta T + \phi_b) + 0.05 V_b cos(2\pi 7 f \Delta T + \phi_b) + \xi_{b,i} \\ v_{c,i} = V_c cos(2\pi f \Delta T + \phi_c - \frac{2\pi}{3}) + 0.06 V_c cos(2\pi 5 f \Delta T + \phi_c) + 0.05 V_c cos(2\pi 7 f \Delta T + \phi_c) + \xi_{c,i} \end{cases} \tag{69}$$

By passing this input signal model through the Clarke's transform, the output will become:

$$V_k = A(1)e^{jwkt} + B(2)e^{jw2kt} + C(3)e^{jw3kt} + D(1)e^{-jwkt} + E(2)e^{-jw2kt} + F(3)e^{-jw3kt} \tag{70}$$

where

$$A(k) = \frac{V_a e^{j\phi_a}}{3} + \frac{V_b e^{j\phi_b}}{6} M(k) - \frac{V_c e^{j\phi_c}}{6} N(k) \tag{71}$$

$$B(k) = \frac{V_a e^{j\phi_a}}{3} + \frac{V_b e^{j\phi_b}}{6} M(k) - \frac{V_c e^{j\phi_c}}{6} N(k) \tag{72}$$

$$C(k) = \frac{V_a e^{j\phi_a}}{3} + \frac{V_b e^{j\phi_b}}{6} M(k) - \frac{V_c e^{j\phi_c}}{6} N(k) \tag{73}$$

$$D(k) = \frac{V_a e^{-j\phi_a}}{3} + \frac{V_b e^{-j\phi_b}}{6} N(k) - \frac{V_c e^{-j\phi_c}}{6} P(k) \tag{74}$$

$$E(k) = \frac{V_a e^{-j\phi_a}}{3} + \frac{V_b e^{-j\phi_b}}{6} N(k) - \frac{V_c e^{-j\phi_c}}{6} P(k) \tag{75}$$

$$F(k) = \frac{V_a e^{-j\phi_a}}{3} + \frac{V_b e^{-j\phi_b}}{6} N(k) - \frac{V_c e^{-j\phi_c}}{6} P(k) \tag{76}$$

and

$$M(k) = \sqrt{3} sin(\frac{2\pi k}{3}) - cos(\frac{2\pi k}{3}) + j[sin(\frac{2\pi k}{3}) + \sqrt{3} cos(\frac{2\pi k}{3})] \tag{77}$$

$$N(k) = \sqrt{3} sin(\frac{2\pi k}{3}) + cos(\frac{2\pi k}{3}) + j[sin(\frac{2\pi k}{3}) - \sqrt{3} cos(\frac{2\pi k}{3})] \tag{78}$$

$$P(k) = \sqrt{3} sin(\frac{2\pi k}{3}) + cos(\frac{2\pi k}{3}) + j[-sin(\frac{2\pi k}{3}) + \sqrt{3} cos(\frac{2\pi k}{3})] \tag{79}$$

By using the same equation (52) to express $V_{k+1}$, the following equations could be obtained:

$$e^{jw\Delta T} = h_k + \frac{D(1)^*}{A(1)} g_k \tag{80}$$

$$e^{-jw\Delta T} = h_k + \frac{A(1)^*}{D(1)} g_k \tag{81}$$

From equation (81), the following equation could be derived:

$$e^{jw\Delta T} = h_k^* + \frac{A(1)}{D(1)^*} g_k^* \tag{82}$$

If $s = \frac{D(1)^*}{A(1)}$, the same equations (56-58) will be obtained again. Therefore, it's proven that theoretically the amplitude unbalanced design is also suitable for the frequency estimation of the 3-phase power system that contains harmonic distortion.

All theories discussed above will be proven by Matlab simulation and the simulation result will be shown in the later chapter.

# 5 Implementation plan

In order to achieve the implementation of different algorithms based on the least square technique, the software, Matlab, is recommended to be used to build up the whole algorithm system and simulate the results. There will be several steps for the implementation:

## 5.1 Balanced system

* First, building up the balanced 3-phase power system model with the proper voltage peak amplitude and frequency.
* Then, building up the Clarke's transform.
* Achieving the complex voltage $V_k = v_{\alpha k} + jv_{\beta k}$.
* Developing the Matlab code for the balanced system based on the ACLMS algorithm in [5].
* Comparing my simulation results with the existing results from [5].
* If the results are different from the research one, modify the coding until the correct results have been achieved.

## 5.2 Amplitude unbalanced system

* By using the previous Clarke's transform, develop the Matlab code for the unbalanced system based on the ACLMS algorithm in [5].
* Comparing my simulation results with the existing results from [5].
* If the results are different from the research one, modify the coding until the correct results have been achieved.
* By using the same unbalanced input signal, compare the estimation result with the one from balanced system.

* By using the previous Clarke's transform, develop the Matlab code for the unbalanced system based on the WLCLMS algorithm in [6].
* Comparing my simulation results with the existing results from [6].
* If the results are different from the research one, modify the coding until the correct results have been achieved.
* By using the same unbalanced input signal, compare the estimation result based on WLCLMS [6] with the one based on ACLMS [5].
* By using the previous Clarke's transform, develop the Matlab code for the unbalanced system based on the CO-CMA algorithm in [6].
* Comparing my simulation results with the existing results from [6].
* If the results are different from the research one, modify the coding until the correct results have been achieved.
* By using the same unbalanced input signal, compare the estimation result based on CO-CMA [6] with the one based on CLMS [5] and the one based on WLCLMS [6].

## 5.3 Phase and amplitude unbalanced system and the unbalanced system with harmonic distortion

* Building up the phase and amplitude unbalanced input voltage signal model.
* Using this input signal to test the ACLMS, WLCLMS and CO-CMA algorithms and comment on the results.
* Building up the amplitude unbalanced input signal model with harmonic distortion.
* Using this input signal to test the ACLMS, WLCLMS and CO-CMA algorithms and comment on the results.

The gantt chart of the overall timeline for the implementation plan will be shown in the chapter "Appendix".

# 6 Evaluation results and discussion

## 6.1 The Clarke's transform

### 6.1.1 Balanced case

For the Clarke's transform in the balanced case, the balanced 3-phase voltage (equation 1) is needed to be set as input. The figure (6.1.1.1) is the plot of the a balanced 3-phase voltage without any noise and with the amplitude of 1 p.u.

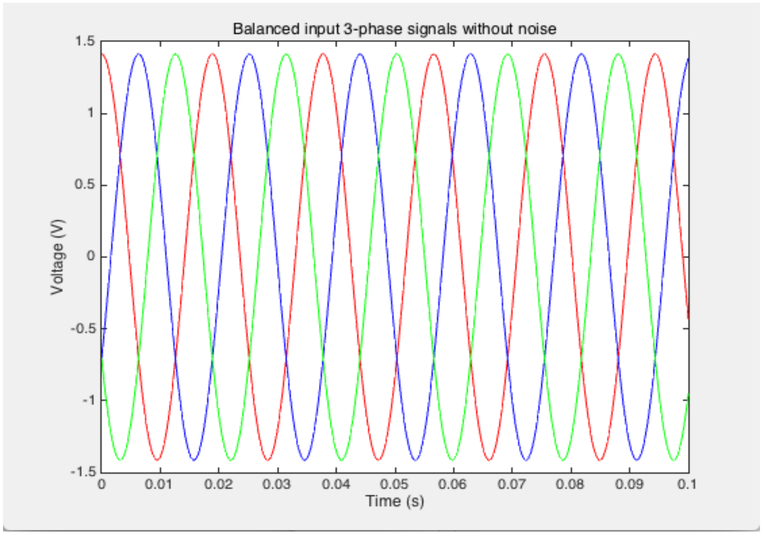



Figure 6.1.1.1

However, in the real world, the power signal will always contain noise. Therefore, to have a more reliable results, the input signal will need to consider the noise. The figure (6.1.1.2) below shows the plot of the balanced input signal with a random noise.

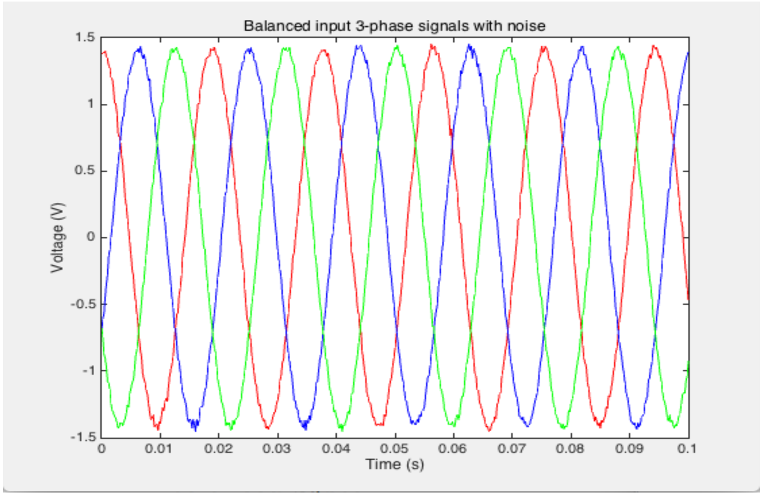



Figure 6.1.1.2

By passing the above input signal with noise through the Clarke's transform, the combined complex output voltage signal is shown below (figure 6.1.1.3).

