Topic Title: Frequency Estimation for Three-Phase Power Systems

Student Name: Tsai-Yen Shih **Student ID: z3451029**

A. Problem statement

When there is deviation in system frequency of a three-phase power system, the load and generation become imbalanced resulting in critical operation conditions. Therefore, to maintain and regulate fast and accurate frequency estimation is the utmost importance as it provides reliable measurements of system parameters. The existing combined complex signal from Clarke's Transform results in partial loss of information and under-performance in unbalanced conditions. In order to increase robustness for frequency estimation in unbalance, the proposed needs to improve the estimation model by evaluating it on each of the three phases. An averaging method needs to be developed to combine the estimated phase frequencies, and thus, to eliminate the effects of the noise variance from the signal.

B. Objective

Implement the Existing Algorithm of a complex signal based AR2 Model estimation
Evaluate Single-Phase Based AR2 model for each of the three phases
Design averaging methods to combine the estimated phase frequencies
Implement and show the proposed is better than the existing.

C. My solution

Single Phase AR2 Model using the sum of the previous term v(n-1) and the next term v(n+1)
Implement Single Phase AR2 Model on the estimators of BCRLS and RTLS
Averaging method 1: Mean
Averaging method 2: Averaging with Amplitude Weight Factor
Amplitude Weighted Averaging is designed to compare the estimated amplitudes and averaging frequencies by placing more weight on the main operating phases
Averaging method 3: Averaging with SNR Weight Factor (SNRW)

D. Contributions

New approach developed the frequency estimation on single phase of three-phase systems
New method focuses on main operating phases when combining frequencies
Efficiently eliminate the effects of noise variance from the signal.
Demonstrated efficacy of new method
Compared proposed and existing methods
Implement different cases in frequency tracking performance
Perform the variance estimation to verify the methods

E. Suggestions for future work

Finding a general average method that deals with all the unbalanced conditions
Implementation in real

While I may have benefited from discussion with other people, I certify that this report is entirely my own work, except where appropriately documented acknowledgements are included.

Signature: ______________________ Date: 27 / 10 / 2016

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Literature:

21	[15] R.Arblouei, K.Dogancay and S.Wemer. 2015
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THE UNIVERSITY OF
NEW SOUTH WALES

SCHOOL OF ELECTRICAL ENGINEERING
AND TELECOMMUNICATION

Frequency Estimation for Three-Phase Power Systems

by

Tsai-Yen Shih

Thesis submitted as a requirement for the degree

Bachelor of Engineering (Electrical Engineering)

Student ID: z3451029

Supervisor: Elias Aboutanios

Submitted: October 27, 2016

Abstract

In smart grid, system frequency typically operates around its nominal values of standard range (49.85 Hz to 50.15 Hz). When there is deviation, the system load and generation become imbalanced resulting in critical operation conditions. Therefore, to maintain and regulate fast and accurate frequency estimation is the utmost importance as it provides reliable measurements of system parameters. The ultimate objective of this thesis is to design a fast and accurate frequency estimation for three-phase power systems under unbalanced conditions (variations of voltage amplitudes). The combined complex signal from Clarke's Transform results in partial loss of information and under-performance in unbalanced conditions. In order to increase robustness for frequency estimation in unbalance, the proposed approach improves the AR2 model by evaluating it on each of the three phases. The amplitude weighted factor is designed to average the phase frequencies , and thus, it eliminates the effects of noise from the signal. Simulation results show that the Amplitude Weighted Average (AW) approach successfully outperforms compared to the existing methods and Bias-Compensated Recursive Least Square (BCRLS) with the AW method performs the most optimally in unbalanced conditions.

Abbreviations

AR Model Auto-Regressive Model

AW Amplitude Weighted Average

BCRLS Bias-Compensated Recursive Least Square

CRB Cramer-Rao Bound

EKF Extended Kalman Filter

LMS Least-Mean-Square

PPL Phase-Locked-Loop

RLS Recursive Least Square

RMSE Root-Mean-Square Error

RTLS Recursive Total Least Square

SNR Signal-to-Noise Ratio

SNRW Signal-to-Noise Ratio Weighted Average

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Chapter 1

Introduction

The Introduction illustrates the setting of frequency estimation and why it is an important aspect of three-phase systems to ensure the optimal outcomes. There is a brief introduction of the thesis problem statement and how it is resolved with the proposed solution.

1.1 Background

Smart grids are the most common hubs for signal processes and communication techniques to be quantified which involve collecting information on consumer behaviours and acting in accordance to the data. With its automated manner, it allows for enhanced approaches to efficiency, reliability, sustainability and consumption of energy. Smart grid is produced at a high three-phase AC voltage since it is a highly complex and fast system. A three-phase system is optimal to operate in this environment and regulates electrical power generation, transmission and distribution[1-3]. In smart grid, system frequency typically operates around its nominal values of standardized range (49.85 Hz to 50.15 Hz)[4]. If there is deviation in frequency, the system load and generation will become imbalanced, resulting in critical operation conditions, e.g. meltdown of the grid[5].

To maintain and regulate an accurate frequency estimation, is the utmost important in smart grids as it provides reliable measurements of system parameters such as; voltages, currents and active and reactive powers. In addition, frequency estimation prevents loss of synchronism for under-frequency relaying and stability in power systems. The frequency transients rapidly in the distribution network and thus, it is very difficult to track quickly and with accuracy[6-7]. Engineers, researchers and power station operators are primary affiliates in relation to the operation of fast and accurate frequency estimations.

There have been a variety of algorithms developed for the sole purpose of increasing the effectiveness of frequency estimation in three-phase power systems. Several techniques have been utilized to estimate power system frequency in accordance with Zero-Crossing technique[8], Phase-Locked Loop (PLL)[9-10], Least-Mean-Square (LMS)[12] and Extended Kalman Filter (EKF)[12]. However, they have been found to be lacklustre in their estimating ability. One reason is that these algorithms rely upon a single phase measurement and hence unreliable in three-phase systems. As a result, Clarke's Transform can be applied to map three-phase information into a single-phase complex signal[13]. Additionally, another reason is that they are mostly built on first-order autoregressive (AR1) models. AR1 models operate best in balanced three-phase systems (i.e. equal valued voltage magnitudes). However, in the case of severe unbalance (e.g. voltage amplitudes of two phases plummet to zero), AR1 based frequency estimators have reduced accuracy. Due to the incompetence of AR1 model, the improved version is the second-order autoregressive (AR2) model which the estimator built on it will be insensitive to the balanced state[14-15].

The existing method[15] utilizes recursive LS (RLS) based AR2 model and has been implemented to eliminate the effects of output noise of AR2 model. However, noise is also observed in the input of AR2 model which makes the LS-based estimation biased. In order to achieve unbiased estimation from the noisy AR2 model, a bias-compensated RLS (BCRLS) algorithm and recursive TLS (RTLS) algorithm are applied. BCRLS is used to gauge the bias and made redundant in biased RLS estimate, while RTLS observes the system parameter's estimation that matches the input to the output with smallest disturbances in the signal.

1.2 Problem Statement

1. Clarke's Transform is able to characterize the voltages of three-phase and produce a complex $\alpha\beta$ signal which can perform well in nominal operating condition (balanced state). However, in unbalanced conditions (unbalance in voltage amplitudes), the complex signal will result in non-unique solutions and affected by severe oscillatory errors. Although it can be improved by using widely-linear models, to model three-phase systems, it lacks the dimensionality and consequently results in partial loss of information and poor performance especially under unbalanced conditions.[11,16] To increase the robustness of estimation in unbalanced state, the proposed approach presented in the thesis is the single-phase based AR2 model with BCRLS and RTLS. This method allows the measurement of instantaneous frequency for each phase, hence it is more robust in measuring the frequency when the amplitudes of phase voltages sags to zero.

2. According to Problem Statement 1, the frequency estimation in unbalanced state is improved. However, the measurement of single phase noises from each phase will be increased when combining the estimated phase frequencies. In order to eliminate effects of the noise, the amplitude and SNR weight factors are applied in averaging the phase frequencies.

In this thesis, the effectiveness of the proposed algorithms examined in frequency estimation under both balanced and unbalanced conditions. In addition, the bias and variance is estimated by using Root-Mean-Square Error (RMSE) and also the tracking ability is validated in simulated experiments. From the simulations, the proposed algorithms in comparison to the Clarke's Transform approach perform with better accuracy during voltage sags.

1.3 Thesis Outline

This thesis is structured as follows. In Chapter 2, the theories and previous developments in relation to frequency estimation for three-phase systems are presented in detail. Chapter 3 includes the thesis aims in steps for improving the accuracy of estimation. Chapter 4 explains the implementation of the existed algorithms – Clarke's Transform based AR2 frequency estimations, in order to be used as comparison in subsequent sections. Chapter 5 reveals the proposed algorithms for a single-phase based AR2 model with frequency estimators and provides approaches to average the estimated phase frequencies. Chapter 6 evaluates the results and the discussion of the variations in phase-voltage amplitudes (unbalance), performance in bias simulation and tracking ability. Chapter 7 provides concluding statements.

Chapter 2

Theories and Literature Reviews

The background reviews are illustrated and evaluated in this chapter. The purpose of it is to establish the foundations of frequency estimation and how previous develops culminates to the proposed.

2.1 Signal Model

Since the input signal is an AC three-phase system, the instantaneous voltages of each phase at time instant n are represented by

$$\begin{aligned} v_a(n) &= V_a(n)\cos(2\pi f\tau n + \theta) + N_a(n); \\ v_b(n) &= V_b(n)\cos(2\pi f\tau n + \theta - \frac{2\pi}{3}) + N_b(n); \\ v_c(n) &= V_c(n)\cos(2\pi f\tau n + \theta + \frac{2\pi}{3}) + N_c(n). \end{aligned} \tag{2.1}$$

where $V_a(n)$, $V_b(n)$, and $V_c(n)$ indicate the amplitudes of the three-phase voltages at time instant n, f is the system frequency, $\tau = 1/f_s$ is the sampling interval with f_s indicating the sampling frequency and θ is an initial phase angle.

$N_a(n)$, $N_b(n)$ and $N_c(n)$ indicate the additive White Gaussian Noises of each phase. We assume these noises are zero-mean i.i.d. with variance σ_a^2, σ_b^2 and σ_c^2.
Thus, the noise signals can be expressed as

$$N_a(n) = \sigma_a \times rand.noise;$$
$$N_b(n) = \sigma_b \times rand.noise;$$
$$N_c(n) = \sigma_c \times rand.noise. \tag{2.2}$$

2.2 Two Ways of Frequency Estimation

Research has found that there are two ways of estimating for frequency in three-phase systems. Firstly, to estimate frequency from a combined signal as demonstrated in Figure 2.1 or secondly, to estimate each phase frequency and combine them to achieve a final estimated frequency as shown in Figure 2.2.

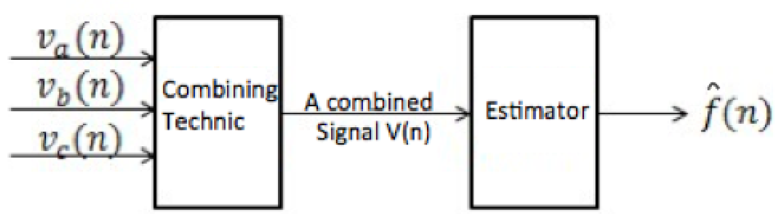

Figure 2.1: Frequency Estimation from a Combined Signal

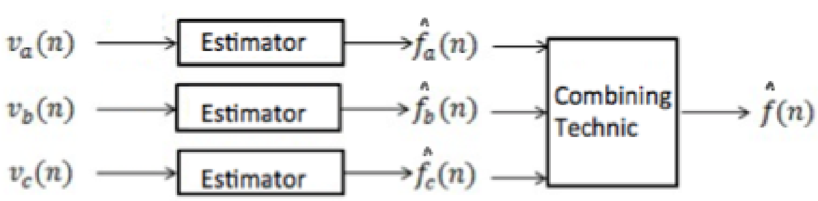

Figure 2.2: Frequency Estimation based on Each Phase

To estimate the frequency as a whole, the use of the combination technique will yield three phase voltages into a complex signal. After combination, frequency estimators for single-phase can be implemented on the complex signal. However, accuracy of the estimation for combined signal could lose some information from the original signals.

