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TOPICS IN ADVANCED TIME SERIES ANALYSIS

Victor Solo

Preface

These notes were prepared for a series of 20 lectures given at the invitation of Professors Rolando Rebolledo and Guido del Pino in July 1982 at the Primera Escuela de Invierno de Probabilidad y Estadistica held at the Universidad Catolica de Chile. I thank them for an enjoyable and useful time in Santiago.

One aim of these lectures is to provide a discussion of some issues not easily found elsewhere. It is hoped some parts of the notes will benefit both theoreticians and practitioners. I have tried as much to put ideas across as to give technical detail: thus where appropriate scalar and bivariate time series have been used to illustrate issues and motivate the general multivariate case.

I would like to thank Marie Sheehan who did an amazing job of typing an enormous amount of material in a very short time.

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