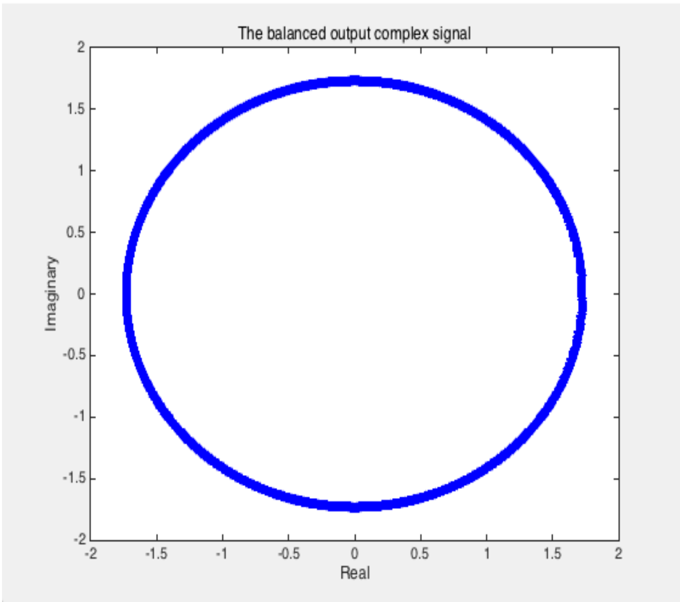



Figure 6.1.1.3

Comparing this result with the simulation result in [7], when the input signal is a balanced signal, then the output of the Clarke's transform should be a circle in the complex plane.

### 6.1.2 Unbalanced case

For the unbalanced case, the input signal with random noise which has the different amplitudes is shown in figure 6.1.1.4 (Va,peak = 1.1 p.u., Vb,peak = 0.7 p.u. and Vc,peak = 0.5 p.u.).

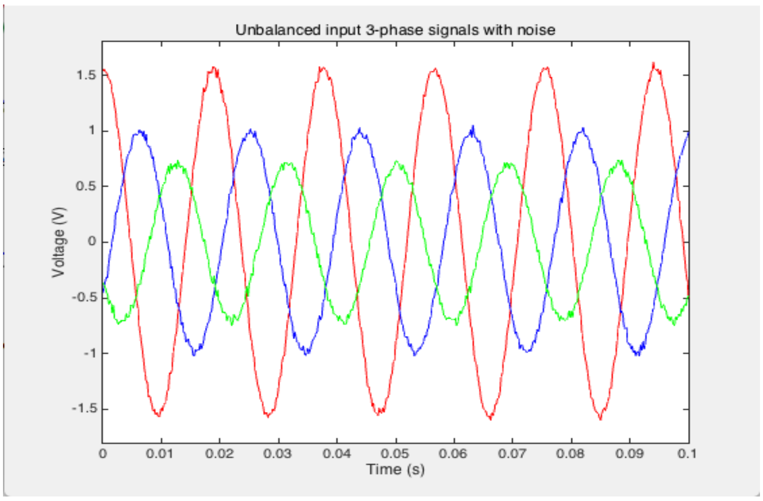



Figure 6.1.1.4

When the above amplitude unbalanced signal is used as input signal, the output combined complex signal of the Clarke's transform will be presented as below (figure 6.1.1.5). Comparing this result with the simulation result in [7], when the input signal is an unbalanced signal, then the output of the Clarke's transform should be an ellipse in the complex plane.

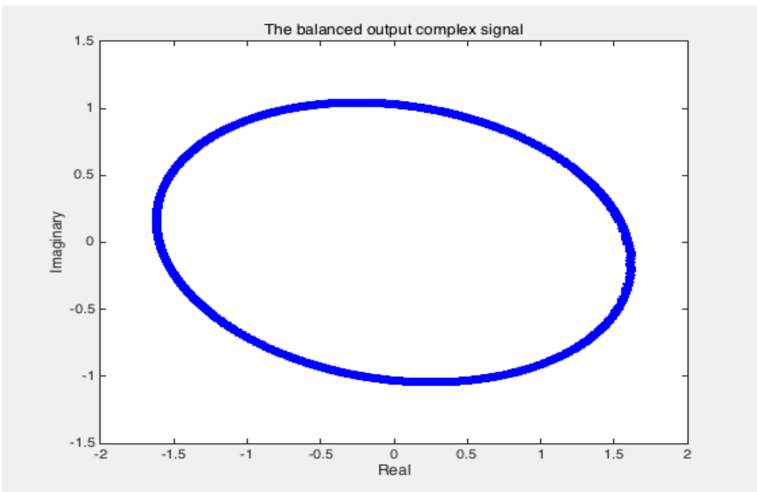



Figure 6.1.1.5

## 6.2 Frequency estimation

For this thesis, four algorithms which are all based on the least square technique have been used to estimate the frequency, ACLMS(balanced design and unbalanced design) [5], WL-CLMS and CO-OMA [6]. Both ACLMS and WLCLMS are designed to estimate the frequency for both balanced and unbalanced 3-phase power system voltage. The third algorithm, CO-CMA is developed based on the WLCLMS, therefore, should have a better performance than the other two.

### 6.2.1 ACLMS (balanced design)

For the balanced design of the ACLMS, it is first tested by the balanced input signal, with the same amplitude of 1 p.u. The frequency estimation is shown below (figure 6.2.1.1).

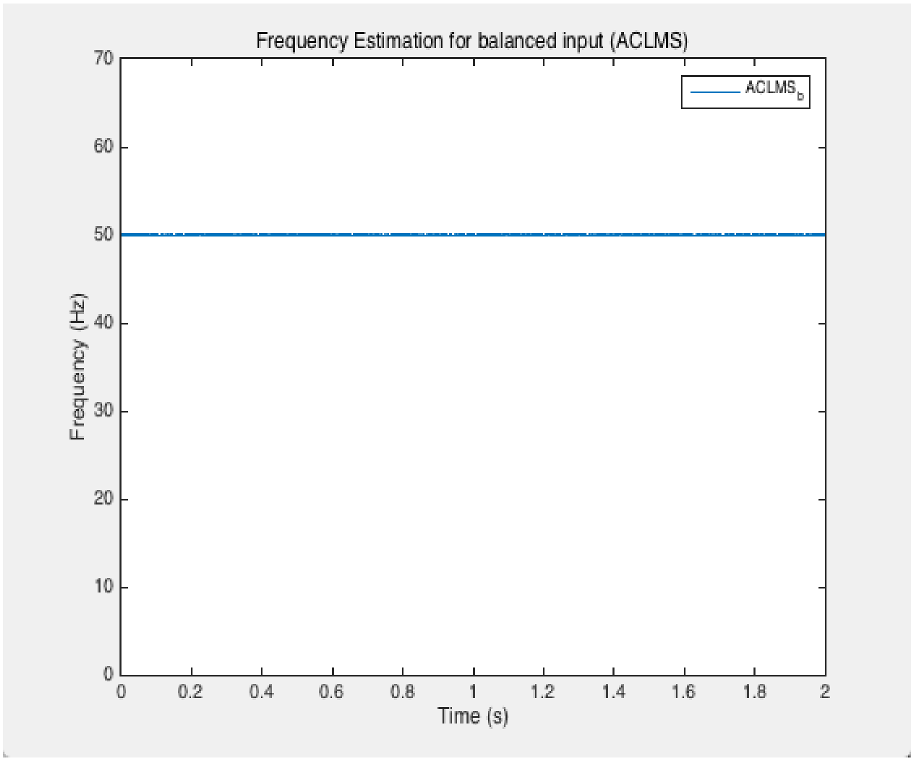



Figure 6.2.1.1

In this case, the frequency of the input signal is set to be 50 Hz, from the simulation plot, it's notice that the frequency estimation error is about 0.02 Hz and the peak noise is about 0.28Hz.

Then, this balanced design is tested by the amplitude unbalanced input signal (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.). The frequency estimation is shown below (figure 6.2.1.2).