An alternative to combination is to estimate the frequencies separately for each phase voltage in three-phase systems and combine the estimated phase frequencies to a final frequency. This will prevent loss of information in the signals. However, this method of frequency estimation is not commonly explored in past literatures. The reason for this is because previous research mainly focused on improvement of accuracy on the combined signal.

2.3 Signal Combination Methods

As mention in the above section (Section 2.2), the combination methods are commonly applied for frequency estimation in three-phase systems. Clarke's Transform is the most widely used in previous research and the new approach is Quaternion.

2.3.1 Clarke's Transform

With the premise that none of the single phases can accurately model the entire three-phase system and its elements. In order for accurate frequency estimation to occur, a robust estimator needs to be implemented. The use of Clarke's transformation in estimation of three phase systems reinforces the single-phase methods with more robustness by computing using information provided by the phase voltages. It produces a complex-valued signal ($\alpha\beta$ signal) and it encompasses the information of all three phases.

$$\begin{bmatrix} v_{\alpha(n)} \\ v_{\beta(n)} \end{bmatrix} + \begin{bmatrix} n_{\alpha(n)} \\ n_{\beta(n)} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \left(\begin{bmatrix} v_a(n) \\ v_b(n) \\ v_c(n) \end{bmatrix} + \begin{bmatrix} n_a(n) \\ n_b(n) \\ n_c(n) \end{bmatrix} \right) \tag{2.3}$$

To model three-phase systems, the two-dimensional $\alpha\beta$ signal lacks the dimensionality which will cause a partial loss of information and negative consequences for the execution, especially under unbalanced operations.[16]

2.3.2 Quaternion

Quaternion is an associative, non-commutative, division algebra and is a relatively new method of creating a new framework for evaluating 3 and 4 dimensional signals. A Quaternion variable q includes a real part and a three-dimensional imaginary part or pure quaternion.

$$\begin{aligned} q(n) &= \Re(q(n)) + \Im(q(n)) \\ &= q_r(n) + iq_i(n) + jq_j(n) + kq_k(n) \end{aligned} \tag{2.4}$$

where q_r, q_i, q_j, $q_k \in \mathbb{R}$

For real time frequency estimation, quaternion method is used extensively. The multi-faceted essence of quaternion can fully define a three-phase power system. The application of quaternion is to develop a unified framework for combining and modelling the voltage measurement on all three phases of the system. The voltages from three phases can form together and generate a pure Quaternion signal:

$$q(n) = iv_a(n) + jv_b(n) + kv_c(n) \tag{2.5}$$

All components in the equation have the same frequency, hence systematic geometry requires an ellipse to be tracked by $q(n)$ in the subspace of the three-dimensional imaginary subspace. The voltage of both balanced and unbalanced states in three-phase systems are represented in Figure 2.3. The blue ellipse represents the voltage for balanced system and the red ellipse is for an unbalanced system.[16-17]

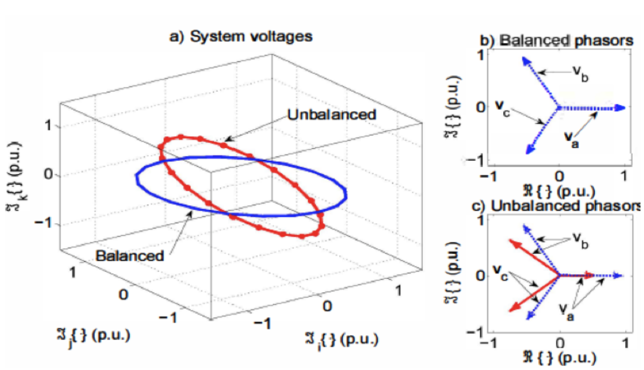

Figure 2.3: Geometric version of system voltage for Quaternion in three-phase system under balanced and unbalanced conditions [16]

2.4 AR Model

AutoRegressive (AR) models are the go-to methods when it comes to estimating system frequency without needing to worry about noise in signals. AutoRegressive first-order (AR1) and second-order (AR2) are such linear predictive models. AR1 model is more applicable in balanced three-phase systems – when phase voltage amplitudes are equal. The model characterizes two successive noiseless signal samples in a balanced system through a single complex parameter which the magnitude is equivalent to 'unity' and the phase angle is given by the system angular frequency ω aggregated by the sampling interval τ. Through the method of widely-linear AR1, frequency estimation is possible to establish in unbalanced. However, this complex-valued technique is still unable to deal with highly unbalanced conditions (e.g. two phases decline to zero) as this lies with the approximate nature of the model[15].

Alternatively, AR2 characterizes three consecutive noiseless signal samples through a real-valued parameter h. The parameter is equivalent to the cosine of the multiples of system angular frequency ω and sampling interval τ. Subsequently, system frequency is evaluated by establishing the AR2 parameter h from the noisy information of the signal and being unaffected by the phase voltage amplitudes or the initial phase angle. AR2 frequency estimation will not be influenced by the balance conditions of three-phase power system since the parameter h only relies on system frequency f and sampling interval τ.

2.5 Frequency Estimators

According to recent literatures, the relatively common and efficient estimators are Extended Kalman Filter (EKF) and Recursive Least Square (RLS) and they are explored in this section in relation to frequency estimation.

2.5.1 Recursive Least Square Algorithm

Least Mean Square (LMS) is an adaptive filter where the filter values are adjusted through the reducing the squared of the error signal to optimal values. LMS is used widely in digital signal processing due to its simple structure, efficiency and robustness computation. However, it has been found to contain the inherent drawbacks of having slow convergence speed and also, it requires to reprocess every time.[11,18] (Figure 2.4)

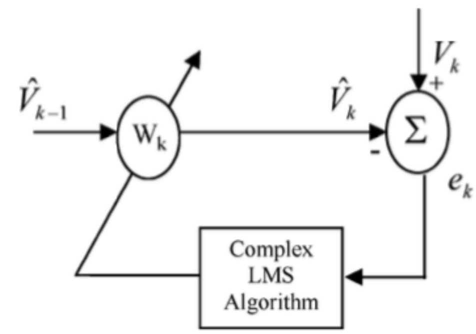

Figure 2.4: LMS Technique [11]

The improved version of LMS is Recursive Least Square (RLS). The algorithm recursively observes the coefficients of reducing the weighted linear LMS cost function of the input signals. In comparison to LMS, RLS algorithm consists of fast convergence rate and is robust and performs well in online estimation. However, its rapid convergence rate comes with high complex computation.[19] The RLS approach is also applicable in minimizing the effects of output noise from a linear model (e.g AR2 model).

An exponentially-weighted LS estimate is obtained by

$$w(n) = \underset{\mathrm{w}}{\operatorname{argmin}} \|y(n) - wx(n)\|^2 \tag{2.6}$$

where $w(n)$ is a parameter which contains frequency information. $x(n)$ and $y(n)$ represent the exponentially-weighted input and output.

The optimization problem of (eqn 2.6) has the recursive solution – RLS estimate:[15]

$$w(n) = \frac{p(n)}{r(n)} \tag{2.7}$$

where $p(n)$ defines the exponentially-weighted time-averaged covariance of the signal input and output:

$$p(n) = x(n)^H y(n) \tag{2.8}$$

and $r(n)$ defines the exponentially-weighted time-averaged variance of the signal input:

$$r(n) = x(n)^H x(n) \tag{2.9}$$

$p(n)$ and $r(n)$ are recursively updated, thus $w(n)$ is a recursive estimate.

2.5.2 Extended Kalman Filter

To understand Extended Kalman Filter (EKF), the basics of Kalman filter will need to be explained first. Figure 2.5 illustrates the Kalman filter. Kalman Filter is a set of recursive equation which update the information in a state space model. The algorithm operates in two stages; in the prediction stage, estimates for the current state variables and the uncertainties are provided by the Kalman Filter. Afterwards, the result of the following measurement is produced, and updated by applying a weighted average. Higher certainty measurements are demonstrated by more weighting for the estimates.

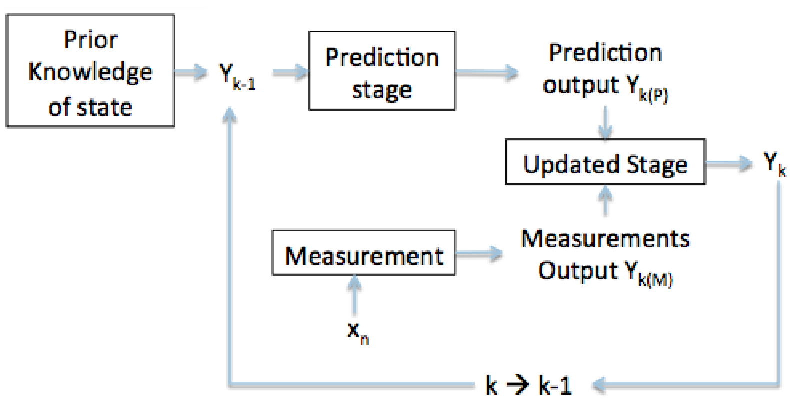

Figure 2.5: Kalman Filter

Kalman filter was governed by non-linear functions, whereas the more advanced version are optimized for linear filters. The Extended Kalman filter is the non-linear version of the Kalman filter, it provides linearity about the current mean and covariance. It is used mainly in three-phase nonlinear systems and works by linearizing the predictions and measurements about their mean.[12,20]

2.6 Estimation Variance

This section outlines Root-Mean-Square error (RMSE) and Cramer-Rao Bound (CRB) in relation to validating the estimators.

2.6.1 Root-Mean-Square-Error

Root-Mean-Square Error (RMSE) evaluates the disparity between the expected value and the measured value. It is used for variance estimation after running the algorithms many times against signal-to-noise ratio (SNR). In this thesis, SNR is in the range of 0 to 60 dB. The experiment will run on 5 dB intervals and each interval will run for 2000 times. The number of runs is validated by; there are too many runs then there will be operational error or too little will lead to inaccuracy of RMSE. The expectations are evaluated with the total average over a substantial number of independent trials.

RMSE is defined as below:

$$RMSE = \lim_{n\to\infty} \sqrt{E[(\hat{f}(n) - f(n))^2]} \tag{2.10}$$

2.6.2 Cramer-Rao Bound

Cramer-Rao Bound (CRB) is the lower limit on the variance which used to compare with the performance of the estimator. The estimator is good if the variance execution is close to the CRB. In this thesis, CRB is used to compare with the RMSE values to determine the validation of the estimators.

By specifying $\eta = [\phi, \omega, a]^T$, a signal can be represented as[21]

$$v = x(\eta) + \epsilon = B(\omega, \phi)a + \epsilon \tag{2.11}$$

The CRB matrix is given by

$$CRB(\eta) = \Big(\frac{3}{2\sigma^2}\mathbb{R}\Big[\frac{\partial x^H(\eta)}{\partial \eta}\frac{\partial x(\eta)}{\partial \eta^T}\Big]\Big)^{-1} \leq RMSE(\eta) \tag{2.12}$$

Chapter 3

Aims

The ultimate objective for this thesis is to design and produce a refined frequency estimation algorithm that is fast and accurate for a three-phase power systems under unbalanced conditions – variations of phase voltage amplitudes. Within the objective, there are the step aims for comparing existing method with the proposed and implementation in a progressive manner.

3.1 Step Aims

1. Implementation of existing algorithms of Clarke's Transform based AR2 model with BCRLS and RTLS estimators.
2. To derive the representation for single-phase based AR2 model.
3. Design optimal weighted average methods in order to obtain an accurate final estimated frequency.
4. To simulate the proposed algorithms under unbalanced conditions based on amplitudes of phase voltage and compare with existing algorithms. To demonstrate the proposed outperforms the existing methods.

Chapter 4

Existing Algorithms

The Clarke's Transform based AR2 model for three-phase systems is observed with noise from both input and output. The recursive least-squares (RLS) is able to eliminate the noise for the output. By applying bias compensated RLS (BCRLS), the estimate bias can be evaluated with a priori knowledge of noise variance σ^2 and subtracted from the RLS estimate. RTLS, Recursive Least-Squares, is another approach to eliminate the noise from the input and output of the AR2 model by implementing the inverse power method. [15] (Figure 4.1)

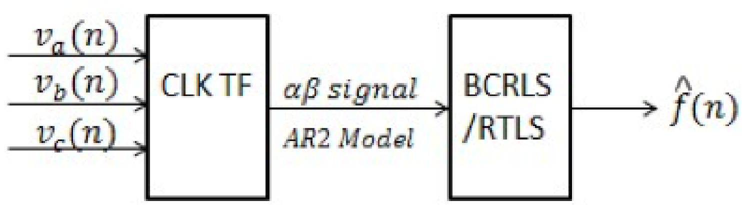

Figure 4.1: Clarke's Transform based AR2 Model with BCRLS/RTLS Estimation

4.1 Clarke's Transform based AR2 Model

According to Section 2.3.1, Clarke's transform is applied to obtain the noisy $\alpha\beta$ signal:

$$v(n) = v_0(n) + N(n) \tag{4.1}$$

where $v_0(n)$ indicates the noiseless $\alpha\beta$ signal,

$$\begin{aligned} v_0(n) &= v_\alpha(n) + jv_\beta(n) \\ &= (A + jB)\cos(2\pi f\tau n + \theta) + (B + jC)\sin(2\pi f\tau n + \theta). \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} A &= \sqrt{\frac{2}{3}}V_a + \frac{1}{2\sqrt{6}}(V_b + V_c); \\ B &= -\frac{1}{2\sqrt{2}}(V_b - V_c); \\ C &= \frac{1}{2}\sqrt{\frac{3}{2}}(V_b + V_c). \end{aligned}$$

and $N(n)$ is the additive complex noise with the noise variance of σ^2 which can be expressed as:

$$\begin{aligned} N(n) &= \sigma \times rand.noise \\ &= N_\alpha(n) + jN_\beta(n) \\ &= \sqrt{\frac{2}{3}}(N_a(n) - \frac{N_b(n)}{2} - \frac{N_c(n)}{2}) + \frac{j}{\sqrt{2}}(N_b(n) - N_c(n)) \end{aligned}$$

Since the noise is assumed to be zero mean i.i.d, the noise variance σ can be calculated as

$$\begin{aligned} \sigma &= E[|N(n)|^2] \\ &= E[\frac{2}{3}(N_a(n) - \frac{N_b(n)}{2} - \frac{N_c(n)}{2})^2 + \frac{1}{2}(N_b(n) - N_c(n))^2] \\ &= \frac{2}{3}(\sigma_a^2 + \frac{\sigma_b^2}{4} + \frac{\sigma_c^2}{4}) + \frac{1}{2}(\sigma_b^2 + \sigma_c^2) \\ &= \frac{2}{3}(\sigma_a^2 + \sigma_b^2 + \sigma_c^2) \end{aligned}$$

Hence, the variance for each phase can be obtained as

$$\implies \sigma_a^2 = \sigma_b^2 = \sigma_c^2 = \frac{\sigma^2}{2} \tag{4.3}$$

The noiseless complex signal can be expressed as an AR2 linear predictive model:

$$\frac{1}{2}(v(n-2) + v(n)) = hv(n-1) \tag{4.4}$$

where

$$h = \cos(2\pi f \tau) \tag{4.5}$$

Due to noisy AR2 model, in order for achieving an accurate parameter h, **a linear estimation technique** ought to be applied. The system frequency $\hat{f}(n)$ can be estimated by evaluating the estimate of h at time instant n, $\hat{h}(n)$:

$$\hat{f}(n) = \frac{1}{2\pi\tau} \cos^{-1}(\hat{h}(n)) \tag{4.6}$$

4.2 Recursive Least Squares Frequency Estimators

As discussed in Section 2.5.1, Recursive Least Square (RLS) algorithm is implemented in order to removing the output noise of the noisy AR2 model. It still produces a biased estimate due to the effect of input noise from the signal. As a result, there are two linear estimators based on RLS algorithm – BCRLS and RTLS, which are able to eliminate the estimation bias.

4.2.1 Bias-Compensated Recursive Least Squares

Based on the AR2 Model (Eqn 4.4), an exponentially-weighted LS estimate of $\hat{h}(n)$ is obtained by:

$$w(n) = \underset{\mathrm{w}}{\operatorname{argmin}} \|y(n) - wx(n)\|^2 \tag{4.7}$$

where

$$\begin{aligned} x(n) &= \sqrt{\lambda} v(n-1) \\ y(n) &= \frac{\sqrt{\lambda}}{2}(v(n-2) + v(n)) \end{aligned}$$

$x(n)$ and $y(n)$ represent the exponentially-weighted input and output and λ
$(0 \ll \lambda < 1)$ is the forgetting factor.
The exponentially-weighted time-averaged variance of the input $r(n)$ is evaluated as:

$$\begin{aligned} r(n) &= x^H(n)x(n) \\ &= \lambda r(n-1) + |v(n-1)|^2 \end{aligned} \tag{4.8}$$

The exponentially-weighted time-averaged covariance of the input and output $p(n)$ is evaluated as:

$$\begin{aligned} p(n) &= x^H(n)y(n) \\ &= \lambda p(n-1) + \frac{1}{2}v^*(n-1)(v(n-2) + v(n)) \end{aligned} \tag{4.9}$$

Therefore, the RLS estimate solution is:

$$w(n) = \frac{p(n)}{r(n)}$$

Bias-compensated RLS estimate (BCRLS) is derived by deducting estimated bias in LS b from the RLS estimation $w(n)$ with the need of the prior knowledge value of the noise variance σ^2:

$$\begin{aligned}\hat{w}(n) &= w(n) - b \\ &= \frac{p(n)}{r(n)} + \frac{\sigma^2}{(1-\lambda)r(n)}\hat{w}(n-1)\end{aligned} \tag{4.10}$$

Table 4.1: Frequency Estimation – BCRLS Algorithm

Initialization:
$r(0) = r(1) = 0$
$p(0) = p(1) = 0$
$\hat{\omega}(0) = \hat{\omega}(1) = 0$
for $n = 2, 3 \ldots$
$r(n) = \lambda r(n-1) + \lvert v(n-1)\rvert^2$
$p(n) = \lambda p(n-1) + \frac{1}{2}v^*(n-1)(v(n-2) + v(n))$
$\hat{\omega}(n) = \frac{p(n)}{r(n)} + \frac{\sigma^2}{(1-\lambda)r(n)}\hat{\omega}(n-1)$
$\hat{f}(n) = \frac{1}{2\pi\tau}\cos^{-1}(\hat{\omega}(n))$

4.2.2 Recursive Total Least Squares

RTLS is the combination of RLS and TLS. This algorithm consists of both qualities of fast convergence and good accuracy.

TLS estimate $\omega(n)$ (LS estimate of $\hat{h}(n)$) matches the input $x(n)$ to the output $y(n)$ with the least disturbances $\varepsilon(n)$ and $\delta(n)$. It is expressed as

$$(x(n) + \varepsilon(n))w(n) = y(n) + \delta(n)$$

By applying the singular value decomposition of the augment $z(n)$ and the weighted data matrix $[x(n), y(n)]T$, the TLS can be obtained as:

$$w(n) = -\frac{z_1(n)}{\gamma z_2(n)} \tag{4.11}$$

$z(n) = [z_1(n), z_2(n)]^T$ indicates the right singular vector relating to the lowest singular value of $[x(n), y(n)]T$, where T is the weight matrix of the differences in the noise variance at the input versus output.

$z(n)$ can be written as its equation of an eigenvector:

$$\begin{aligned}\Psi(n) &= T \begin{bmatrix} x^H(n) \\ y^H(n) \end{bmatrix} [x(n), y(n)]T \\ &= \begin{bmatrix} r(n) & \sqrt{2}p(n) \\ \sqrt{2}p^*(n) & 2s(n) \end{bmatrix}\end{aligned}$$

where p(n) and r(n) are provided by (Eqn 4.8) and (Eqn 4.9) respectively. $s(n)$ represents the exponentially-weighted time-averaged variance of the output and is evaluated as

$$\begin{aligned}s(n) &= y^H(n)y(n) \\ &= \lambda s(n-1) + \frac{1}{4}|v(n-2) + v(n)|^2\end{aligned} \tag{4.12}$$

The eigen-decomposition of $\Psi(n)$ provides $z(n)$. In each time instant, $z(n)$ is updated by performing the inverse power method with a single iteration – observing an eigenvector by knowing an approximation to its eigenvalue. As the eigenvector belongs to the smallest eigenvalue, the approximation to the eigenvalue is given by zero. Hence, the recursion for $z(n)$ through the inverse power method – Recursive TLS (RTLS) is evaluated as:

$$z(n) = \Psi(n)^{-1} z(n-1) \tag{4.13}$$

By multiplying both sides with $\Psi(n)/(\sqrt{2} z_2(n-1) z_2(n))$:

$$\Psi(n) \begin{bmatrix} w(n) \\ -1/\sqrt{2} \end{bmatrix} = \frac{z_2(n-1)}{z_2(n)} \begin{bmatrix} w(n-1) \\ -1/\sqrt{2} \end{bmatrix}$$

$$\implies w(n) = \frac{p(n) + 2s(n)w(n-1)}{r(n) + 2p^*(n)w(n-1)} \tag{4.14}$$

Table 4.2: Frequency Estimation – RTLS Algorithm

Initialization
$r(0) = r(1) = 0$
$s(0) = s(1) = 0$
$p(0) = p(1) = 0$
$w(0) = w(1) = 0$
for $n = 2, 3, \ldots$
$r(n) = \lambda r(n-1) + \lvert v(n-1)\rvert^2$
$p(n) = \lambda p(n-1) + \frac{1}{(}2) v^*(n-1)(v(n-2) + v(n))$
$s(n) = \lambda s(n-1) + \frac{1}{4} \lvert v(n-2) + v(n)\rvert^2$
$w(n) = \frac{p(n)+2s(n)w(n-1)}{r(n)+2p^*(n)w(n-1)}$
$\hat{f}(n) = \frac{1}{2\pi\tau} cos^{-1}(w(n))$

4.3 Simulated Results

In the following simulation performance, BCRLS and RTLS estimations on a Clarke's Transform based AR2 model are compared. In this three-phase power system, $f = 50Hz$, $\tau = 2ms$, $f_s = 500Hz$, voltage amplitudes of three phases at balanced condition $V_a(n) = V_b(n) = V_c(n) = 230V$, $\theta = 0$, $\sigma^2 = 0.01$, thus, $\sigma_a^2 = \sigma_b^2 = \sigma_c^2 = \sigma^2/2$ and $\lambda = 0.999$.

The phase voltages experience progressive drops during the simulation to reach severe unbalanced conditions as shown in Figure 4.2. During the initial 0.25 seconds, the system is balanced. The voltage amplitude of Phase C reduces to half at 0.25 seconds. At 0.5 seconds, voltage of Phase A drops to 0 (one phase voltage = 0). This is when serious unbalance occurs. At 0.75 seconds, voltage of Phase B declines to 0 (voltages of two phases = 0).