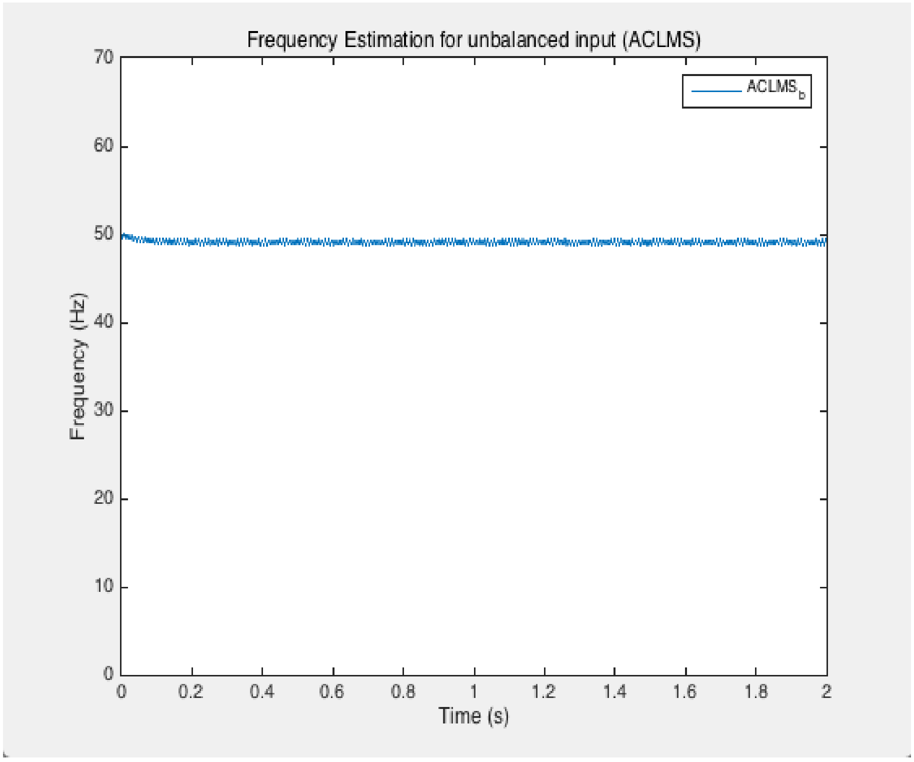



Figure 6.2.1.2

In this case, the frequency of the input signal is set to be 50 Hz, from the simulation plot, it's notice that the frequency estimation error is about 0.915 Hz and the peak noise is about 0.83 Hz. This result shows that this balanced design does not work well for the unbalanced input signal. Therefore, we need to have the unbalanced design for the unbalanced 3-phase signal which will be shown in the next section.

### 6.2.2 ACLMS (amplitude unbalanced design)

For the unbalanced design based on ACLMS, it's tested by the amplitude unbalanced 3-phase input under the same condition as the one for the balanced design (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.). The figure below (figure 6.2.2.1) shows the frequency estimation of the unbalanced input signal.

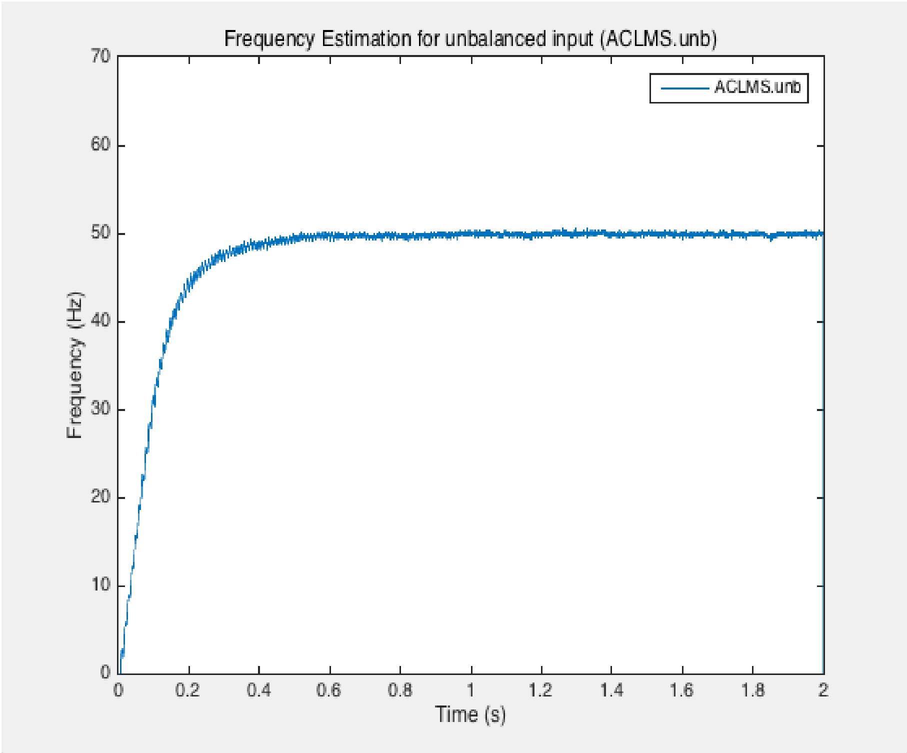



Figure 6.2.2.1

In this case, the frequency of the input signal is set to be 50 Hz, from the figure above, it's notice that the frequency estimation error is about 0.115 Hz and the peak noise is about 1.01 Hz. Comparing this result with the previous balanced design, it's noticed that the frequency estimation error decreases from 0.915 Hz to 0.115 Hz. In other words, the unbalanced design has been proven to have a better performance (more accurate) for unbalanced cases. However, for unbalanced design, the estimated frequency always has to start from zero, as a result of that, there is a 0.29s rising time.

### 6.2.3 WLCLMS

For the WLCLMS algorithm, in order to compare with the previous algorithms, the same amplitude unbalanced input signal is used again. The result is shown below (figure 6.2.3.1).

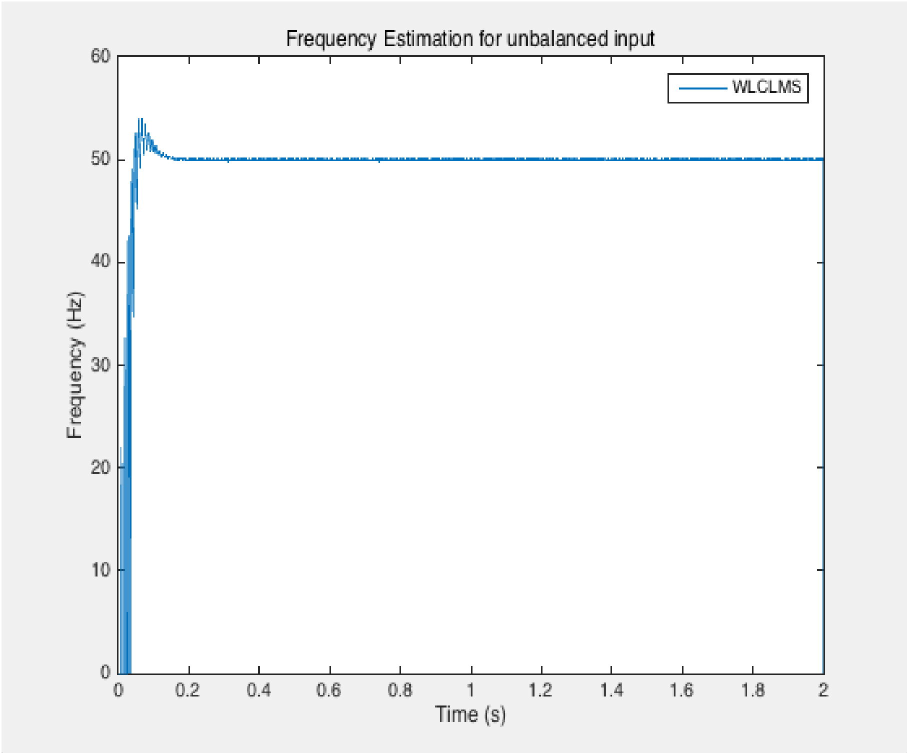



Figure 6.2.3.1

In this case, the frequency is set to be 50 Hz, and from the simulation plot, it's notice that the frequency estimation error is about 0.02 Hz with a rising time of 0.046s and the peak noise is about 0.36 Hz which is not only more accurate, but also has smaller noise than the previous algorithms for the unbalanced signal.

### 6.2.4 CO-CMA

The CO-CMA is developed based on the above algorithm (WLCLMS), therefore, it's expected to have a better performance. Again, the same unbalanced input signal is used and the result is shown in figure 6.2.4.1.

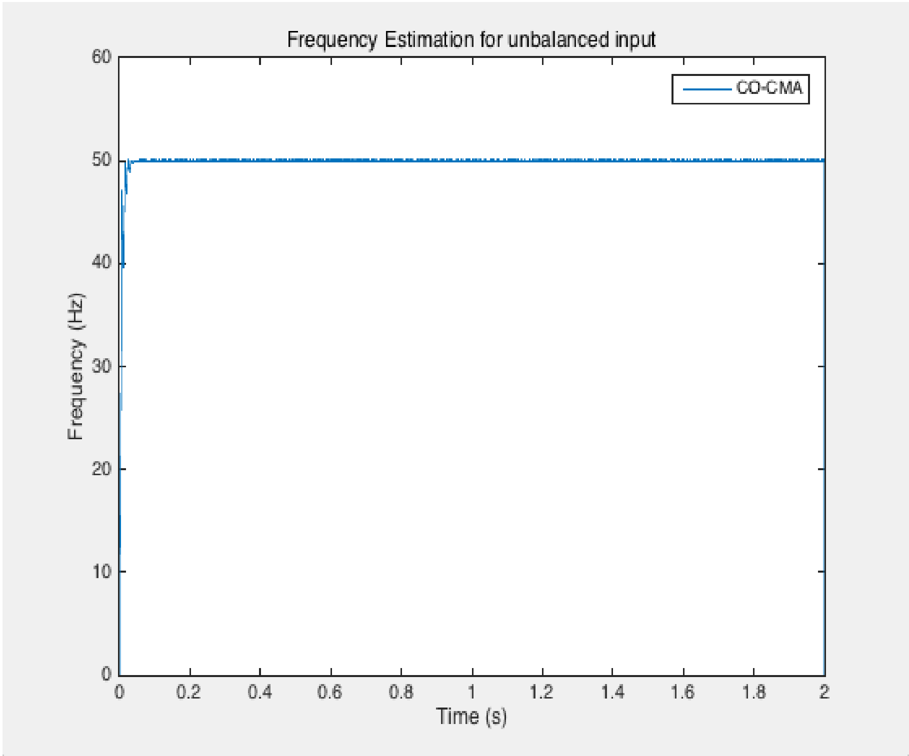



Figure 6.2.4.1

In this case, again, the the frequency is set to be 50 Hz. From the plot, it's notice that the frequency estimation error is about 0.01 Hz with a rising time of 0.0354s and the peak noise is about 0.22 Hz. As expectation, the CO-CMA has a better performance than not only WLCLMS, but all the above algorithms.

## 6.3 Comparison for different frequency cases

In the previous section, the 4 algorithms are discussed separately. In this section, all estimation results under different frequency cases for the 4 algorithms are discussed together for comparison purpose.

### 6.3.1 Estimate constant frequency

The figure below shows the constant frequency estimation of the balanced 3-phase power system.

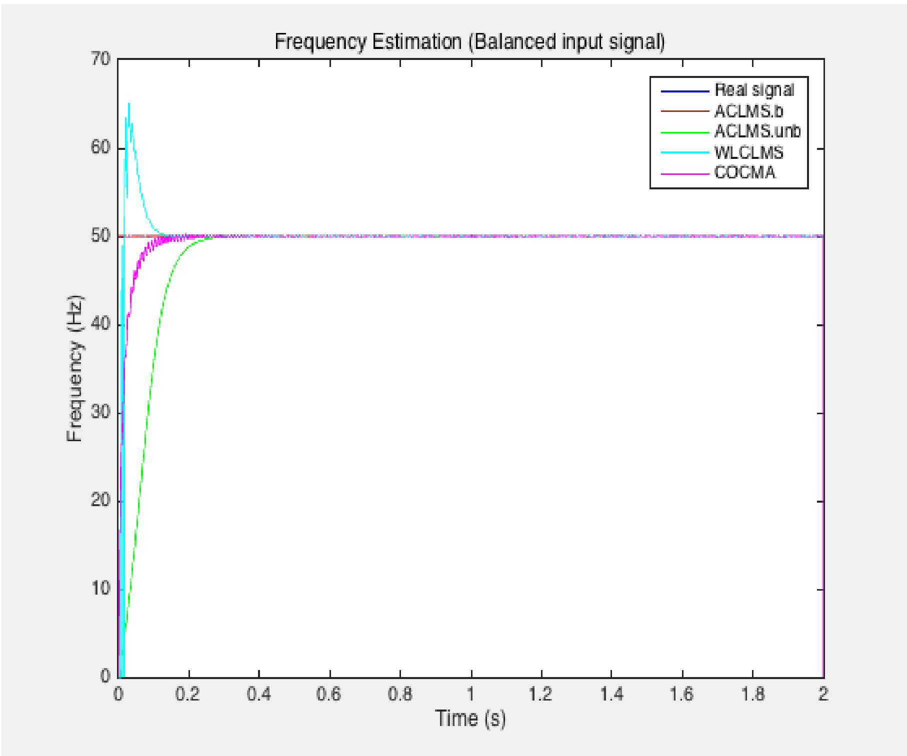



Figure 6.3.1.1

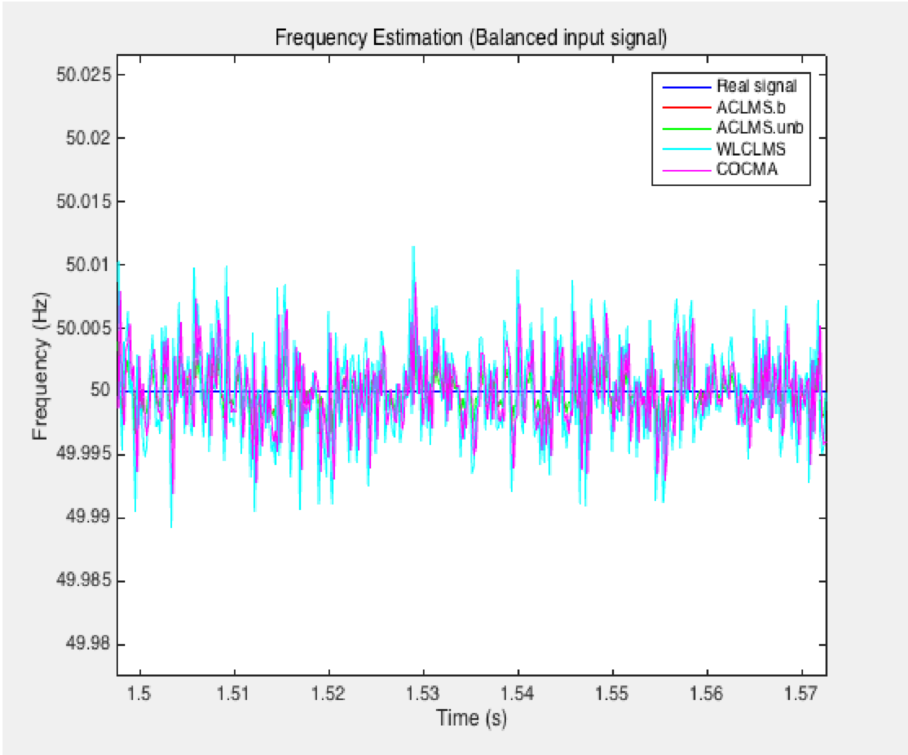



Figure 6.3.1.2

As we can see, for balanced systems, all 4 algorithms have very accurate estimation results which all have a very small output noise (shown in figure 6.3.1.2). For comparison, the balance design of the ACLMS has the best performance in balance case with the relative small noise. The WLCLMS and CO-CMA has the similar converging time. The unbalance design of the ACLMS has the worse converging time, but it has a better noise performance (smaller) than WLCLMS and CO-CMA.

As for the unbalanced case, the figure below shows the constant frequency estimation of the amplitude (only) unbalanced system (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.) by using all four algorithms discussed before.

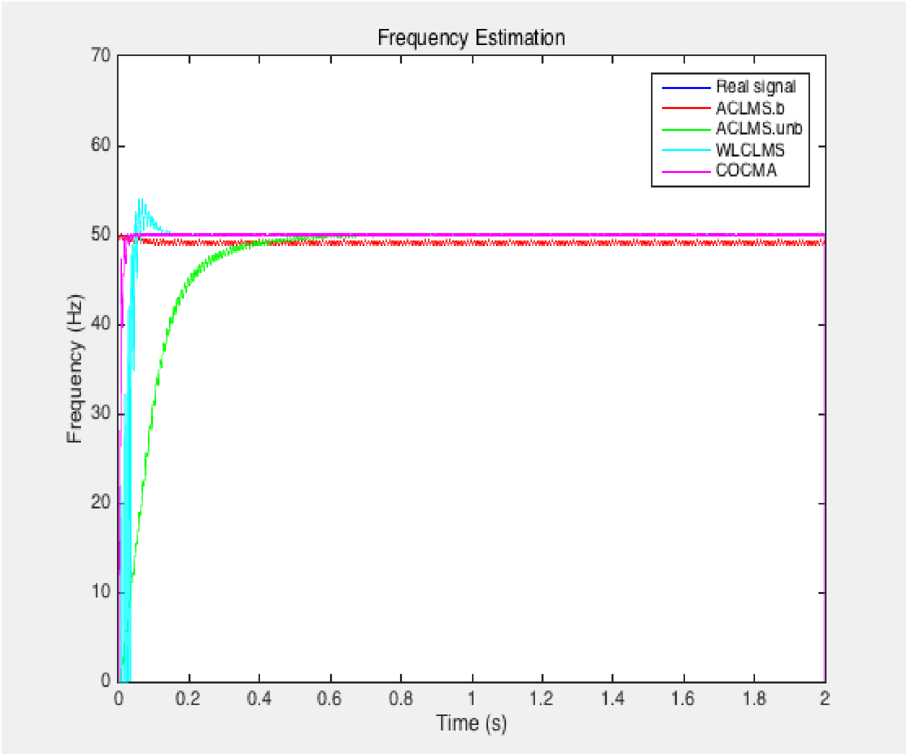



Figure 6.3.1.3

For the constant frequency estimation, CO-CMA has the best performance which is due to the different models that are used by CO-CMA. For the first 3 algorithms (WLCLMS, balanced ACLMS and unbalanced ACLMS), they all use the same voltage model (equation 6). However, for CO-CMA, it has the different voltage model (equation 44). As a result of that, it will increase the speed to reach the final state. For this amplitude unbalanced system, the amount of imbalance is small, so all 4 algorithms seems to work fine and have no large difference to each other.