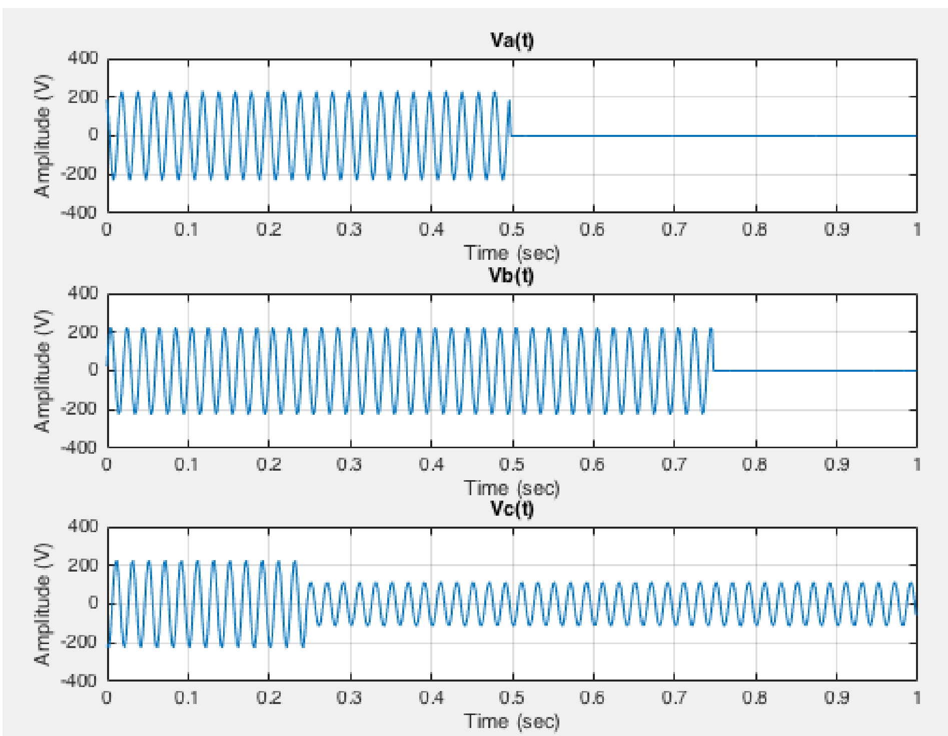

Figure 4.2: Several voltage sags occur in a three-phase system

Figure 4.3 shows the performance of $\alpha\beta$ signal based AR2 model with the algorithms of BCRLS and RTLS which undergoes the voltage sags. In Table 4.3, the estimated frequencies at different conditions are listed.

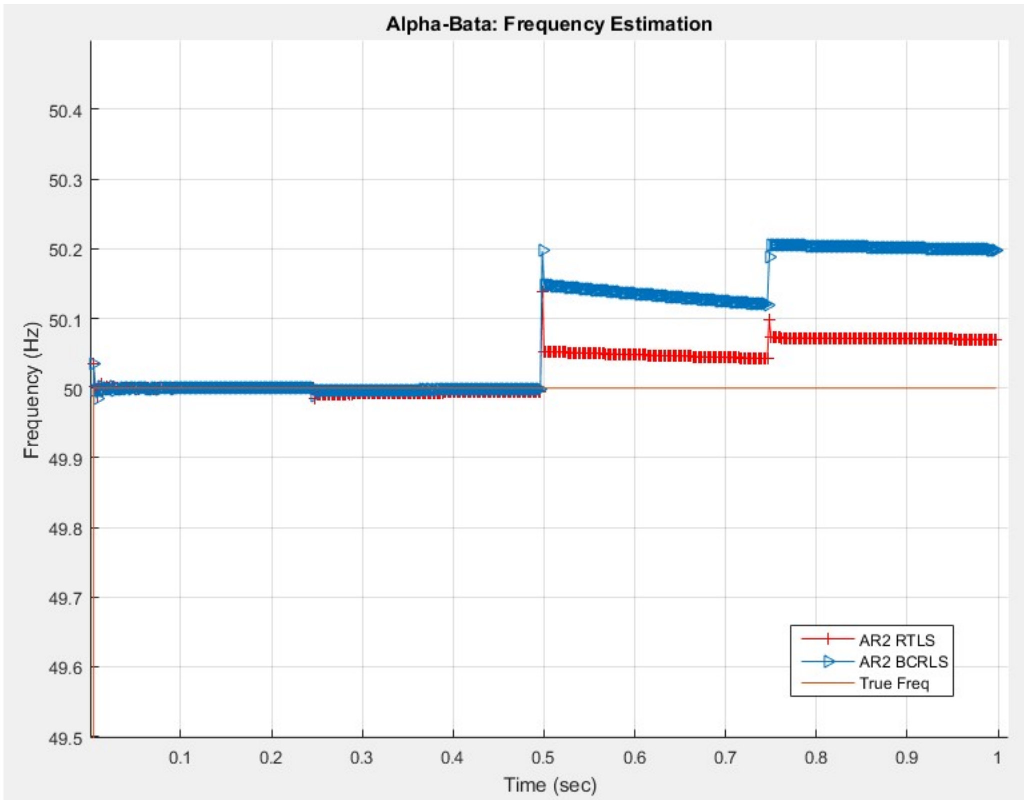

Figure 4.3: Frequency Estimation - $\alpha\beta$ signal based AR2 with BCRLS/RTLS

Time (sec)	0.15 (Balanced)	0.25 (1ph$\rightarrow$ 50%)	0.5 (1ph$\rightarrow$0)	0.75 (2phs$\rightarrow$0)
BCRLS(Hz)	50	50	50.15	50.21
RTLS(Hz)	50	49.99	50.05	50.07

Table 4.3: Estimated Frequencies at different voltage sags

4.4 Discussion and Summary

According to the simulation results, both algorithms – BCRLS and RTLS converge fast and stable with accuracy under balanced condition. When the system is suffering from severe unbalancedness, BCRLS and RTLS are able to estimate the frequency in a stable manner. It can be observed that RTLS is more efficacious under highly unbalanced cases. However, the accuracy of frequency estimations still need improvement in order to reach the level close to the real frequency (50 Hz), especially under unbalance.

Chapter 5

Proposed Algorithm

The measurements with the complex signal from Clarke's Transform perform exceptionally in balanced three-phase systems. However, this complex signal can be considered as an element that causes the inaccuracy of the frequency estimation during unbalanced, since it lacks the dimensionality for a three-phase system and results in partial loss of information. In order to increase the robustness of estimation techniques under unbalance, the proposed approach in Figure 5.1 improves the AR2 model by evaluating it on each of the three phases. This allows the estimate of the instantaneous frequency for each phase. To combine the estimated phase frequencies, we need to design a suitable averaging method that can eliminate noise effects when a three-phase system experiences voltage sags.

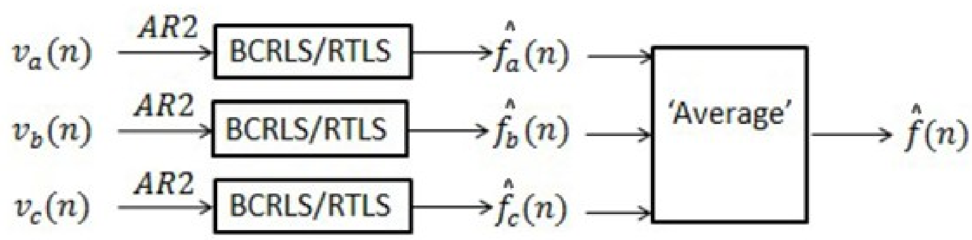

Figure 5.1: Applying AR2 with BCRLS/RTLS on each phase signal and Average

5.1 Single-Phase based AR2 Frequency Estimation

AR2 model is able to be built on one arbitrary noiseless phase of a three-phase system. The computations are shown below.

For phase A at time instant n, $n+1$ and $n-1$:

$$\begin{aligned}
v_a(n) &= V_a(n)\cos(2\pi f\tau n+\theta) \\
v_a(n+1) &= V_a(n)\cos(2\pi f\tau n+\theta+2\pi f\tau) \\
&= V_a(n)\cos(2\pi f\tau n+\theta)\cos(2\pi f\tau) - V_a(n)\sin(2\pi f\tau n+\theta)\sin(2\pi f\tau) \\
v_a(n-1) &= V_a(n)\cos(2\pi f\tau n+\theta-2\pi f\tau) \\
&= V_a(n)\cos(2\pi f\tau n+\theta)\cos(2\pi f\tau) + V_a(n)\sin(2\pi f\tau n+\theta)\sin(2\pi f\tau)
\end{aligned}$$

By summing above $v_a(n+1)$ with $v_a(n-1)$:

$$\begin{aligned}
v_a(n+1)+v_a(n-1) &= 2V_a(n)\cos(2\pi f\tau n+\theta)\cos(2\pi f\tau) \\
&= 2\cos(2\pi f\tau)v_a(n); \qquad (5.1)
\end{aligned}$$

Similarly, phase B and phase C can be calculated as

$$v_b(n+1)+v_b(n-1) = 2\cos(2\pi f\tau)v_b(n); \qquad (5.2)$$

$$v_c(n+1)+v_c(n-1) = 2\cos(2\pi f\tau)v_c(n). \qquad (5.3)$$

Rearrange the AR2 models (Eqn 5.1),(Eqn 5.2) and (Eqn 5.3), we can obtain:

$$\begin{aligned}
\frac{1}{2}(v_a(n-2)+v_a(n)) &= hv_a(n-1); \\
\frac{1}{2}(v_b(n-2)+v_b(n)) &= hv_b(n-1); \\
\frac{1}{2}(v_c(n-2)+v_c(n)) &= hv_c(n-1). \qquad (5.4)
\end{aligned}$$

where

$$h = \cos(2\pi f\tau)$$

Therefore, similar to (Eqn 4.6), the phase frequencies can be estimated separately via a **linear estimation technique (Section 4.2: BCRLS/RTLS)**,

$$\begin{aligned}
\hat{f}_a(n) &= \frac{1}{2\pi\tau}\cos^{-1}(\hat{h}_a(n)); \\
\hat{f}_b(n) &= \frac{1}{2\pi\tau}\cos^{-1}(\hat{h}_b(n)); \\
\hat{f}_c(n) &= \frac{1}{2\pi\tau}\cos^{-1}(\hat{h}_c(n)). \qquad (5.5)
\end{aligned}$$

5.2 Average of Estimated Frequencies

An averaging tool is needed to obtain the final estimated frequency for the three-phase system since none of the single phase can represent the entire system.

In order to achieve an accurate measurement, the following three cases of averaging are designed.

5.2.1 Mean

This is a simple approach of averaging the estimated frequencies by their mean:

$$\hat{f}(n) = \sum_{x=a,b,c} \frac{\hat{f}_x(n)}{3} \qquad (5.6)$$

where $\hat{f}(n)$, the final estimated frequency is obtained by dividing the sum of the three estimated phase frequency $\hat{f}_a(n)$, $\hat{f}_b(n)$ and $\hat{f}_c(n)$ by the total number of 3.

However, this method will enlarge the effects of noise variance from each of the three phases under unbalanced conditions. Therefore, this averaging approach may not be an improvement on the accuracy for the frequency estimation.

5.2.2 Apply Amplitude Weight Factor (AW Method)

To reduce the effects of signal noise during voltage sags, the amplitude weight factor is designed when averaging the estimated frequencies. It is an efficient way of averaging frequencies by placing more weight on the main operating phases.

There are three steps to reach the final estimated instantaneous frequency for the system:

Step 1. Amplitudes Estimation:

According to [22], by rearranging the signal model, the signal in vector notation can be expressed as

$$v(n) = a(n)Z(\hat{f}(n)) + N(n) \tag{5.7}$$

where $a(n)$ is the amplitude vector, $Z(\hat{f}(n))$ is $\cos(2\pi f\tau n + \theta + \phi)$ and $N(n)$ is the signal noise.