When the amount of imbalance increases, for example, Va=1 p.u., Vb=0.5 p.u. and Vc=0.4 p.u., the output will be shown below:

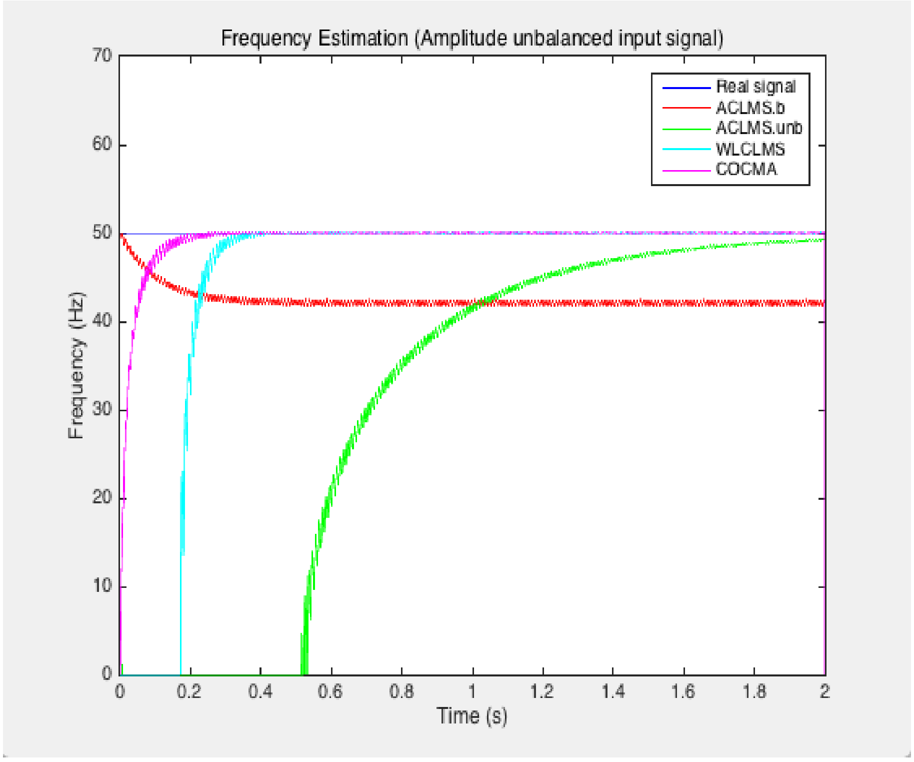



Figure 6.3.1.4

From the simulation result, we could see that as the amount of imbalance increases, the balanced design of ACLMS will no longer be accurate anymore, which proves that it's not suitable for the frequency estimation of the unbalanced system. As for the unbalanced design of the ACLMS and WLCLMS, the converge time increases, especially for ACLMS. However, for CO-CMA, it still has the best performance comparing to others.

### 6.3.2 Step frequency tracking

In the real world, the frequency of the 3-phase power system will not always be a constant, therefore, tracking for different frequencies is very important. For this reason, all four algorithm are also tested by the same amplitude unbalanced system as before (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.) but with varied frequency. The result will be shown below (figure 6.3.2.1).

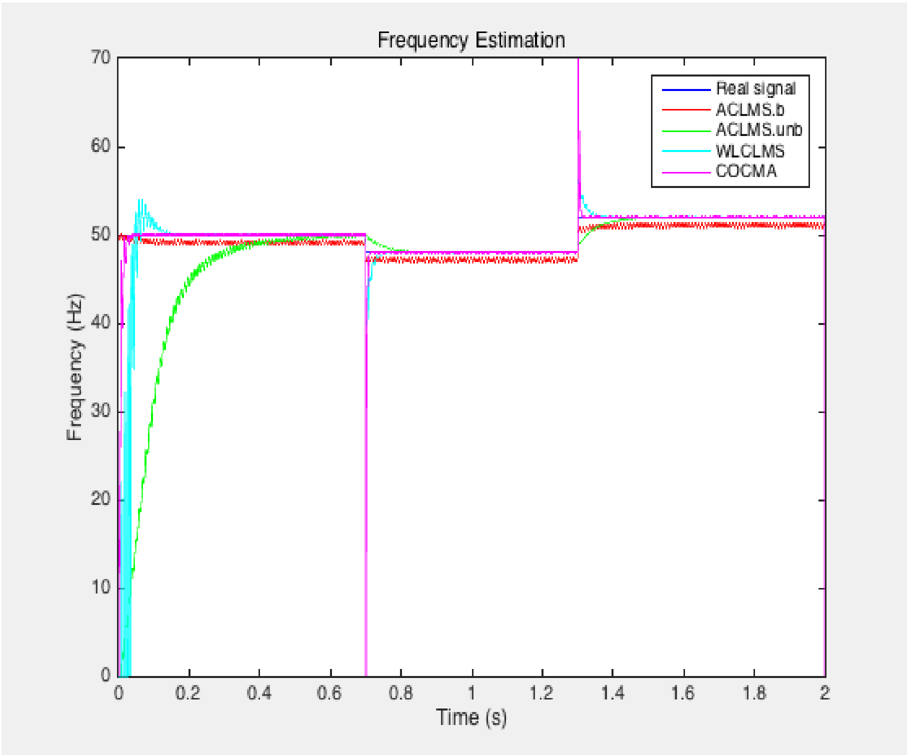



Figure 6.3.2.1

In this case, the frequency is set to be 50 Hz for the first 0.7s, and then the frequency is set to be 48 Hz for the next 6s. In the end, the frequency is set to be 52 Hz for the rest of the time. According to the result above, the CO-CMA algorithm is the most accurate, but has the highest peak when the frequency changes. The WLCLMS has the second best accurate performance, but still has a relative high peak comparing with the ACLMS. As for ACLMS, it has the no peak when the frequency changes, however, the estimated frequency is not as accurate as other algorithms.

### 6.3.3 Linear frequency tracking

In order to test the tracking ability of the increasing frequency, all four algorithms are tested by the increasing frequency for the same amplitude unbalanced system (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.). The result is shown in figure 6.3.3.1.

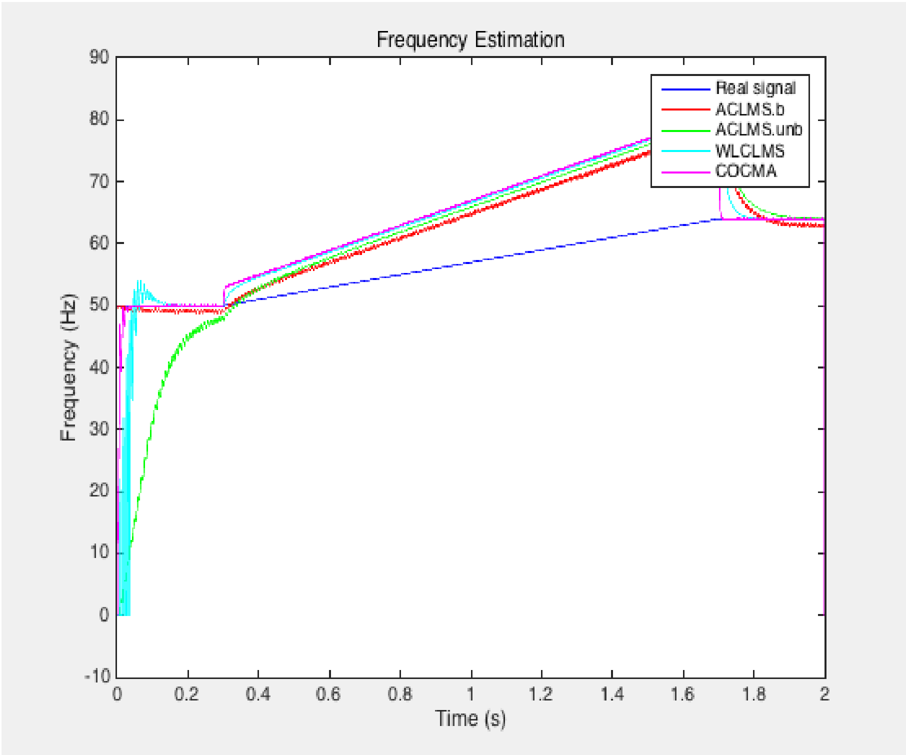



Figure 6.3.3.1

This input voltage signal has a frequency of 50 Hz for the first 0.3s and increase for the next 14s with a gradian of 0.002. For the last 3s, the frequency is set to be 64 Hz. From the result above, all algorithms do not have strong tracking ability for the increasing frequency estimation. Furthermore, CO-CMA has the worst ability for the increasing frequency estimation and the balanced design based on ACLMS has the relative better performance than other algorithms.

## 6.4 Further testing for the phase unbalanced case

In the real world, the unbalanced 3-phase voltage signal will not only be amplitude unbalanced, it could also have the situation where the phase is also unbalanced. For the previous 4 algorithms, they only consider the amplitude unbalanced input voltage signal. Therefore, it's decided to test these algorithms with the input signal whose amplitude and phase are both unbalanced. The simulation results are shown in figure 6.4.1.

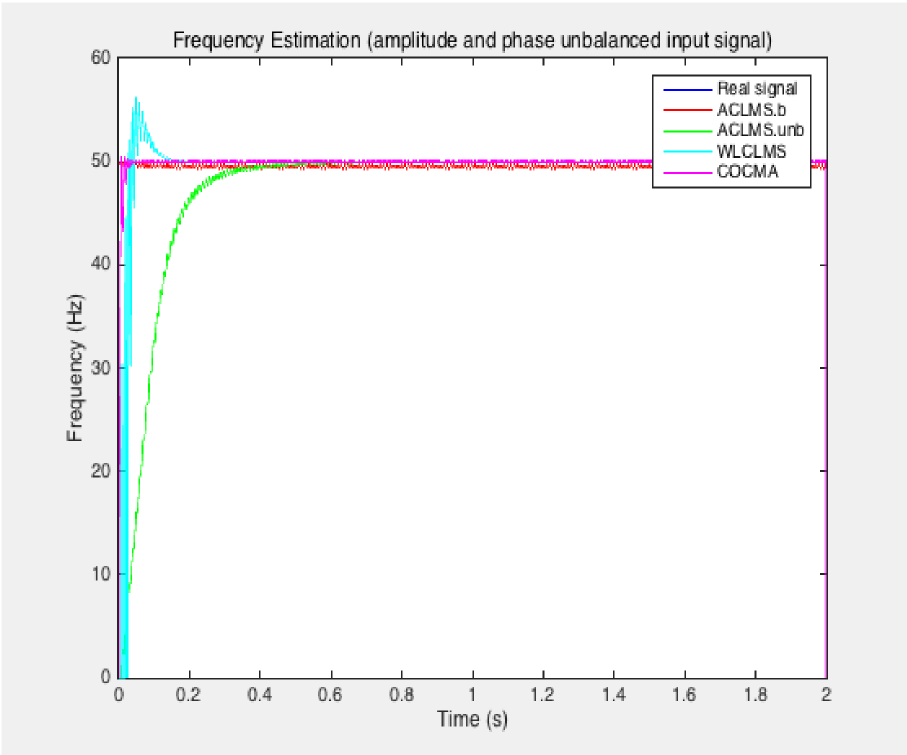



Figure 6.4.1

Again, base on the same amplitude unbalanced values as before (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.), the phase unbalanced values are added into the input signal ($Ph_a = 0rad$, $Ph_b = \frac{\pi}{60}rad$ and $Ph_b = -\frac{\pi}{60}rad$).

Comparing this results with figure 6.3.1.1, we can see that they are very similar. In other words, the previous amplitude unbalanced design algorithms are all suitable for the frequency estimation of the phase unbalanced system.

If the amount of imbalance increases, for example, Va=1 p.u., Vb=0.5 p.u., Vc=0.4 p.u., $Ph_a = 0rad$, $Ph_b = \frac{\pi}{6}rad$ and $Ph_b = \frac{\pi}{4}rad$. The simulation result will be shown below:

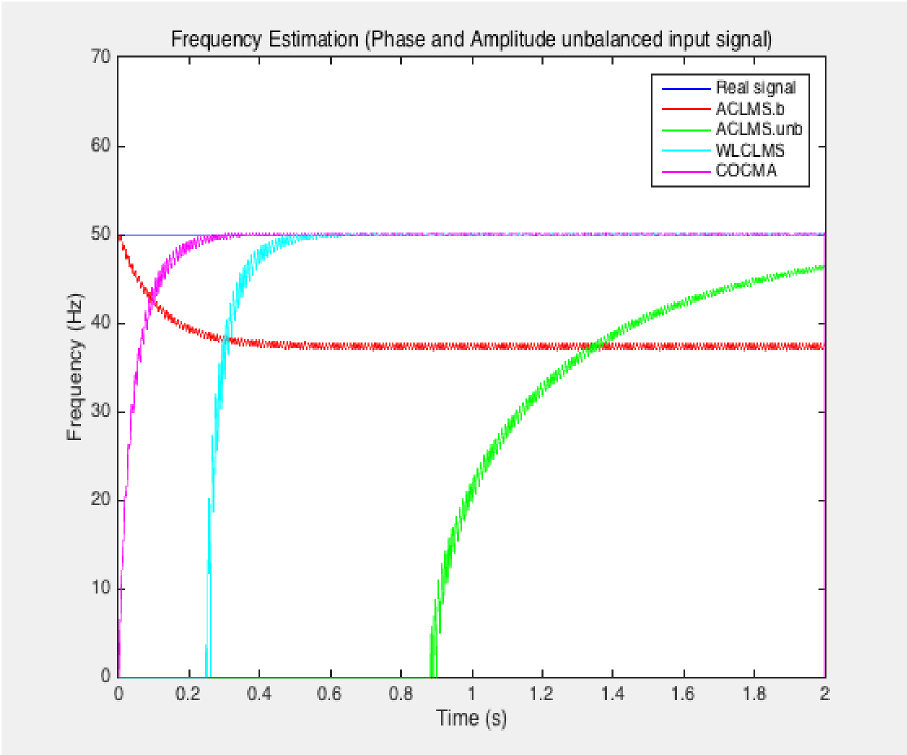



Figure 6.4.2

As the imbalance becomes larger for the amplitude and phase unbalanced case, the balanced design of ACLMS has a very poor performance (does not work anymore). The converge time of other 3 algorithms increase a lot, especially for the unbalanced design of the ACLMS. However, among all algorithms, CO-CMA still has the best performance.

## 6.5 Further testing for the harmonic distortion

### 6.5.1 Amplitude unbalanced signal with harmonic distortion

It's also decided to test the previous four algorithms with the amplitude unbalanced input signal that contains the harmonic distortion. The simulation results are shown below:

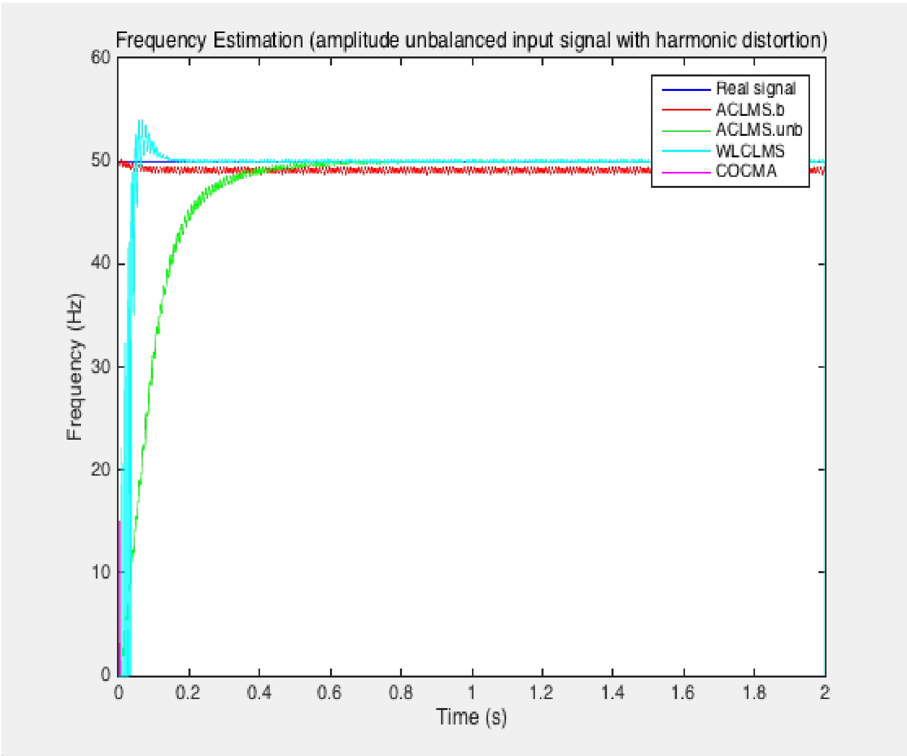



Figure 6.5.1.1

With the same amplitude unbalanced values like before (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.) and the additional 5th and 7th order of harmonic distortion, the estimated results are similar to the one before (figure 6.3.1.1). This means that, the previous amplitude unbalanced designs are also suitable for the frequency estimation of the signal with harmonic distortion.