By solving the LS problem in (Eqn 5.7), the amplitude vector $\hat{a}(n)$ can be estimated as

$$\begin{aligned} \hat{a}(n) &= \underset{\mathrm{a}}{\operatorname{argmin}} \|v(n) - Z(\hat{f}(n))a(n)\| \\ &= [Z^H(\hat{f}(n))Z(\hat{f}(n))]^{-1}Z^H(\hat{f}(n))v(n) \end{aligned} \tag{5.8}$$

Therefore, the estimated amplitudes of three phases $\hat{a}_a(n)$, $\hat{a}_b(n)$ and $\hat{a}_c(n)$ can be expressed as:

$$\begin{aligned} \hat{a}_a(n) &= [Z_a^H(\hat{f}_a(n))Z(\hat{f}_a(n))]^{-1}Z_a^H(\hat{f}_a(n))v_a(n); \\ \hat{a}_b(n) &= [Z_b^H(\hat{f}_b(n))Z(\hat{f}_b(n))]^{-1}Z_b^H(\hat{f}_b(n))v_b(n); \\ \hat{a}_c(n) &= [Z_c^H(\hat{f}_c(n))Z(\hat{f}_c(n))]^{-1}Z_c^H(\hat{f}_c(n))v_c(n). \end{aligned} \tag{5.9}$$

Step 2. Amplitude Weight Factors $W_a(n)$, $W_b(n)$, $W_c(n)$:

Compare the estimated amplitudes $\hat{a}_a(n)$, $\hat{a}_b(n)$ and $\hat{a}_c(n)$, the largest amplitude will have the weight value of 1 and weight factors for the other two will be $\hat{a}_x(n)/\hat{a}_{largest}(n)$

For instance, if $\hat{a}_a(1)$ is the largest among the three estimated amplitudes,

$$\text{weight factor of Phase A: } W_a(1) = 1$$
$$\text{weight factor of Phase B: } W_b(1) = \frac{\hat{a}_b(1)}{\hat{a}_a(1)}$$
$$\text{weight factor of Phase C: } W_c(1) = \frac{\hat{a}_c(1)}{\hat{a}_a(1)}$$

Step 3. Frequencies Average with the Amplitude Weighted Factors:

The averaged frequency $\hat{f}(n)$ at each time instant n by using the amplitude weight factors can be obtained as:

$$\hat{f}(n) = \frac{W_a(n)\hat{f}_a(n) + W_b(n)\hat{f}_b(n) + W_c(n)\hat{f}_c(n)}{W_a(n) + W_b(n) + W_c(n)} \tag{5.10}$$

5.2.3 Apply SNR Weight Factor (SNRW Method)

SNR Weighted Averaging (SNRW) is a similar method to Amplitude Weighted Averaging (AW) (Section 5.2.2). There are four steps to approach the final estimated frequency:

Step 1. Noiseless Signals:

$$
\begin{aligned}
v_{a0}(n) &= \hat{a}_a(n) Z(\hat{f}_a(n)); \\
v_{b0}(n) &= \hat{a}_b(n) Z(\hat{f}_b(n)); \\
v_{c0}(n) &= \hat{a}_c(n) Z(\hat{f}_c(n)).
\end{aligned} \tag{5.11}
$$

$v_{a0}(n)$, $v_{b0}(n)$ and $v_{c0}(n)$ are the pure signals, where $\hat{a}(n)$ is the estimated amplitude and $Z(\hat{f}(n)) = \cos(2\pi f \tau n + \theta + \phi)$.

Step 2. Noise Estimation:

Subtracting (Eqn.5.11) from the noisy signals (input signals), we can obtain the estimated noise signals $\hat{n}(n)$:

$$
\begin{aligned}
\hat{n}_a(n) &= v_a(n) - v_{a0}(n); \\
\hat{n}_b(n) &= v_b(n) - v_{b0}(n); \\
\hat{n}_c(n) &= v_c(n) - v_{c0}(n).
\end{aligned} \tag{5.12}
$$

Step 3. SNR Estimation:

Since $SNR = 10\log_{10}(Amplitude^2/Noise^2)$, the estimated SNR for each phase can be expressed as,

$$
\begin{aligned}
SNR_a(n) &= 10 \times log_{10}(\frac{\hat{a}_a^2(n)}{\hat{n}_a^2(n)}); \\
SNR_b(n) &= 10 \times log_{10}(\frac{\hat{a}_b^2(n)}{\hat{n}_b^2(n)}); \\
SNR_c(n) &= 10 \times log_{10}(\frac{\hat{a}_c^2(n)}{\hat{n}_c^2(n)}).
\end{aligned} \tag{5.13}
$$

Step 4. Frequencies average with SNR Weight Factors $Wsnr_a(n)$, $Wsnr_b(n)$, $Wsnr_c(n)$

Similarly in Section 5.2.2, by comparing the estimated SNR: $SNR_a(n)$, $SNR_b(n)$ and $SNR_c(n)$, the largest SNR will have the weight value of 1 and weight values for the other two will be $SNR_x(n)/SNR_{largest}(n)$

Therefore,the averaged frequency by applying the SNR weight factors can be evaluated as:

$$\hat{f}(n) = \frac{Wsnr_a(n)\hat{f}_a(n) + Wsnr_b(n)\hat{f}_b(n) + Wsnr_c(n)\hat{f}_c(n)}{Wsnr_a(n) + Wsnr_b(n) + Wsnr_c(n)} \tag{5.14}$$

Chapter 6

Evaluation

In this chapter, the performance of frequency estimation is presented to discuss Single phased based AR2 model frequency estimation with Amplitude Weighted Averaging (AW) method and SNR Weighted Averaging (SNRW) method compare with the existing estimation methods by using BCRLS and RTLS. In addition, the simulation of estimators tracking abilities in several cases and variance estimation are used to examine and validate the proposed averaging methods. The weighted averaging methods are found to perform better in unbalanced conditions compared to the existing methods – Clarke's transform AR2 model with BCRLS and RTLS. Overall performance, it is found AW method performs outstanding estimation in frequency.

6.1 Simulated Results

6.1.1 Performance of Voltage Sags

In this simulation, the set-up is based on Section 4.3 as shown in Table 6.1, so that the results can be compared under the same environment. The simulation code can be found in Appendix A.

f	τ	f_s	θ	λ	σ^2	σ_x^2	$V_x(n)$ (balanced)
50Hz	2ms	500Hz	0	0.999	0.01	0.005	230V

Table 6.1: Simulation Set-up ($x = a, b, c$)

The amplitudes of the system phase voltages drop every 0.25 seconds during the performance and the system progressively reach severe unbalanced conditions as shown below:

<ins>Balanced state</ins>:

Voltage amplitudes $V_a(n) = V_b(n) = V_c(n)$ $= 230V$ from 0 to 0.25 seconds.

<ins>Unbalanced state</ins>:

start after 0.25 seconds

Phase C: Voltage amplitude drops to half $V_c(n) = 115V$ at 0.25 seconds

Phase A: Voltage amplitude drops to zero $V_a(n) = 0V$ at 0.5 seconds

Phase B: Voltage amplitude drops to zero $V_b(n) = 0V$ at 0.75 seconds

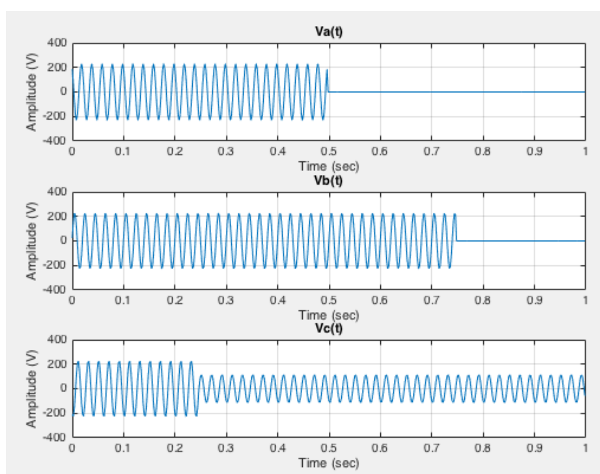

Figure 6.1: Voltage Sags

Figure 6.2 shows the performance of Single phase AR2 frequency estimations (BCRLS/RTLS) with amplitude weighted averaging (AW) method. Table 6.2 lists the estimated frequencies at different balance states.

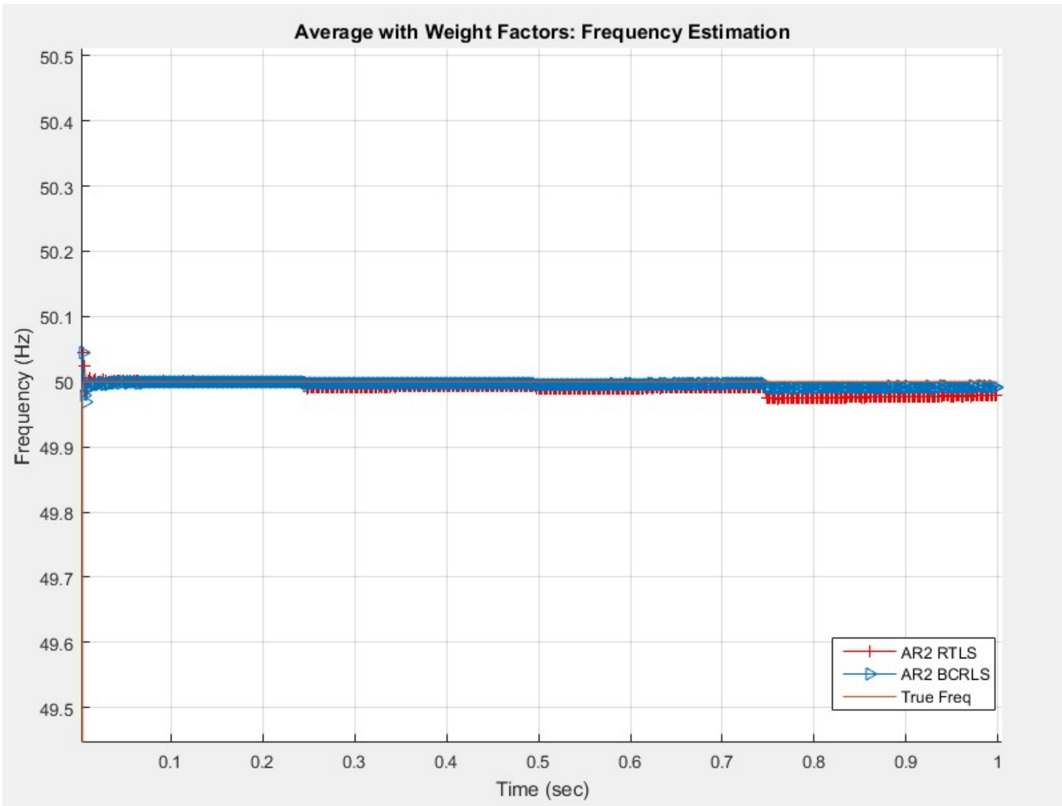

Figure 6.2: Frequency Estimation - AW for single phase AR2 with BCRLS/RTLS

Time (sec)	0.15 (Balanced)	0.25 (1ph→ 50%)	0.5 (1ph→0)	0.75 (2phs→0)
BCRLS (Hz)	50	50	50	49.99
RTLS (Hz)	50	49.99	49.99	49.97

Table 6.2: AW Method – Estimated Frequencies at different voltage sags

Figure 6.3 shows the performance of Single phase AR2 frequency estimations (BCRLS/RTLS) with SNRW method. Table 6.3 lists the estimated frequencies at different voltage sags.