### 6.5.2 Amplitude and phase unbalanced signal with harmonic distortion

In this case, both amplitude and phase unbalanced 3-phase voltage input signal are considered together with the harmonic distortion. By using this complicated input model, all algorithms are test and the results are shown below:

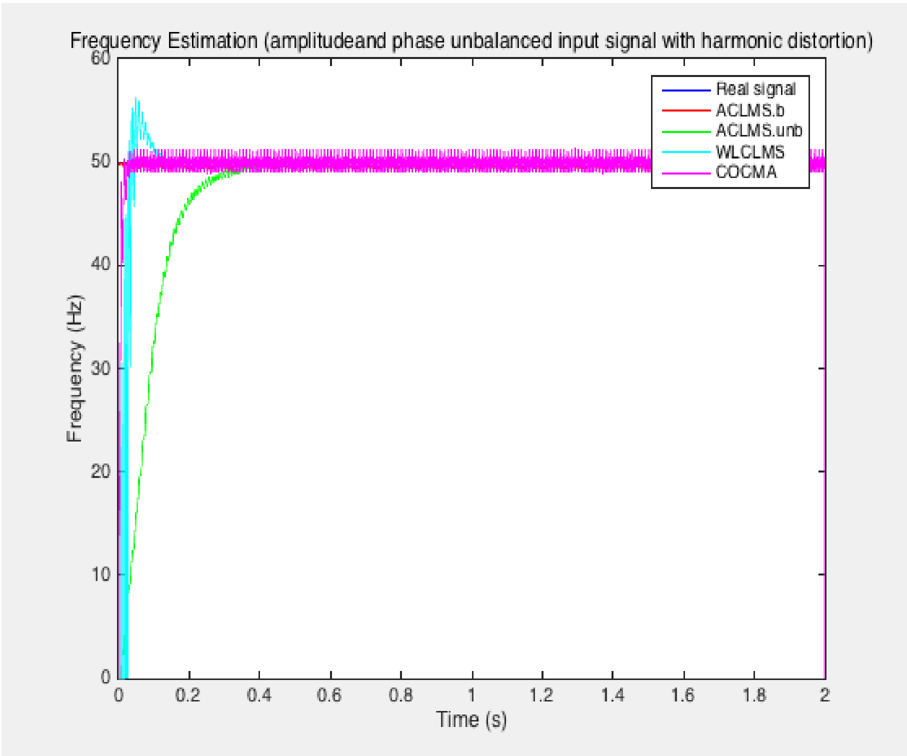



Figure 6.5.2.1

Same amplitude unbalanced values (Va=1.1 p.u., Vb=0.8 p.u. and Vc=0.9 p.u.) and phase unbalanced values ($Ph_a = 0rad$, $Ph_b = \frac{\pi}{60}rad$ and $Ph_b = -\frac{\pi}{60}rad$) are used again in this case with the additional harmonic distortion.

From the simulation results (Figure 6.5.2.1), we could see that most of the previous algorithms are suitable for dealing with the harmonic distortion except CO-CMA. For CO-CMA in this case (Figure 6.5.2.2.), the algorithm still has the shortest rising time (0.045 second), but with a very large noise which is about 2.23 Hz. The only different between CO-CMA and other algorithms is the additional matrix coefficient inside the output of the Clark's transform (44) which is more complicated than the normal one that is used for other algorithms, so when it's used to estimate the system with the harmonic distortion, the complicated input signal will increase the complexity of the estimation system which will cause the unstable result for the estimated frequency (output with larger noise).

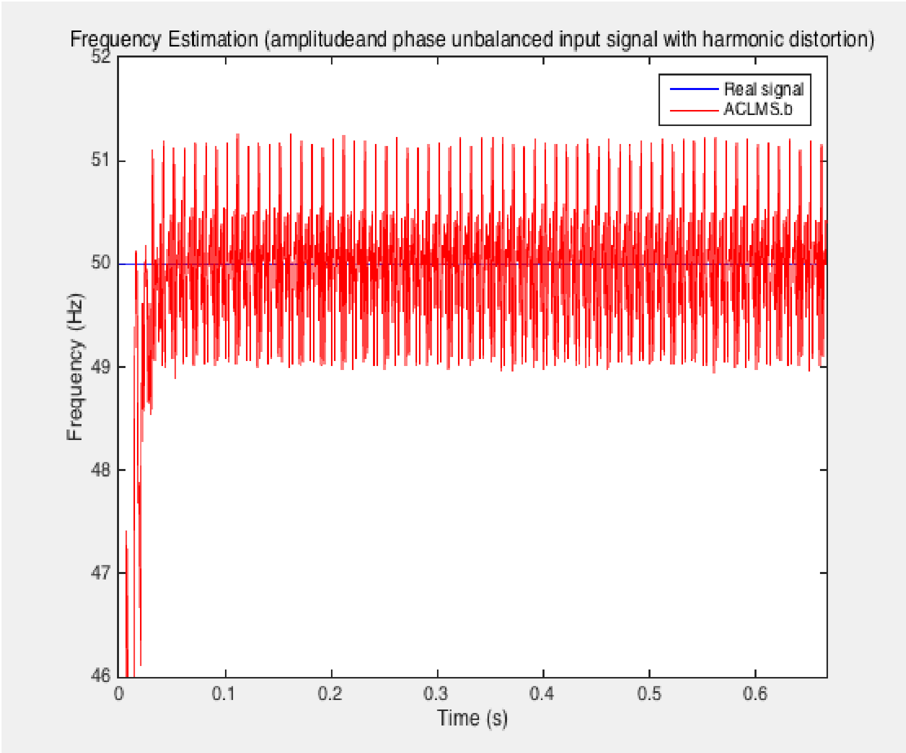



Figure 6.5.2.2

Therefore, the CO-CMA algorithm could be improved further to have a better performance when dealing with the amplitude and phase unbalanced signal which contains the harmonic distortion.

# 7 Conclusion

In this thesis report, it introduces the thesis topic and explain the reason why people from power industry will be interested in it. Also, it contains the background research which explains the latest algorithms that could estimate the frequency for both balanced and unbalanced 3-phase power system, such as, the algorithm by using the extended complex Kalman filter, least mean square algorithm, extended least square algorithm, weighted-least-square algorithm with adaptive FIR filter, correntropy adaptive filter and coupled orthogonal constant modulus algorithm.

For those existing algorithms that have been reviewed and discussed in this report, they include the algorithm based on both Kalman filter and least square technique which are suitable for the frequency estimation of both balanced and amplitude unbalanced 3-phase power system. From the research [3], least square method has a slightly better performance than the Kalman filter, but the design based on the least square technique does not consider the phase unbalanced case and the harmonic distortion. Therefore, 4 new fomulae for both algorithm and the Clarke's transform are being developed base on the least square method for each factors that mentioned before.

As the decision that is mentioned above, 4 algorithms (balanced ACLMS, unbalanced ACLMS, WLCLMS and CO-CMA) in paper [5], [6] and [7] are been simulated in Matlab. According to the simulation results, CO-CMA has the best performance for balanced, amplitude and phase unbalanced voltage input signal with noise. However, for the further consideration about the harmonic distortion, CO-CMA has the worst performance (relatively short rising time but with large noise). Therefore, to deal with the harmonic distortion for CO-CMA is considered to be the future work, as this method has the best performance among all the method that is based on the least square techniques.

In the later section of this report, the gantt chart for the work done during this thesis period is available in the Appendix.

# 8 Appendix

## 8.1 Matlab code

### 8.1.1 3-phase power system model with noise and the Clarke's transform

```
clc;
clear all;

%Create the 3-phase power system with random noise
%Initializing the parameters
Va = 1*sqrt(2);
Vb = 0.5*sqrt(2);
Vc = 0.4*sqrt(2);
Ph_a = 0;
Ph_b = pi/6;
Ph_c = pi/4;
fs = 5000;                                  %sampling frequency
T = 1/fs;                                   %sampling period
t = 2;                                      %sampling time (s)
num_samples = t*fs;                         %number of samples
n = (0 : num_samples-1);                    %sampling sequence
fc = 50*ones(1,num_samples);                %input frequency
time = linspace(0,t,num_samples);

%noise generation
SNR = 60;
sigma = sqrt((10^(-SNR/10))*Va^2);
random_noise = (sigma/2)*randn(3,num_samples);

%Generating the balanced 3-phase input and plot them
va=Va*cos(2*pi/fs*fc.*n+Ph_a)+random_noise(1,:);
vb=Vb*cos(2*pi/fs*fc.*n+Ph_b-2*pi/3)+random_noise(2,:);
vc=Vc*cos(2*pi/fs*fc.*n+Ph_c+2*pi/3)+random_noise(3,:);

%Passing the previous input into the Clarke's transform
V_alpha = (2/3)^(0.5)*[1 -0.5 -0.5]*[va;vb;vc];
V_beta = (2/3)^(0.5)*[0 (3/4)^(0.5) -(3/4)^(0.5)]*[va;vb;vc];
```