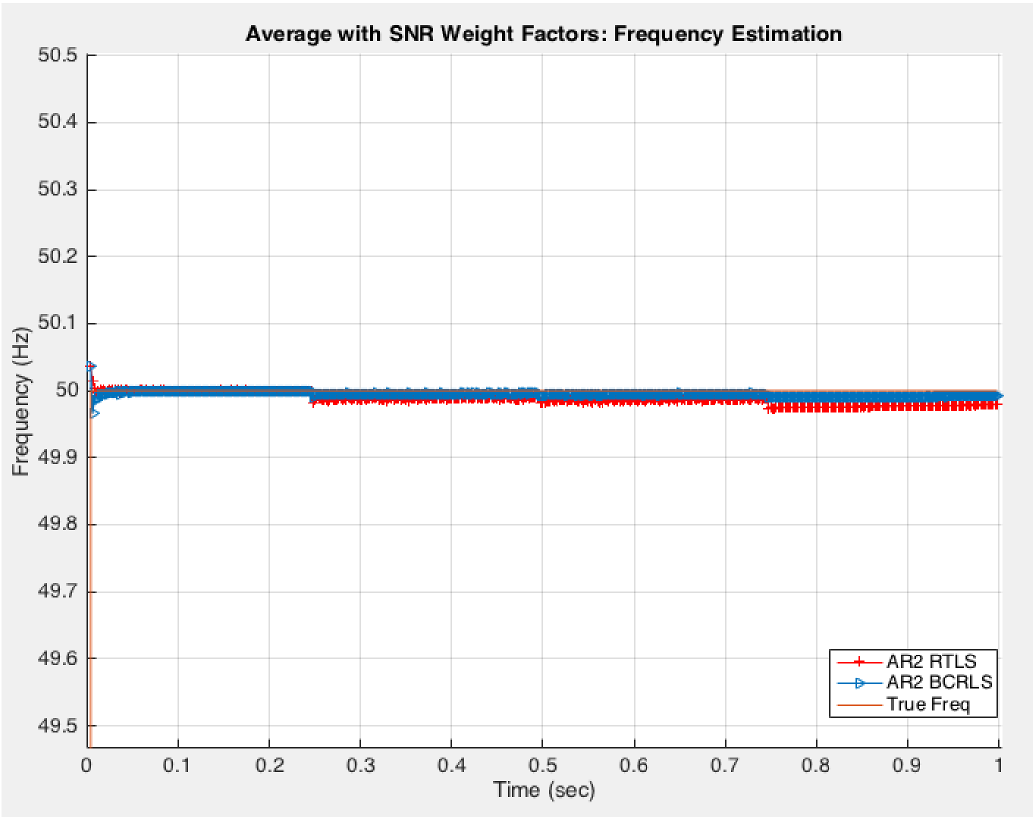

Figure 6.3: Frequency Estimation - SNRW for single phase AR2 with BCRLS/RTLS

Time (sec)	0.15 (Balanced)	0.25 (1ph→ 50%)	0.5 (1ph→0)	0.75 (2phs→0)
BCRLS (Hz)	50	49.99	49.99	49.99
RTLS (Hz)	50	49.99	49.98	49.97

Table 6.3: SNRW Method – Estimated Frequencies at different voltage sags

Figure 6.4 and Table 6.4 show the performance for the algorithms of BCRLS and RTLS implement on the complex signal, single phase AR2 with Mean, AW and SNRW methods.

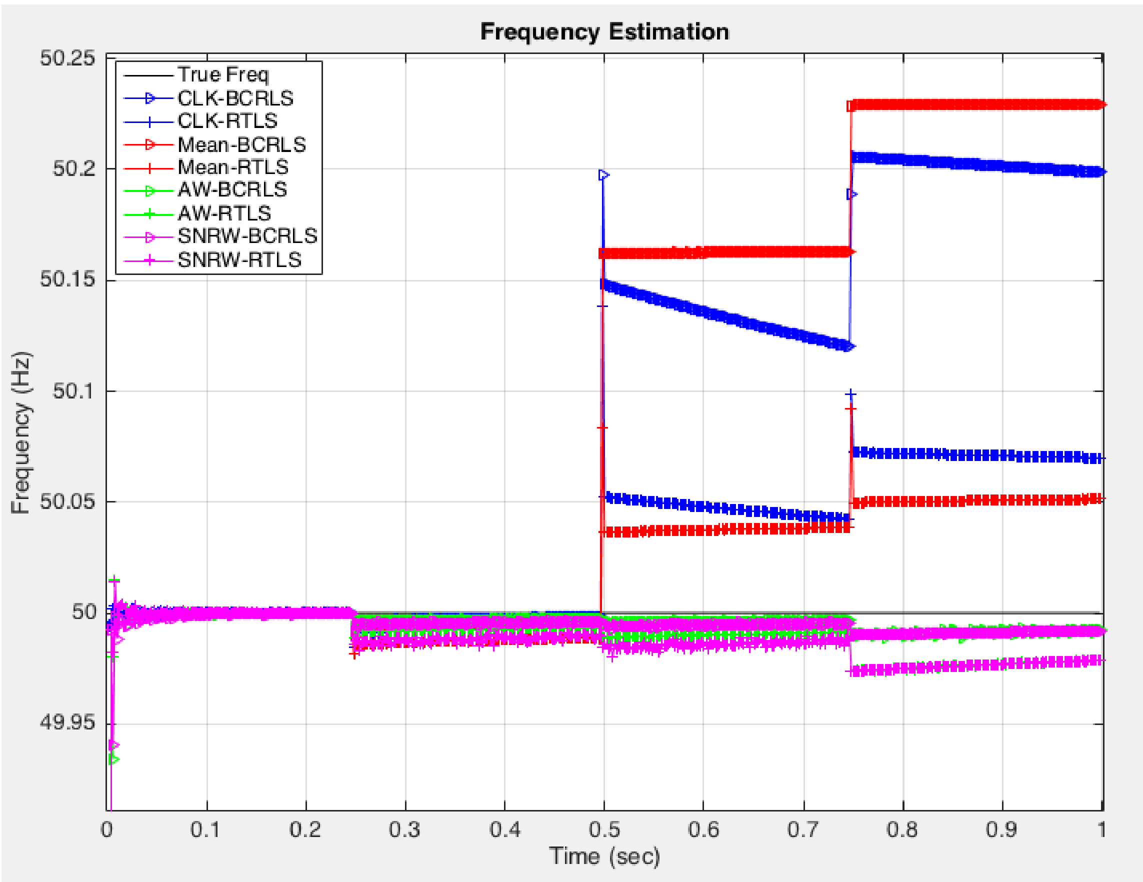

Figure 6.4: Frequency Estimation - All the simulations with BCRLS/RTLS

Time (sec)	0.15 (Balanced)	0.25 (1ph→ 50%)	0.5 (1ph→0)	0.75 (2phs→0)
$\alpha\beta$ signal - BCRLS (Hz)	50	50	50.15	50.21
$\alpha\beta$ signal - RTLS (Hz)	50	49.99	50.05	50.07
Mean - BCRLS (Hz)	50	49.99	50.16	50.23
Mean - RTLS (Hz)	50	49.99	50.04	50.05
AW - BCRLS (Hz)	50	50	50	49.99
AW - RTLS (Hz)	50	49.99	49.99	49.97
SNRW - BCRLS (Hz)	50	49.99	49.99	49.99
SNRW - RTLS (Hz)	50	49.99	49.98	49.97

Table 6.4: All the cases – Estimated Frequencies at different voltage sags

In all simulations, the estimated frequencies have been found to converge rapidly. In balanced condition, all estimators provide consistent and accurate estimation. This proves that Clarke's Transforms is able to perform well in balance. For unbalanced scenarios, the estimators are also found to operate with good stability. Through applying the Mean methods, the performance of algorithms improves minimally in RTLS while becoming more inaccurate in BCRLS due to the effects of noise variances. The AW and SNR methods are almost unaffected by the balance state of the system since it clearly outperforms $\alpha\beta$ signal based estimators and Mean method. RTLS is found to outperform BCRLS in the first four cases of Table 6.4. However, for the weighted averaging methods – last 4 cases, BCRLS is more accurate in frequency estimation than RTLS. As a result, BCRLS with the weighted average is more effective in eliminating the noise effect. In weighted averaging methods, AW is more accurate and stable than SNRW. Even though the estimated frequencies are similar in value, in the figures shown above, SNRW fluctuates during the frequency estimation since it is very sensitive to detect the signal noise. SNRW method is not considered as the best improvement of frequency estimation in this simulation environment of the same noise variance in each phase. Therefore, the best estimation is to apply AW-BCRLS, since it produces the most stable and accurate simulated result of frequency estimation.

6.1.2 Tracking ability

The proposed weighted averaging estimations are examined under various conditions of frequency – Square Wave, Slant line and Sine Wave to test the tracking ability compared to other methods.

Square Wave

In Figure 6.5, the estimated and tracked frequency is plotted by applying different algorithms when a balanced system experiences a square wave with the period $T = 4.5$ seconds in the system frequency at 1 second. The peak of the square wave is 1 Hz and the maximum change rate is 2 Hz/s.

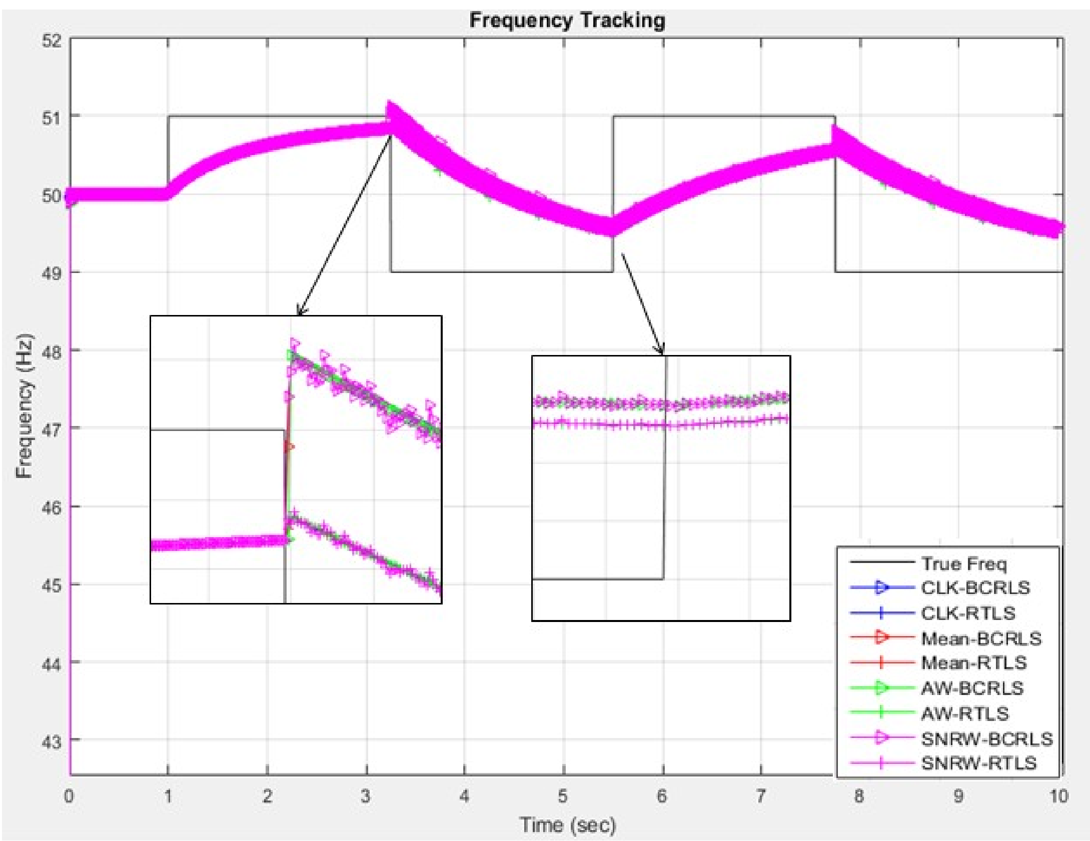

Figure 6.5: Frequency Tracking - a square wave occurs in frequency of a balanced system

Figure 6.5 illustrates RTLS has a greater tracking ability than BCRLS. AW method has good tracking ability in square wave consistent with other methods. However, SNRW has the most unsteady tracking ability as it is shown to fluctuate along the estimated frequencies from other methods.

Slant Line

In Figure 6.6, the estimated and tracked frequency is plotted with different algorithms when a balanced system experiences a constant decline in the system frequency from 51 Hz to 49 Hz during the performance. The slope of the tracking line is $k = -2$.