### 8.1.2 Set up for different frequency estimation cases

```
% The step frequency.........................................
N1 = 0.7*fs;   % number of sampling points form 0s to 0.3s
N2 = 0.6*fs;    % number of sampling points form 0.3s to 0.6s
N3 = num_samples-N1-N2;
f1 = 50 * ones(1,N1);  %the frequency sequency from 0 to 0.3s
f2 = 48 *ones(1,N2); %the frequency sequency from 0.3 to 0.8s
f3 = 52 * ones(1,N3);  %the frequency sequency from 0.8 to 0.9s
fc = [f1,f2,f3];  %the combined frequency sequency

% The linear frequency........................................
N1 = 0.3*fs;
N3 = 0.3*fs;
f1 = 50 * ones(1,N1);
f2 = 50 * ones(1,num_samples-N1-N3);
f3 = 64 * ones(1,N3);
for i=1:num_samples-N1-N3
    f2(i)=50+i*0.002;
end
fc = [f1, f2, f3];
% ..................................................................
```

### 8.1.3 Matlab code for the balanced design of ACLMS

```
% (1) Develop the ACLMS system for balanced design..................
% Initializing the parameters

y=zeros(1,num_samples+1);
e=zeros(1,num_samples);
w_h=zeros(1,num_samples+1);
w_g=zeros(1,num_samples+1);
w=zeros(1,num_samples);
f_ACLMS_b=zeros(1,num_samples);

f_ACLMS_b(1)=50;
u_CLMS=1.5e-3;
u_pro=0.06;
w(1)=exp(1i*2*pi*f_ACLMS_b(1)/fs);

for k=1:num_samples-1
    y(k+1)=w(k)*v(k);
    e(k)=v(k+1)-y(k+1);
    w(k+1)=w(k)+u_CLMS*e(k)*conj(v(k));
    f_ACLMS_b(k+1)=asin(imag(w(k+1)))/(2*pi*T);
end
```

### 8.1.4 Matlab code for the unbalanced design of ACLMS

```
% (2)Develop the ACLMS system for unbalanced design

y=zeros(1,num_samples+1);
e=zeros(1,num_samples);
w_h=zeros(1,num_samples+1);
w_g=zeros(1,num_samples+1);
w=zeros(1,num_samples);
h=zeros(1,num_samples);
g=zeros(1,num_samples);
s1=zeros(1,num_samples);
f_ACLMS_unb=zeros(1,num_samples);

u_h=1.5e-3;
u_g=3e-4;

Ker = 1;                              %kernal size

for k=1:num_samples-1
    y(k+1)=v(k)*h(k)+conj(v(k))*g(k);
    e(k)=v(k+1)-v(k)*h(k)-conj(v(k))*g(k);
    h(k+1)=h(k)+u_h*e(k)*conj(v(k))...
        *exp(-(abs(e(k)))^2/(2*Ker^2));
    g(k+1)=g(k)+u_g*e(k)*v(k)...
        *exp(-(abs(e(k)))^2/(2*Ker^2));
    s1(k)=(-1i*imag(h(k))+1i*sqrt((imag(h(k)))^2-(abs(g(k)))^2))...
        /g(k);
    f_ACLMS_unb(k)=asin(imag(h(k)+s1(k)*g(k)))/(2*pi*T);
end
```

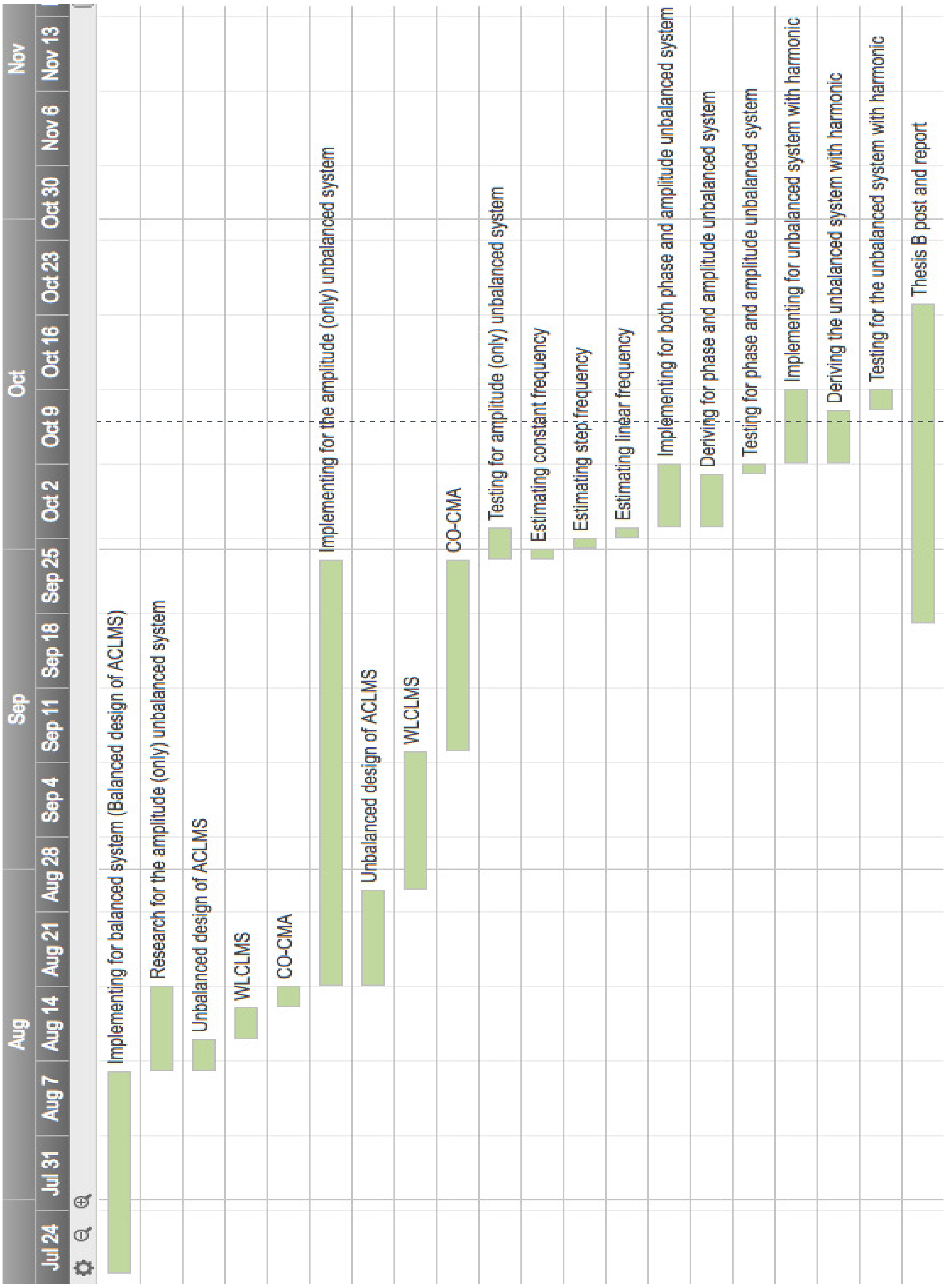
### 8.1.5 Matlab code for WLCLMS

```
% (3)Develop the WLCLMS algorithm

y=zeros(1,num_samples+1);
e=ones(1,num_samples);
w_h=zeros(1,num_samples+1);
w_g=zeros(1,num_samples+1);
w=zeros(1,num_samples);
f_WLCLMS=zeros(1,num_samples);
h=zeros(1,num_samples);
g=zeros(1,num_samples);
a1=zeros(1,num_samples);
u=4e-3;

for k=1:num_samples-1
    y(k+1)=v(k)*h(k)+conj(v(k))*g(k);
    e(k)=v(k+1)-y(k+1);
    h(k+1)=h(k)+u*e(k)*conj(v(k));
    g(k+1)=g(k)+u*e(k)*v(k);

    a1(k)=(-1i*imag(h(k))+1i*sqrt((imag(h(k)))^2-(abs(g(k)))^2))...
        /g(k);
    f_WLCLMS(k)=asin(imag(h(k)+a1(k)*g(k)))/(2*pi*T);
end
```

### 8.1.6 Matlab code for CO-CMA

```
% (4)Develop the CO-CMA algorithm
V = [V_alpha ; 1i*V_beta];

%Develop the ACLMS system for balanced input
%Initializing the parameters

y=zeros(1,num_samples+1);
e=zeros(1,num_samples);
f_COCMA=zeros(1,num_samples);
h=ones(1,num_samples);
r=zeros(1,num_samples);
ep=zeros(1,num_samples);
R=[0 1; 1 0];
R1=R;
Vr=1;

W1=1;
W2=1;
W=[W1 W2]';
Wr=W;

u=0.009;
lambda=0.8;

warning off;
r(1)=W'*V(:,1);
for k=2:num_samples-1
    r(k)=W'*V(:,k);
    e(k)=h(k)-r(k)*conj(r(k-1));
    h(k+1)=h(k)-u*e(k);
    ep(k)=Vr^2-r(k)*conj(r(k));
    R1 = lambda*R + V(:,k)*V(:,k)'*r(k)*conj(r(k));
    R = R1;
    Wr = W + u*ep(k)*conj(r(k))*inv(R)*V(:,k);
    W= Wr;
    f_COCMA(k)=asin(imag(h(k))/abs(h(k)))/(2*pi*T);
end
```

## 8.2 Gantt chart for Thesis B



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