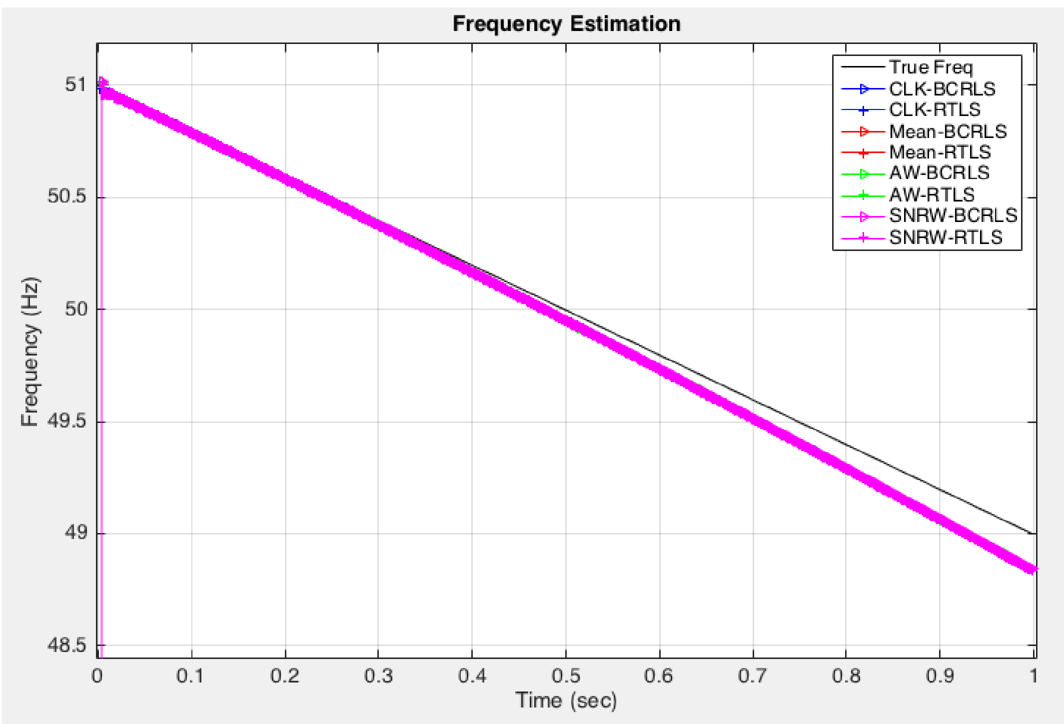

Figure 6.6: Frequency Tracking - a constant reduce in frequency of a balanced system

According to Figure 6.6, the estimators have good tracking performance from 0 second to 0.5 second. Afterwards, the tracking ability loses accuracy but still manageable.

Sine Wave

In Figure 6.7, the estimated and tracked frequency is plotted by applying different approaches when a balanced system experiences a sinusoidal oscillation with a period $T = 0.335$ seconds in the system frequency at 0.33 seconds. The peak of the square wave is 1 Hz and the maximum change rate is 2 Hz/s.

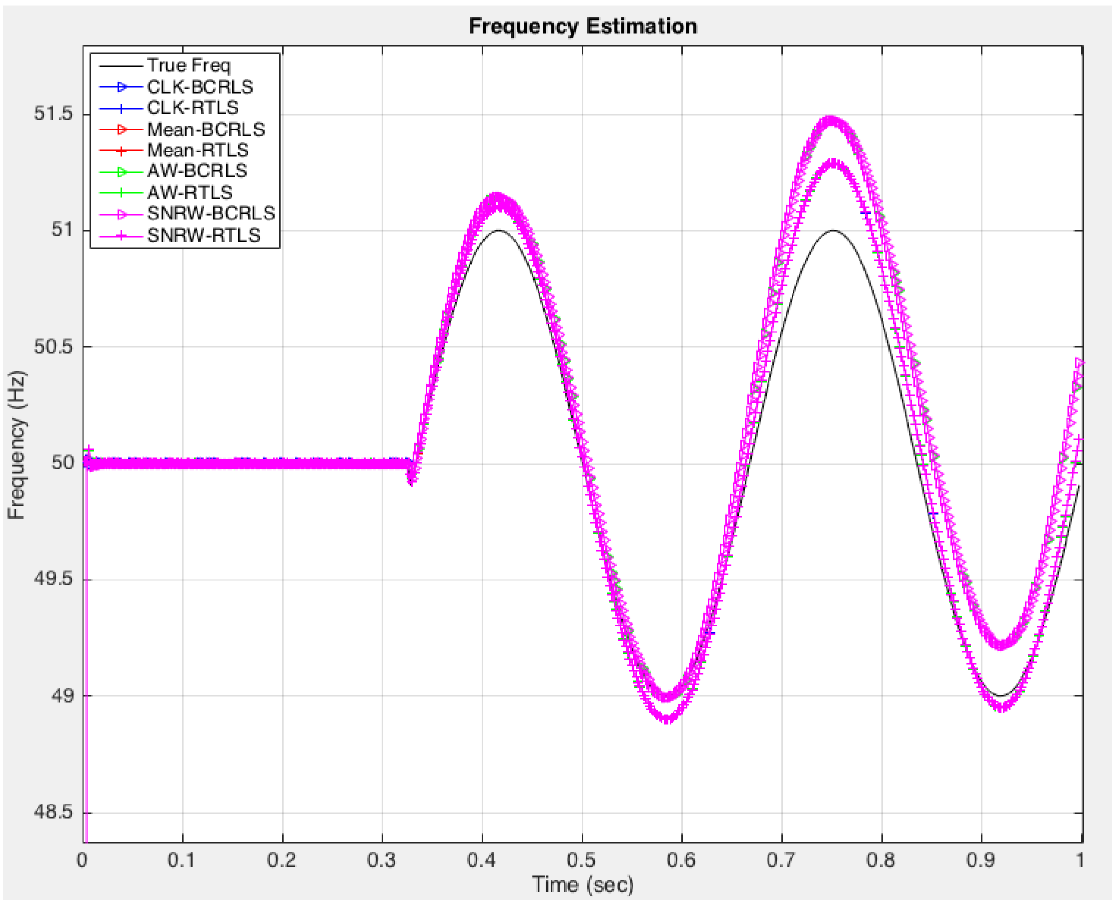

Figure 6.7: Frequency Tracking - a sinusoidal oscillation occurs in frequency of a balanced system

From Figure 6.7, all estimators have satisfactory tracking performance in the sinusoidal oscillation. Additionally, RTLS generally performs better than BCRLS.

6.1.3 Variance Performance

To validate the estimations, the simulated results of variance estimate with different algorithms is shown in Figure 6.7. In the simulation, system frequency is set at $f = 50$ Hz, the sampling frequency is $fs = 3000$ Hz, which indicates the sample per cycle to be $N = 60$. The amplitudes of phase voltages are set as 230V and initial phase angle is $\theta = 0$ (balanced system). To reach the accurate estimate, 2000 runs are averaged for each point. The results are evaluated as the Root-Mean-Square Error ($RMSE = \lim_{n\to\infty} \sqrt{E[(\hat{f}(n) - f(n))^2]}$). In Figure 6.7, it shows the RMSE of different algorithms corresponding to the signal-to-ratio ($SNR = 10\log_{10}(A^2/\sigma^2)$). SNR is in the range of 0 to 60 dB and the experiment runs on 5 dB intervals. The values of RMSE are compared with CRB in order to prove the estimation efficiency.

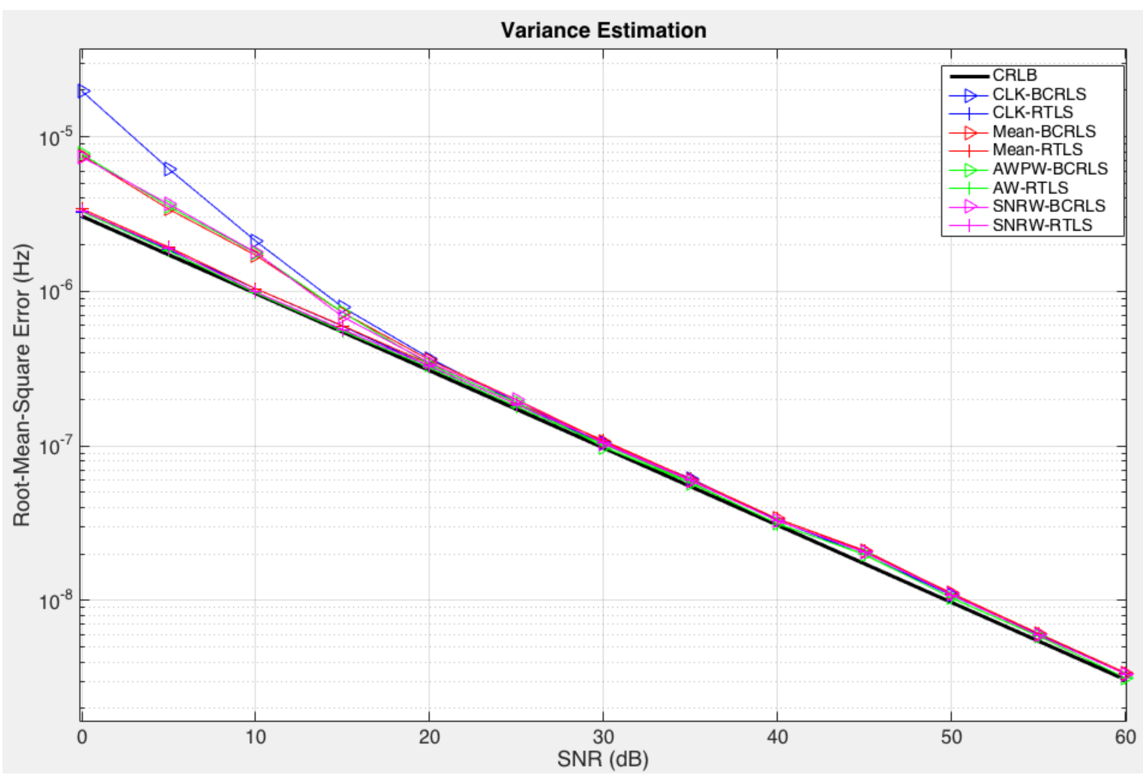

Figure 6.8: Variance estimation - RMSE of the algorithms as a function of the SNR versus CRB. $fs = 3000$ Hz and average of 2000 independent trails

The proposed weighted averaging methods provide the more accurate variance estimation while RTLS with weighted factor is considered as the best performing algorithm for overall performance. It is able to successfully remove the estimation bias and estimate very close to CRB. The Estimators become more accurate as SNR increases.

6.2 Discussion

According to the simulated results displayed in the previous sections, the outcomes of the proposed will be defined below:

- All simulated results are found to converge rapidly.
- All the estimators perform well under balanced condition
- In Unbalanced States:
  - The Mean method has no improvement on the performance
  - The proposed weighted averaging methods (AW and SNRW) outperform the existing method, especially under severe unbalanced conditions.
  - SNRW estimated frequency fluctuates during unbalanced conditions since it is susceptible to sense the noise from the phase signals. Hence, it is not effective in this condition of same phase variance in the system.
  - BCRLS combined with AW averaging had the best performance since it is more efficient with eliminating the effects of signal noise.
- The proposed has good tracking ability even when there are varying changes occurring with the system frequency. However, SNRW averaging under-performs when the system frequency is suffered from a square wave.
- The proposed successfully removes the estimation bias during the variance estimation.

Subsequently, the proposed AW method is found to have better and more accurate estimation when a three-phase system suffers from voltage sags.

Chapter 7

Conclusion

In this thesis, the proposed method aims to design and develop a robust and accurate frequency estimation method to account for unbalanced conditions of voltage sags. In order to experiment the extent of the estimator, a number of methods were simulated. The AR2 model was found to be more efficient operating under unbalance. As a result, Single-phase AR2 model was proposed since it is able to include all the information from the signal. With the application of Amplitude Weighted Average (WA), the effects from the signal noise is reduced because the method allows the estimation to be implement on the main operating phases. The proposed AW method increases the accuracy in estimating frequency for three-phase power systems and outperforms other methods in severe voltage sags. It is also assessed by its tracking performance and validated by the variance estimation.

7.1 Future Work

From this thesis, the estimators can be developed more in-depth by including a general average method that deals with all the unbalanced conditions; e.g. phase shifts. In addition, it will be expected to implement the approaches in real.

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Appendix A. Simulation Code

This Appendix provides the main code with the explanation for implementing frequency estimation with different methods under the progressive voltage sags (Section 6.1.1) in a three-phase system.

```
% Modifying "Frequency estimation of three-phase power systems"
% Author: Tsai-Yen Shih z3451029
% Last modified date: 26 Oct 2016

%------------------------------------------------------------------------
% Set up
fc = 50; %frequency @ 50Hz
Fs = 500; % # of samples
dt = 1/Fs; % Time increment per sample
StopTime = 1;
t = (0:dt:StopTime-dt);
L1 = length(t);
lambda = 0.999; % forgetting factor
sigma = 0.01; % variance

% noise of zero mean i.i.d for each phase
noise = sqrt(sigma/2)*randn(L1,3);
```

```
% Progressive Voltage Sags
    for n = 1 : length(t) % time increment

        % Signal Model:
        % 3 phases are balanced before 0.25s
        % Phase C: 50% drops @ 0.25s
        if n < 125
            Vc(n) = 230*cos(2*pi*fc*dt*n + 2*pi/3)+noise(n,3);
        else
            Vc(n) = 230/2*cos(2*pi*fc*dt*n + 2*pi/3)+noise(n,3);
        end

        % Phase A: drops to 0 @ 0.5s
        if n < 250
            Va(n) = 230*cos(2*pi*fc*dt*n)+noise(n,1);
        else
            Va(n) = 0*cos(2*pi*fc*dt*n)+noise(n,1);
        end

        % Phase B: drops to 0 @ 0.75s
        if n < 375
            Vb(n) = 230*cos(2*pi*fc*dt*n - 2*pi/3)+noise(n,2);
        else
            Vb(n) = 0*cos(2*pi*fc*dt*n - 2*pi/3)+noise(n,2);
        end

        % Clarke's Transform: alpha-beta signal
        Valpha(n)= sqrt(2/3)*(Va(n)- 1/2*Vb(n) - 1/2*Vc(n));
        Vbeta(n) = sqrt(2/3)*(sqrt(3)/2*Vb(n) - sqrt(3)/2*Vc(n));
        V = complex(Valpha,Vbeta);

        % For Weighted Averaging:
        % define values of Z
        Za(n) = cos(2*pi*fc*dt*n);
        Zb(n) = cos(2*pi*fc*dt*n - 2*pi/3);
        Zc(n) = cos(2*pi*fc*dt*n + 2*pi/3);
```

```
        % Amplitude Estimation:
        Amp_a(n) = (1/(Za(n)*conj(Za(n)))) * conj(Za(n)) * Va(n);
        Amp_b(n) = (1/(Zb(n)*conj(Zb(n)))) * conj(Zb(n)) * Vb(n);
        Amp_c(n) = (1/(Zc(n)*conj(Zc(n)))) * conj(Zc(n)) * Vc(n);

        % Noiseless Signal
        siga(n) = Amp_a(n).*(Za(n));
        sigb(n) = Amp_b(n).*(Zb(n));
        sigc(n) = Amp_c(n).*(Zc(n));

        % SNR Estimation: 10*log10(A^2/n^2)
        SNRa(n) = 10*log10((Amp_a(n)).^2/(Va(n)-siga(n)).^2);
        if (SNRa(n) <= 0) || (Amp_a(n) <= 1)
            SNRa(n) = 0;
        else
            SNRa(n) = SNRa(n);
        end

        SNRb(n) = 10*log10((Amp_b(n)).^2/(Vb(n)-sigb(n)).^2);
        if SNRb(n) <= 0 || (Amp_b(n) <= 1)
            SNRb(n) = 0;
        else
           SNRb(n) = SNRb(n);
        end

        SNRc(n) = 10*log10((Amp_c(n)).^2/(Vc(n)-sigc(n)).^2);
        if (SNRc(n) <= 0) || (Amp_c(n) <= 1)
            SNRc(n) = 0;
        else
            SNRc(n) = SNRc(n);
        end
    end
```

```
%===================================================
% Frequency Estimation:
% BCRLS & RTLS Algorithms:
for i = 3:length(t)

    % sample frequency @ 50Hz
    Fc(i) = 50;

    % BCRLS:
    % Alpha-Beta Signal:
    r(i) = lambda*r(i-1) + abs(V(i-1))^2;
    p(i) = lambda*p(i-1) + conj(V(i-1))*(V(i-2)+V(i))/2;
    w_BCRLS(i) = (p(i)/r(i)) + (sigma)/((1 - lambda)*r(i))*w_BCRLS(i-1);
    f_BCRLS(i) = 1/(2*pi*dt)*acos(real(w_BCRLS(i)));

    % Single Phase:::::::::::::::::::::::::::::::::::
    % Phase A
    ra(i) = lambda*ra(i-1) + (Va(i-1))^2;
    pa(i) = lambda*pa(i-1) + (Va(i-1))*(Va(i-2)+Va(i))/2;
    wa_BCRLS(i) = (pa(i)/ra(i)) + (sigma)/((1 - lambda)*ra(i))*wa_BCRLS(i-1);
    fa_BCRLS(i) = 1/(2*pi*dt)*acos(real(wa_BCRLS(i)));

    % Phase B
    rb(i) = lambda*rb(i-1) + (Vb(i-1))^2;
    pb(i) = lambda*pb(i-1) + (Vb(i-1))*(Vb(i-2)+Vb(i))/2;
    wb_BCRLS(i) = (pb(i)/rb(i)) + (sigma)/((1 - lambda)*rb(i))*wb_BCRLS(i-1);
    fb_BCRLS(i) = 1/(2*pi*dt)*acos(real(wb_BCRLS(i)));

    % Phase C
    rc(i) = lambda*rc(i-1) + (Vc(i-1))^2;
    pc(i) = lambda*pc(i-1) + (Vc(i-1))*(Vc(i-2)+Vc(i))/2;
    wc_BCRLS(i) = (pc(i)/rc(i)) + (sigma)/((1 - lambda)*rc(i))*wc_BCRLS(i-1);
    fc_BCRLS(i) = 1/(2*pi*dt)*acos(real(wc_BCRLS(i)));

    % Mean of 3 single phase signals................
    favg_BCRLS(i) = (1/3) * (fa_BCRLS(i) + fb_BCRLS(i) + fc_BCRLS(i));
```

```
%------------------------------------------------------------
% RTLS
% Alpha-Beta Sigal:
s(i) = lambda*s(i-1) + (abs(V(i-2)+V(i))^2)/4;
w_RTLS(i) = (p(i) + 2*s(i)*w_RTLS(i-1))/(r(i) + 2*conj(p(i))*w_RTLS(i-1));
f_RTLS(i) = 1/(2*pi*dt)*acos(real(w_RTLS(i)));

% Single Phase:::::::::::::::::::::::::::::::::::::
% Phase A
sa(i) = lambda*sa(i-1) + (Va(i-2)+Va(i))^2/4;
wa_RTLS(i) = (pa(i) + 2*sa(i)*wa_RTLS(i-1))/(ra(i) + 2*pa(i)*wa_RTLS(i-1));
fa_RTLS(i) = 1/(2*pi*dt)*acos(real(wa_RTLS(i)));

% Phase B
sb(i) = lambda*sb(i-1) + (Vb(i-2)+Vb(i))^2/4;
wb_RTLS(i) = (pb(i) + 2*sb(i)*wb_RTLS(i-1))/(rb(i) + 2*(pb(i))*wb_RTLS(i-1));
fb_RTLS(i) = 1/(2*pi*dt)*acos(real(wb_RTLS(i)));

% Phase C
sc(i) = lambda*sc(i-1) + (Vc(i-2)+Vc(i))^2/4;
wc_RTLS(i) = (pc(i) + 2*sc(i)*wc_RTLS(i-1))/(rc(i) + 2*(pc(i))*wc_RTLS(i-1));
fc_RTLS(i) = 1/(2*pi*dt)*acos(real(wc_RTLS(i)));

% Mean of 3 single phase Signals................
favg_RTLS(i) = (1/3) * (fa_RTLS(i) + fb_RTLS(i) + fc_RTLS(i));
```

```
%*******************************************
%  Amplitude Weighted Averaging (AW):
% Compare estimated amplitudes and assign the values for the weight factors
% Phase A is largest:
if (Amp_a(i) >= Amp_b(i)) && (Amp_a(i) >= Amp_c(i))
    wamp_a(i) = 1;
    wamp_b(i) = Amp_b(i)/Amp_a(i);
    wamp_c(i) = Amp_c(i)/Amp_a(i);

 % Phase B is largest:
elseif (Amp_b(i) >= Amp_a(i)) && (Amp_b(i) >= Amp_c(i))
    wamp_a(i) = Amp_a(i)/Amp_b(i);
    wamp_b(i) = 1;
    wamp_c(i) = Amp_c(i)/Amp_b(i);

 % Phase C is largest:
elseif (Amp_c(i) >= Amp_a(i)) && (Amp_c(i) >= Amp_b(i))
    wamp_a(i) = Amp_a(i)/Amp_c(i);
    wamp_b(i) = Amp_b(i)/Amp_c(i);
    wamp_c(i) = 1;
end

% Average by applying the Amplitude weight factors:
    fw_BCRLS(i) = (1/(wamp_a(i)+wamp_b(i)+wamp_c(i)))
     * (wamp_a(i)*fa_BCRLS(i) + wamp_b(i)*fb_BCRLS(i) + wamp_c(i)*fc_BCRLS(i));
    fw_RTLS(i) = (1/(wamp_a(i) + wamp_b(i)+wamp_c(i)))
     * (wamp_a(i)*fa_RTLS(i) + wamp_b(i)*fb_RTLS(i) + wamp_c(i)*fc_RTLS(i));
```

```
    %++++++++++++++++++++++++++++++++++++++++++++++++++++
    % SNR Weighted Averaging (SNRW)
    % Compare estimated SNR and assign the values for the weight factors
    % Phase A is largest:
    if (SNRa(i) >= SNRb(i)) && (SNRa(i) >= SNRc(i))
        wSNRa(i) = 1;
        wSNRb(i) = SNRb(i)/SNRa(i);
        wSNRc(i) = SNRc(i)/SNRa(i);

    % Phase B is largest:
    elseif (SNRb(i) >= SNRa(i)) && (SNRb(i) >= SNRc(i))
        wSNRa(i) = SNRa(i)/SNRb(i);
        wSNRb(i) = 1;
        wSNRc(i) = SNRc(i)/SNRb(i);

   % Phase C is largest:
    elseif (SNRc(i) >= SNRa(i)) && (SNRc(i) >= SNRb(i))
        wSNRa(i) = SNRa(i)/SNRc(i);
        wSNRb(i) = SNRb(i)/SNRc(i);
        wSNRc(i) = 1;
    end

    % Average by applying the SNR weight factors:
        fwSNR_BCRLS(i) = (1/(wSNRa(i) + wSNRb(i) + wSNRc(i)))
        * (wSNRa(i)*fa_BCRLS(i) + wSNRb(i)*fb_BCRLS(i) + wSNRc(i)*fc_BCRLS(i));
        fwSNR_RTLS(i) = (1/(wSNRa(i) + wSNRb(i) + wSNRc(i)))
        * (wSNRa(i)*fa_RTLS(i) + wSNRb(i)*fb_RTLS(i) + wSNRc(i)*fc_RTLS(i));
end
```